Abstracts for the MAA Undergraduate Poster Session

Boston, MA
January 6, 2012

MAA
MATHEMATICAL ASSOCIATION OF AMERICA

MAA Undergraduate Poster Session

Veterans Auditorium
January 6, 2012

313	312/311	309/308	307/306	305/304	

303/302	301/300	299/298	297/296	295/294	293/292

268/269	270/271	272/273	274/275	276/277	278/279
267/266	265/264	263/262	261/260	259/258	257/256

280/281	282/283	284/285	286/287	288/289	290/291
255/254	253/252	251/250	249/248	247/246	245/244

220/221	222/223	224/225	226/227	228/229	230/231
219/218	217/216	215/214	213/212	211/210	209/208

232/233	234/235	236/237	238/239	240/241	242/243
207/206	205/204	203/202	201/200	199/198	197/196

172/173	174/175	176/177	178/179	180/181	182/183
171/170	169/168	167/166	165/164	163/162	161/160

184/185	186/187	188/189	190/191	192/193	194/195
159/158	157/156	155/154	153/152	151/150	149/148

124/125	126/127	128/129	130/131	132/133	134/135
123/122	120/119	118/117	116/115	114/113	112/111

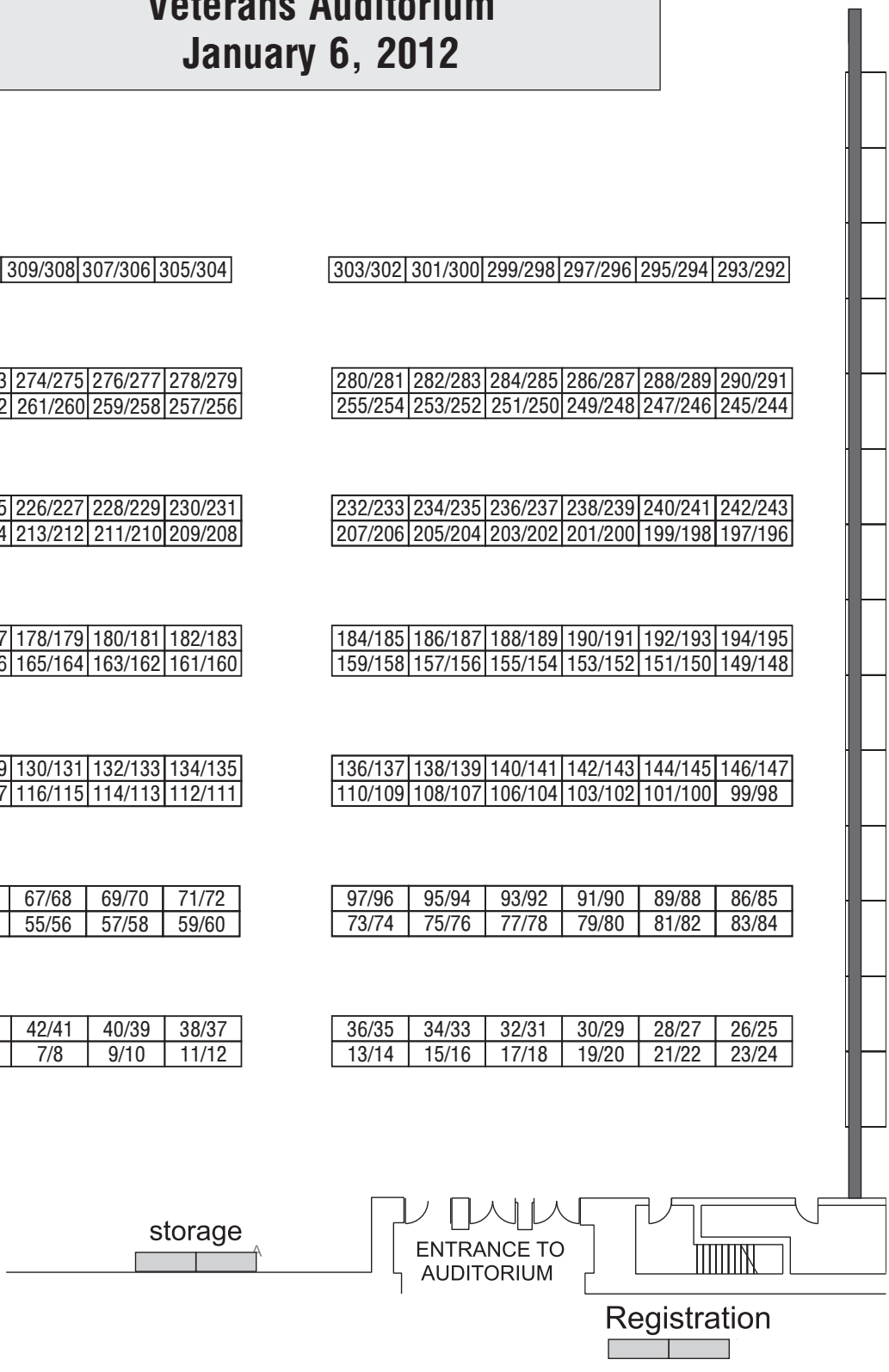
136/137	138/139	140/141	142/143	144/145	146/147
110/109	108/107	106/104	103/102	101/100	99/98

