A Convergence of Lives—Sophia Kovalevskaia: Scientist, Writer, Revolutionary. By Ann Hibner Koblitz. Birkhäuser, Boston, 1983. xx + 305 pp.

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Sonya Kovalevsky was the greatest woman mathematician prior to the twentieth century.

E. E. Kramer [7]

Kowalewsky is one of the few women mathematicians of distinction.

M. Kline [**6**, p. 702]

At the time of her death, Kovalevskaia was indeed considered the equal of anyone of her generation. This included Poincaré, Picard, and Mittag-Leffler.

Ann Hibner Koblitz (p. 250)

Sofia V. Kovalevskaia*, the object of these effusions, was born in Moscow in 1850 to a wealthy landowning family. She grew up on their provincial estate, ineffectively shielded from the intellectual and political ferment then sweeping Russia. She managed to receive tutoring in mathematics, at which she excelled. To gain the independence from her family necessary to enable her to pursue further mathematical studies (in German universities, Russian universities being closed to women), she entered at the age of 18 into a fictitious marriage with V. O. Kovalevskii. The ploy proved successful. One year later she had mastered the bureaucratic obstacles posed by the University of Heidelberg and was attending lectures on mathematics and physics there. The following year she moved to Berlin to study with Weierstrass. When her application for admission to the University was denied because of her sex (despite her excellent references), Weierstrass undertook to give her private tutoring. Thus began a warm and enduring friendship between Kovalevskaia and Weierstrass. By the age of 24, Kovalevskaia completed three dissertations for Weierstrass (one being her proof of the Cauchy-Kovalevskaia Theorem). Weierstrass wangled a doctorate for her from Göttingen, the first doctorate in mathematics for a woman since the Renaissance.

Kovalevskaia and her husband, himself newly armed with a doctorate in paleontology, returned to Russia, but failed to find suitable positions, she because of her sex and he because of his arrogance. They were each seduced from their scientific studies by the social and financial whirl of St. Petersburg. In the ensuing years a daughter was born to the Kovalevskii's, Kovalevskaia returned to mathematics, and her husband committed suicide. Her widowhood gave Kovalevskaia both the freedom to pursue her scientific interests and the social respectability necessary to enter the masculine world of the European university. A tenacious campaign by Mittag-Leffler resulted in her appointment first as privat docent at the age of 33 and later as professor at the age of 39 at the University of Stockholm. There she carried out her prize-winning work on the motion of a rigid body about a fixed point. At the age of 41 she succumbed to pneumonia.

Ann Hibner Koblitz presents Kovalevskaia's fascinating biography in a style that is generally lively and graceful. The author is at her best in capturing the social and cultural milieu in which Kovalevskaia moved: Kovalevskaia was on familiar terms with many of the premier mathematicians of the time, her husband and his brother were distinguished scientists, and her path crossed those of Dostoevskii, Chekhov, George Eliot, and Darwin. Koblitz's treatment of Dostoevskii's association with Kovalevskaia's family is particularly noteworthy.

**Kovalevskaia* is a standard English transliteration of the nominative of the feminine form of *Kovalevskii*, her husband's family name (just as *Karenina* is that for *Karenin*). Kovalevskaia, however, published her scientific work in German and French under the transliteration *Kowalevski* of her husband's name. Thus in Latin alphabets her name appears in at least a dozen variants. To complicate matters further, she was affectionately called *Sonia*, the diminutive of *Sofia*.

Throughout her life Kovalevskaia was supremely conscious of her pioneering role as a woman scientist, the foremost of her century. Though endowed with her share of human frailties, she conducted her scientific life with a combination of daring, perserverence, and tactful circumspection. Koblitz carefully documents Kovalevskaia's sympathies for liberalism and for what was then deemed revolutionary. But the hyperbole of the book's title notwithstanding, the author admits (p. 111), "[Kovalevskaia] was not a revolutionary herself."

Koblitz points out that scarcely any of the obstacles Kovalevskaia encountered on account of her sex were erected by mathematicians. Indeed, Königsberger, Hermite, Weierstrass, and especially Mittag-Leffler took great pains to combat the reactionaries opposing her rise to her rightful position.