61/62	63/64	65/66	67/68	69/70	71/72
49/50	51/52	53/54	55/56	57/58	59/60

97/96	95/94	93/92	91/90	89/88	86/85
73/74	75/76	77/78	79/80	81/82	83/84

48/47	46/45	44/43	42/41	40/39	38/37
1/2	3/4	5/6	7/8	9/10	11/12

36/35	34/33	32/31	30/29	28/27	26/25
13/14	15/16	17/18	19/20	21/22	23/24



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Organized by

Joyati Debnath
Winona State University

Organized by the MAA

Committee on Undergraduate Student Activities and Chapters

and

CUPM Subcommittee on Research by Undergraduates

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Dear Students, Advisors, Judges and Colleagues,

If you look around today you will see over 300 posters and 525 presenters|record numbers, once again. It is so rewarding to see this session, which offers such a great opportunity for interaction between students and professional mathematicians, continue to grow.

The judges you see here today are professional mathematicians from institutions around the world. They are advisors, colleagues, new Ph.D.'s, and administrators. We have acknowledged many of them in this booklet; however, many judges here volunteered on site. Their support is vital to the success of the session and we thank them.

We are supported financially by several organizations that are committed to undergraduate research in mathematics. These are the National Science Foundation, Educational Advancement Foundation, American Mathematical Society, and Pi Mu Epsilon.

Our online submission system and technical support is key to managing the ever-growing number of poster entries we receive. Thanks to MAA staff, especially Grace Cunningham and Maia Henley, for their work setting up and managing the system this year. Preparation of the abstract book is a time-consuming task. Thanks to Steven Schlicker and Anthony Tongen for cleaning up the abstracts, and to Beverly Ruedi for doing the final production work.

There are many details of the poster session that begin with putting out the advertisement in FOCUS in February, ensuring students have travel money and organizing tables in the room we are in today that are attributed to Gerard Venema (MAA Associate Secretary), Michael Pearson, Donna Salter and Peter Smith (AMS). Michael Dorff (Brigham Young University) rallied volunteers to judge the session and Lisa Marano (West Chester University) coordinated the judge assignments.

Thanks to all the students, judges, volunteers, and sponsors. I hope you have a wonderful experience at this year's poster session!

Joyati Debnath
Winona State University

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Titles, Authors, Advisors and Abstracts

1. Completions of Hypersurface Domains

Ji Won Ahn Williams College
Feiqi Jiang University of Michigan-Ann Arbor
Giang Tran Bard College
Advisor(s): Susan Loepp, Williams College

Let V be a complete regular local ring and $f \in V$ a nonunit. We find necessary and sufficient conditions for $\frac{V}{fV}$ to be the completion of a domain of the form $\frac{S}{gS}$, where S is a regular local ring whose completion is V and $g \in S$. In addition, if $\frac{V}{fV}$ contains the rationals, we give necessary and sufficient conditions for $\frac{V}{fV}$ to be the completion of an excellent local domain of the form $\frac{S}{fS}$, where S is a regular local ring whose completion is V.

2. The Noncommutative Geometry of Conway's Game of Life

Nadine Amersi University College London
Advisor(s): Javier López Peña, University College London

In classical geometry there is a duality between commutative rings and certain families of topological and geometrical spaces. This duality, however, fails to provide any useful algebraic information about certain "bad quotients" such as the space of leaves of a foliation. Noncommutative geometry, pioneered by A. Connes, provides a way around this issue by associating to the ill-behaved space a noncommutative algebra instead of a commutative one. John Conway's "Game of Life" is an example of a space equipped with a natural equivalence relation, but where the the classical commutative setting fails to capture useful algebraic information. We explore finite boards following Connes' approach (i.e., with groupoid algebras), as well as by means of path algebras.

3. Lower Central Series of Free Associative Algebras over the Integers and Finite Fields

Surya Bhupatiraju Lexington High School
William Kuszmaul Lexington High School
Advisor(s): David Jordan, University of Texas, Austin

Noncommutative algebras are ubiquitous in mathematics. They arise as algebras of observables in quantum physics, where commutativity fails. A basic example of a noncommutative algebra is the free algebra A_n generated by n letters $x_1 \dots x_n$. Interesting objects attached to $A = A_n$ are members of its lower central series, $L_i = L_i(A)$, defined as $L_1 = A$, $L_{i+1} = [A, L_i]$, and its associated graded components $B_i = B_i(A)$ defined as $B_i = L_i/L_{i+1}$. These quotients B_i for $i \geq 2$, and the reduced quotient $\bar{B}_1 = B_1/AL_3$, exhibit a rich geometric structure, as shown by Feigin and Shoikhet and later authors.

We study the same problem over the integers $\mathbb{Z}$ and finite fields $\mathbb{F}_p$. New phenomena arise, namely, torsion in B_i over $\mathbb{Z}$, and jumps in dimension over $\mathbb{F}_p$. We describe the torsion in the reduced quotient $\bar{B}_1$ and (conjecturally) B_2 geometrically in terms of de Rham cohomology of $\mathbb{Z}^n$. As a corollary we obtain a complete description of $\bar{B}_1(A_n(\mathbb{Z}))$ and $\bar{B}_1(A_n(\mathbb{F}_p))$, and conjecturally of $B_2(A_n(\mathbb{Z}))$ and $B_2(A_n(\mathbb{F}_p))$.

4. Finding Factors of Factor Rings over the Eisenstein Integers

Valmir Bucaj Texas Lutheran University
Advisor(s): William Hager, Texas Lutheran University

Eisenstein integers are defined to be the set $Z[\omega] = \{a + b\omega : a, b \in \mathbb{Z}\}$ where $w = (-1 + i\sqrt{3})/2$. This set lies inside the set of complex numbers $\mathbb{C}$ and they also form a commutative ring in the algebraic number field $\mathbb{Q}(\omega)$. In this paper we prove a few results related to the factor rings over the Eisenstein integers. In particular we show that the ring $\mathbb{Z}[\omega]$ factored by an ideal generated by any element $m + n\omega$ of this ring, where $g.c.d(m, n) = 1$ is isomorphic to the ring $\mathbb{Z}_{N(m+n\omega)}$, where N is the norm function given by $N(m + n\omega) = (m + n\omega)(m + n\bar{\omega}) = m^2 + n^2 - mn$. This result helps us quickly answer questions about the number of elements of the factor ring $\mathbb{Z}[\omega]/\langle m + n\omega \rangle$. Then, we give a representation for the factor ring $\mathbb{Z}[\omega]/\langle m + n\omega \rangle$ in terms of simpler rings. At the end we give a few applications to elementary number theory. More specifically, we use some of our results to derive information about $\mathbb{Z}$.

5. Directed Graphs of Commutative Rings

Seth Hausken University of St. Thomas
Jared Skinner University of St. Thomas
Advisor(s): Michael Axtell, University of St. Thomas

The directed graph of a commutative ring is a graph representation of its additive and multiplicative structure. Using the mapping $(a, b) \to (a + b, a \cdot b)$ one can create a directed graph for every commutative ring. We examine the properties of directed graphs of commutative rings, with emphasis on the information the graph gives about the ring.