Most American mathematicians know of Kovalevskaia's life through Bell's [1] superficial portrait, which lacks any bibliographical citation. In contrast, Koblitz's full-scale biography is based upon examination of Russian and Swedish archives and is written with an outward adherence to the norms of historiography. Nevertheless, I found errors of fact and changes of nuance in the interpretation of some quotations from her sources:

(i) The famous episode in which Kovalevskaia persuaded the misogynous chemist Bunsen to admit women to his laboratory is described by Koblitz (p. 89) in a manner totally at variance with that of Bell's amusing tale [1, pp. 424–5], in which Bunsen is portrayed as a harmless crank. The account of Koblitz, citing both Bell and Mittag-Leffler [11, p. 135] is much closer to that of Mittag-Leffler in its lack of theatricality than is Bell's. But Koblitz's statement, "Bunsen... later spread scandalous stories about [Kovalevskaia]" has a flavor different from that of Mittag-Leffler's statement, "[Bunsen] circulait à ce moment des bruits de toutes sortes et non de plus avantageux sur le compte des étudiantes russes qui avaient leur principale résidence à Zuerich." (Although some of Kovalevskaia's Russian friends lived in Zürich, the careful chronology of Koblitz's book gives no indication that she ever did; and she only visited Zürich in 1873.)

(ii) Koblitz asserts that Kovalevskaia was unaware of the efforts of Hermite and Bertrand to rig the Prix Bordin for her: "Kovalevskaia had no way of knowing the behind-the-scenes maneuverings of her French colleagues." (p. 207.) A letter of June 1886 from Kovalevskaia to Mittag-Leffler details how she actually participated in precisely those maneuverings:

> Bertrand always demonstrates towards me an extraordinary benevolence. Imagine what he thought up: next Monday these gentlemen are supposed to gather, to propose a theme for the grand academic prize for the year 1886. Bertrand got the idea of proposing as a theme precisely the problem of rotation of a rigid body. In this way I shall have some chances of getting the prize. You can imagine how much this thought tempts me. Yesterday Hermite, Bertrand, Camille Jordan and Darboux, who are all members of that commission, discussed this project together with me. They made me expose to them once again in detail the results of my work and again heard everything, so that they think, that this work has many chances of being crowned. The only inconvenience is the fact, that I shall have to postpone the publication in that case until 1888. You can imagine how much this project appeals to me. But in that case I shall not be able to make the publication of my work in Christiania in that year... *

(iii) Felix Klein's [4] account of Kovalevskaia's work is branded (along with Bell's) as "unreliable" (p. 279). Koblitz says, "But [Klein] also implies that most of her work was probably done by Weierstrass, and he disparages her later papers," (p. 279). The relevant passage of Klein [4, p. 294] is not so unambiguous: "... ihre Arbeiten in enger Anlehnung und ganz im Stil von Weierstrass geschrieben sind, so dass man nicht sieht, wie weit sie unabhängige, eigene Gedanken enthalten." The first part of this quotation mirrors the view of Mittag-Leffler [10, p. 388]:

*This quotation from [2, p. 223] is apparently an "English" translation of a Russian translation of the French original. I have not been able to see either the original or the Russian translation.

"Comme mathematicien, Sophie Kovalevsky appartient entièrement à l'école de Weierstrass." It is true, however, that Klein did disparage both Kovalevskaia's faulty work on diffraction and her prize-winning work on rigid body rotation, the latter with egregious innuendo: "Ebenso ist man mit der Arbeit über die Rotation nicht durchweg zufrieden." [4, p. 295] What is remarkable about this view, besides the absence of any supporting justification, is that Klein, a coauthor of the monumental treatise on tops [5], was the mathematician of his era best qualified to evaluate Kovalevskaia's work. We shall return to this question below.

In discussing Kovalevskaia's literary works, Koblitz forms her own opinions from her reading of the original Russian versions and expresses them forcefully and authoritatively. But in place of reading Kovalevskaia's mathematical works and related studies, the author relies entirely on others' opinions to evaluate their scientific merit. The progression of favorable quotations, accepted at face value, which form the brief chapter on Kovalevskaia's mathematics soon becomes tiresome. The danger with proof by quotation is apparent from the epigraphs: The first is virtually a tautology, the second might have been true fifty years ago, but is not true now, and the third, couched in the same irresponsible passive voice as used by Klein, is absurd.

Rather than retailing superlatives about Kovalevskaia's genius, it is more satisfying to discuss her mathematical accomplishments. (Koblitz's book, which is addressed to the layman, gives but a superficial account of Kovalevskaia's work.) Kovalevskaia wrote only ten scientific papers, the most important being those devoted to the Cauchy-Kovalevskaia Theorem and to rigid body motion.