6. The Solutions of Nonlinear Rational Difference Equations

Fakui Li Queensborough Community College
Minzi Mei Queensborough Community College
Advisor(s): Haishen Yao, CUNY-QCC

In this poster, we will introduce a method to find out the solutions of certain type of rational difference equations. A rational difference equation is a nonlinear difference equation. The solution of the so called Riccati difference equation was studied by Brand, Louis, "A sequence defined by a difference equation," American Mathematical Monthly 62, September 1955, 489{492. Our method improves Brand's method.

7. Solution to the matrix equation $AX + X^\star B = 0$.

Nicolas Reyes University of Massachusetts, Amherst
Nathan Guillery University of California, Santa Barbara
Advisor(s): Fernando De Teran Vergara, Universidad Carlos III de Madrid

The matrix equation $AX + X^\star B = 0$, where $A \in \mathbb{C}^{m\times n}$ and $B \in \mathbb{C}^{n\times m}$ are arbitrary and $(\cdot)^\star$ denotes either the transpose or the conjugate transpose, has been of interest to researchers since at least the 1960s, but has remained unsolved until now. This equation appears similar to the Sylvester Equation, $CX + XD = 0$, which, in contrast, has been solved since the 1950s. Solving it reduces to solving the simpler equation $J_C X + XJ_D = 0$ where the matrices J_C and J_D are the Jordan Canonical Forms of C and D respectively, and then recovering solutions to the original equation from solutions of this simpler one. The equation $AX + X^T A = 0$ was solved in 2011 by analogous methods, using the Canonical Form for Congruence, rather than the Jordan Canonical Form. Unfortunately, we may not assume our coefficient matrices are in either of these Canonical Forms when considering the equation $AX + X^\star B = 0$. Instead, we must consider matrix pencils, tools for coupling our coefficient matrices, A and B, and then consider the Kronecker Canonical Form (KCF) of matrix pencils under strict equivalence. We explicitly solve this equation in terms of the KCF of the pencil associated with the pair $(A, B^\star)$.

8. Lie Algebra Change of Basis

Kermit Sharp Grand Valley State University
Advisor(s): Firas Hindeleh, Grand Valley State University

We present our work in Lie algebras change of basis. We investigate the problem of low dimensional matrix Lie algebras, and present an algorithm that works for Lie algebras with given structure equations. The results are helpful for studying special Lie algebra extensions and subalgebras.

9. Idempotent Matrix Enumeration and a Putnam Problem

Praveen Venkataramana MIT
Advisor(s): Mohan Venkataramana, Syntonix

A problem on a recent Putnam competition concerned matrices M of real numbers with the property that $(M^k)_{i;j} = ((M)_{i;j})^k$. I call these *entrywise exponentiable matrices* (EEM), and in this poster I provide a characterization of these matrices in terms of idempotent $\{0, 1\}$-matrices. A neat corollary is that the number of $d \times d$ EEMs with entries taken from a finite set $\{0, a_1, \cdots, a_n\}$ is:

$$B(d, n) = \sum_{r=0}^{d} \binom{d}{r} I(d, d - r)(n - 1)^r$$

where $I(d, d-r)$ is the number of $d \times d$ idempotent $\{0,1\}$-matrices with rank r and decreasing diagonals. In particular, $B(d,n,r) = \binom{d}{r} I(d,d-r)(n-1)^r$ is the number of $d \times d$ EEM's with entries in a set of $n+1$ elements and rank r.

This enumeration formula begs several questions on the growth of the functions B and I. In this poster, I show that although the growth of $B(d,n)$ is superexponential (trivially greater than $C(n)2^{d^2/4}$ for large d, for a function $C(n) > 0$), the generating functions

$$F_r(x) = \sum_{d \geq 1} I(d, d-r)x^r \qquad \text{and} \qquad G_r(x) = \sum_{d \geq 1} I(d,r)x^r$$

are both rational, for every integer r. I prove this using a probabilistic argument that can be used to directly, albeit slowly, compute F_r and G_r.

10. The Conjugacy Problem — Theory and Applications

Chao Xu Stony Brook University
Jonathan Siegel University of California, Santa Cruz
Hannah Lewis Dixie State College of Utah
Advisor(s): Jens Harlander, Boise State University

The standard algorithms by Garside and Thurston do not provide efficient solutions for solving the conjugacy problem in braid groups, not even in the three strand braid group B_3. However, we have found other algorithms that provide linear time solutions for both the word and conjugacy problems in B_3. These algorithms rely on special combinatorial and topological features of B_3. Our poster combines and summarizes the complexities of known algorithms that solve the conjugacy problem in the braid group with our new solutions. We also explain the applications of our finding to the field of cryptography.

11. Poisson homology in characteristic p

Michael Zhang Belmont High School
Yongyi Chen Bedford High School
Advisor(s): Pavel Etingof, MIT

A Poisson algebra is a commutative algebra with a Lie bracket $\{,\}$ satisfying the Leibniz rule. Such algebras appear in classical mechanics. Namely, functions on the phase space form a Poisson algebra, and Hamilton's equation of motion is $\frac{df}{dt} = \{f, H\}$, where H is the Hamiltonian (energy) function. An important invariant of a Poisson algebra A is its zeroth Poisson homology $HP_0(A) = A/\{A, A\}$. It characterizes densities on the phase space invariant under all Hamiltonian flows. Also, the dimension of $HP_0(A)$ gives an upper bound for the number of irreducible representations of any quantization of A.

We study $HP_0(A)$ when A is the algebra of functions on an isolated quasihomogeneous surface singularity. Over $\mathbb{C}$, it's known that $HP_0(A)$ is the Jacobi ring of the singularity whose dimension is the Milnor number. We generalize this to characteristic p. In this case, $HP_0(A)$ is a finite (although not finite dimensional) module over A^p. We give its conjectural Hilbert series for Kleinian singularities and for cones of smooth projective curves, and prove the conjecture in several cases.

12. New series for π via polynomial approximations to arctangent

Colleen Bouey Loyola Marymount University
Erika Meza Loyola Marymount University
Advisor(s): Herbert A. Medina, Loyola Marymount University

Using rational functions of the form $\left\{ \dfrac{t^{km}(t-\beta)^{lm}}{1+t^2} \right\}_{m \in \mathbb{N}}$ for different values of k, l, and β, we produce different families of efficient polynomial approximations to arctangent on the interval $[0, \beta]$, and hence, provide approximations to π via known arctangent values. The polynomials produce approximations to π that require only the computation of a single square root ($\sqrt{3}$); moreover, on the interval $[0, \beta]$, they are more accurate than Maclaurin polynomials and other approximations to arctangent recently studied. We turn the approximations of π into series that yield the

same number of digits of accuracy as the corresponding polynomial approximations. In particular, we produce a new series that gives 21 more decimal places of accuracy for π with each successive term. We provide numerical comparisons of the new series to other well-known ones.

13. Classification of Replicator Dynamics in Evolutionary Game Theory

Tara Gangarossa Adelphi University
Advisor(s): Lee Stemkoski, Adelphi University

We examine a mathematical model of population dynamics using a system of three replicator equations: differential equations frequently used in evolutionary game theory. We classify phase portraits of the systems that arise from variations on the Prisoners' dilemma by changing the payoff matrix and the strategies used by each of the three populations.

14. On Li-Yorke Measurable Sensitivity

Jared Hallett Williams College
Lucas Manuelli Princeton University
Advisor(s): Cesar Silva, Williams College

The notion of Li-Yorke sensitivity has been studied extensively in the case of topological dynamical systems. We introduce a measurable version of Li-Yorke sensitivity, for nonsingular (and measure-preserving) dynamical systems and compare it with various mixing notions. It is known that in the case of nonsingular dynamical systems, ergodic Cartesian square implies double ergodicity, which in turn implies weak mixing, but the converses do not hold in general, though they are all equivalent in the finite measure-preserving case. We show that for nonsingular systems, ergodic Cartesian square implies Li-Yorke measurable sensitivity, which in turn implies weak mixing. As a consequence we obtain that, in the finite measure-preserving case, Li-Yorke measurable sensitivity is equivalent to weak mixing. We also show that with respect to totally bounded metrics, double ergodicity implies Li-Yorke measurable sensitivity, and we extend the known result that weak mixing implies measurable sensitivity for finite measure-preserving systems to the case of infinite measure-preserving systems.