The Cauchy-Kovalevskaia Theorem asserts that a partial differential equation involving only analytic functions subjected to analytic initial conditions prescribed on an analytic noncharacteristic initial surface has an analytic solution near that surface that satisfies the initial conditions. On one hand the theorem is general because it applies to equations of all types. On the other hand, it is restrictive because it requires the analyticity of everything in sight, thereby excluding many physical applications, and it is of limited utility because the existence is asserted only on a neighborhood of the initial surface.

Kovalevskaia and Darboux each published proofs of the "Cauchy-Kovalevskaia" Theorem in 1875, whereupon it was discovered that Cauchy had already done so in 1842. Kovalevskaia's proof is the most detailed of the three, those of Cauchy and Darboux appearing in brief notes in the Comptes Rendus. In a letter of recommendation for Kovalevskaia, Hermite commented (p. 241) that Kovalevskaia's paper would be the point of departure for all future research in partial differential equations. Though this theorem has since been invoked from time to time in research work, it has never come near fulfilling Hermite's expectations. Its proof can be found in many books on partial differential equations.

Of more interest is Kovalevskaia's work on the unsymmetrical top, the significance of which has eluded some commentators. To set the stage for a discussion of it, we outline the formulation of the equations of motion of a rigid body about a fixed point taken to be at the origin $\mathbb{O}$ of Euclidean three-space. (Cf. [2], [5], [9].) Let $\mathbb{e}_1(t), \mathbb{e}_2(t), \mathbb{e}_3(t)$ be the principal axes of inertia of the body about $\mathbb{O}$ at time t, which can be taken to be a right-handed orthonormal system fixed in the body. Let I_1, I_2, I_3 be the corresponding principal moments of inertia. Since the body is rigid, I_1, I_2, I_3 are constants. To find the motion of the body we need only find $\mathbb{e}_1, \mathbb{e}_2, \mathbb{e}_3$. The properties of $(\mathbb{e}_1, \mathbb{e}_2, \mathbb{e}_3)$ imply that there is a vector $\omega(t) = \Sigma\omega_j(t)\mathbb{e}_j(t)$, called the *angular velocity* of the body at time t, such that

$$(1) \qquad \dot{\mathbb{e}}_j = \omega \times \mathbb{e}_j .$$

Here the superposed dot denotes the time derivative. All summations Σ are taken over 1, 2, 3. The angular momentum of the body about $\mathbb{O}$ is $\Sigma I_j\omega_j\mathbb{e}_j$. Let $\rho(t) = \Sigma\rho_j\mathbb{e}_j(t)$ be the position of the mass center of the body at time t. The rigidity of the body implies that ρ_1, ρ_2, ρ_3 are constants. Suppose that the only forces acting on the body are the force of gravity $-mg\mathbf{k}$ and the reaction at the support. Here $\mathbf{k}$ is a fixed unit vector in the vertical direction, g is the acceleration due to gravity, and m is the mass of the body. Suppose that there are no couples applied to the body. (A couple is a pure torque that is not the moment of a force). Then Euler's Law of Motion requiring that the time derivative of the angular momentum about $\mathbf{0}$ equal the resultant torque about $\mathbf{0}$ reduces to

$$(2) \qquad \frac{d}{dt}\Sigma I_j \omega_j(t)\mathbf{e}_j(t) = \boldsymbol{\rho}(t) \times [-mg\mathbf{k}].$$

Let

$$(3) \qquad \mathbf{k} = \Sigma k_j(t)\mathbf{e}_j(t).$$

Differentiating this equation with respect to time and using (1) we get

$$(4) \qquad \Sigma \dot{k}_j \mathbf{e}_j + \boldsymbol{\omega} \times \mathbf{k} = \mathbf{0},$$

or equivalently

$$(5) \qquad \dot{k}_1 + \omega_2 k_3 - \omega_3 k_2 = 0, \ldots,$$

(the ellipses denoting the two equations derived from the exhibited equation by cyclic permutation of the indices). Similarly (2) reduces to

$$(6) \qquad I_1\dot{\omega}_1 + (I_3 - I_2)\,\omega_2\omega_3 = mg(k_2\rho_3 - k_3\rho_2), \ldots .$$

Equations (5) and (6) represent a formidable nonlinear sixth-order system of ordinary differential equations for $\omega_1, \omega_2, \omega_3, k_1, k_2, k_3$. (Once these are found, $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ can be determined from additional equations not shown here.)

By dotting (2) with $\boldsymbol{\omega}$ and using (4) we obtain the energy integral:

$$(7) \qquad \frac{1}{2}\Sigma I_j \omega_j^2 + mg\boldsymbol{\rho} \cdot \mathbf{k} = \text{constant}.$$

By dotting (2) with $\mathbf{k}$ and using (4) we obtain the conservation of angular momentum about $\mathbf{k}$:

$$(8) \qquad \Sigma I_j \omega_j k_j = \text{constant}.$$

Since $\mathbf{k}$ is a unit vector we have a third integral

$$(9) \qquad \Sigma k_j^2 = 1.$$

Since t does not appear explicitly in (5) and (6), it can be shown that this system can be reduced to quadratures, i.e., can be solved explicitly in terms of known functions and functions defined by indefinite integrals provided that a fourth integral can be found. There are two important cases in which the fourth integral is readily accessible:

(i) $\boldsymbol{\rho} = \mathbf{0}$ (so that the mass center is at the point of support). Gyroscopes and planets can be described by this case. By multiplying the first equation of (6) by $I_1\omega_1$, the second by $I_2\omega_2$, and the third by $I_3\omega_3$, and then adding the resulting equations we obtain an equation whose integral is $\Sigma I_j^2 \omega_j^2 = \text{constant}$.

(ii) $I_1 = I_2, \rho_1 = 0 = \rho_2$. These conditions describe a symmetric top or pendulum. The third equation of (6) immediately yields the integral $\omega_3 = \text{constant}$.

A degenerate case in which the analysis is elementary is

$$(\text{iii}) \qquad I_1 = I_2 = I_3.$$

The collection of integrals for cases (i) and (ii) can be manipulated algebraically to produce explicit representations for the triad $\mathbb{e}_1, \mathbb{e}_2, \mathbb{e}_3$ in terms of elliptic functions so dear to the heart of the nineteenth century scientist. To the scientist of the twentieth century the virtue of these integrals is that they yield a simple yet complete qualitative picture of all possible motions, thereby exempting students of rigid body mechanics from the rigors of the theory of elliptic functions.

Being one of the leading experts in the theory of elliptic functions and aware of their role in these simple problems of rigid body motion, Weierstrass proposed (cf. [2, p. 222]) that Kovalevskaia investigate other problems of rigid body motion where her expertise in elliptic functions and more generally in Abelian integrals might be brought to bear. She succeeded in obtaining the fourth integral

$$(10) \qquad \left(\omega_1^2 - \omega_2^2 - mg\rho_1 k_1/I_3\right)^2 + \left(2\omega_1\omega_2 - mg\rho_1 k_2/I_3\right)^2 = \text{constant}.$$

in the case that

$$(\text{iv}) \qquad I_1 = I_2 = 2I_3, \quad \rho_2 = 0 = \rho_3.$$

(The deduction of (10) from (iv) is not obvious, although it is easy to verify that (10) is an integral.) Using (10) Kovalevskaia was able to carry out the full integration of (5) and (6) by an impressively intricate analysis exploiting hyperelliptic integrals. Cf. [2], [9]. For parts of this work she received the Prix Bordin.

Several observations should be made about this work (a) to explain why it is justifiably ignored in many texts on rigid body motion, (b) why it was nevertheless deserving of the Prix Bordin, and (c) how it is influencing some modern work on differential equations.

(a) While conditions (i) and (ii) are frequently encountered in applications, conditions (iv) are very artificial. (The requirement that $\rho_2 = 0$ can be suspended.) This fact and the complicated analysis necessary to solve the problem in terms of special functions have properly led to the relegation of "Kovalevskaia's top" to the rank of a mere curiosity in texts on mechanics.

(b) Conditions (iv) were not a fortuitous result of a search for algebraic integrals. Rather they were the by-product of a systematic analysis of the full system of equations (5), (6). Recall that the solutions in cases (i) and (ii) can be expressed in terms of elliptic functions. When regarded as functions of a complex (time) variable, they have simple poles as their only singularities. Motivated by this observation, Kovalevskaia sought general solutions of (5), (6) that are analytic functions of complex t in a punctured neighborhood of $t = 0$ and that have poles at $t = 0$. These requirements restrict the possible values of the parameters $I_1, I_2, I_3, \rho_1, \rho_2, \rho_3$. Kovalevskaia discovered that cases (i)–(iv) are compatible with her representation. Later Liapunov proved Kovalevskaia's assertion that cases (i)–(iv) are exhaustive. Thus Kovalevskaia's work had the coherence, novelty, and scope necessary to enlist the enthusiastic support of the jury for the Prix Bordin. (Klein's dissatisfaction may have been directed at the gap filled by Liapunov and may further have been influenced by the lack of physical significance of (iv).) Kovalevskaia's work has led to a classification of the elementary cases of (5), (6). Cf. [9, Sects. 7, 8]. (The general problem of rigid body motion is only now beginning to yield to powerful modern methods of analysis. Cf. [3].)

(c) Kovalevskaia's work strongly suggested that one consider the related problem of determining when (5), (6) admits a fourth (independent) integral. According to Leimanis [9, Sect. 7.2], Husson and Burgatti proved that a fourth algebraic integral of (5), (6) exists exactly in the cases (i)–(iv). (If the motion is constrainted, then integrals are known to exist in other cases.) It is remarkable that the purely local and seemingly old-fashioned approach of Kovalevskaia is so intimately related to the question of integrability, which supports modern global qualitative approaches to the same problem. In recent years it has become evident that completely integrable nonlinear ordinary and partial differential equations, which describe important physical processes, admit very detailed analyses and have solutions with remarkable properties. Simple tests now being developed for determining whether a system is completely integrable can be traced back through the work of Painlevé to works of Fuchs and to Kovalevskaia's local study of the complexified problem of rotation. There still remain, however, deep questions on the relationship of her approach to integrability theory. Cf. **[8]** and the references cited therein. (It is interesting to note that Emmy Noether, Kovalevskaia's successor as the commonly acknowledged greatest female mathematician, made major contributions to the problem of integrability in her 1918 study of invariant variational problems.)

Koblitz's book gives a very readable account of Kovalevskaia's life. It was a life filled with personal drama. The author has uncovered much new material about Kovalevskaia. But the author's admiration for her subject and her lack of expertise in mathematics have led her to color many episodes of Kovalevskaia's mathematical life and assessments of her work in ways more favorable to Kovalevskaia and less favorable to her ostensible opponents than a sober evaluation of the evidence and its context warrants. Kovalevskaia does not require such assistance. She was the author of influential papers and was respected by the mathematical community for mathematical abilities beyond those reflected in her research. She was recognized for her literary efforts. But perhaps her greatest legacy is that today there are many female mathematicians whose *mathematical* accomplishments have surpassed hers.

References

1. E. T. Bell, Men of Mathematics, Simon & Schuster, 1937.

2. Y. V. Golubev, Lectures on Integration of the Equations of Motion of a Rigid Body about a Fixed Point, U.S. Dept. Comm., 1960. (English transl. of Russian edition of 1953.)

3. P. J. Holmes and J. E. Marsden, Horseshoes and Arnold diffusion for Hamiltonian systems on Lie groups, Indiana Univ. Math. J., 32 (1983) 273–309.

4. F. Klein, Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert, Heft I, Springer, 1927.

5. F. Klein and A. Sommerfeld, Über die Theorie des Kreisels, Teubner, 1897–1910.

6. M. Kline, Mathematical Thought from Ancient to Modern Times, Oxford Univ. Press, 1972.

7. E. E. Kramer, Sonya Kovalevsky, Dictionary of Scientific Biography, vol. VII, Scribner's, 1973, pp. 447–480.

8. M. Kruskal, Generalized Painlevé tests for integrability of differential equations, in preparation.

9. E. Leimanis, The General Problem of the Motion of Coupled Rigid Bodies about a Fixed Point, Springer-Verlag, 1965.

10. G. Mittag-Leffler, Sophie Kovalevsky, notice biographique, Acta Math., 16 (1983) 385–392.

11. G. Mittag-Leffler, Weierstrass et Sonja Kowalewsky, Acta Math., 39 (1923) 133–198.

ANSWERS TO "NAME THIS BOOK" ON PAGE 138

1. Roots.
2. Fathers and Sons.
3. Hard Times.
4. The Right Stuff.
5. Westward ho!
6. Far from the Madding Crowd.
7. From Here to Eternity.
8. The End of the Tether.