15. Cloud Cryptography

Kelsey Rauber New York City College of Technology
Advisor(s): Delaram Kahrobaei, New York City College of Technology

As kids we often wonder what it would feel like to touch a cloud, play on it, maybe even build a living environment on it. We now have found a way to step into one: Cloud computing! To a kid, this statement might evoke false hope, but to scientists all over the world, we are a step closer to making the impossible possible. Cloud computing is the new hot topic everyone is talking about. What will it offer, how will we use it? How is it being used right now? This paper focuses on the key element with whose absent the entire system could fall apart: Security! What are the main components that need to be secured? How will cloud computing revolutionize our computer experience? Where do Security-As-A-Service, Homomorphic Encryption and Functional Encryption fit in? How do they work and how will they keep our information secure? We also explore a mathematical aspects of cloud cryptography.

16. Colorado State University Math Club Trebuchet Modeling

John Bloemker Colorado State University
Kyle Strand Colorado State University
Nicholas Kaufhold Colorado State University
Advisor(s): Patrick Shipman, Colorado State University

In this poster, we explore the use of a Medieval-style trebuchet as a teaching implement for introducing and motivating the study of differential equations and optimization problems in a high school or undergraduate setting. The trebuchet is a particularly effective teaching tool for two reasons: first, because it is relatively simple to build an actual full-size or scaled-down replica of the historical siege engine and test it, and second, because there are a number of ways to add or remove complexity to the basic model, permitting instructors to use it either as an introduction to the desired topics or as a more thought-provoking series of examples. Our poster explores the mathematics of trebuchet dynamics, methods of building model trebuchets, and potential teaching applications.

17. A Look on Scheduling Classes at Fitchburg State

Jared Weed Fitchburg State University
Robert Patenaude Fitchburg State University
Advisor(s): Peter Staab, Fitchburg State University

Class scheduling at Fitchburg State is a tedious and complicated task often left up to individual department chairs. In this project we examine how the Mathematics Department schedule the different ways professors can be matched to classes. With the class matching currently done by hand, it is possible to improve (and/or algorithmically optimize) the matching with various restraints or stressors. Problems we will attempt to cover include limited resources within the department, professor dependency, block-scheduling conflicts, professor enthusiasm, class availability, and 4-year student plans. We will analyze these problems and propose an optimizing solution that can be used to match professors to classes more efficiently. This matching optimization will most likely result in an improvement to overall enthusiasm among professors, increase the quantity of classes offered per semester, and decrease block-scheduling conflicts occurring for mathematics students. In the future, we anticipate our research could also benefit other departments within the university.

18. PA ÷ 18 = ?

John Hocker Shippensburg University
Advisor(s): Ben Galluzzo, Shippensburg University

The United States conducts a census every ten years in order to redistribute the U.S. House of Representatives "fairly" amongst all states in the union. After the 2010 census, all states that experienced a significant (relative) loss or gain in population were given new congressional allotments and are required to create new congressional districts for 2012 elections. This project uses accessible technology and graph theoretical techniques to develop methods for creating district maps in some of the affected states. This project is also being used to introduce a "real world" topic to the high school classroom. The poster presentation will cover a brief history of redistricting, the motivation for the research, the maps and mathematical processes used to create new maps, and the approach used to bring this problem into the high school classroom.

19. Group-Theoretic Solutions to Rubik-Style Permutation Puzzles and Run-Time Analysis

Nicholas Miceli Adelphi University
Advisor(s): Lee Stemkoski, Adelphi University

Various methods of solution to Rubik's cube and related puzzles (constructed on alternative polyhedra) have been developed during the past three decades. An easily-understood algorithm from abstract algebra involves solving corner pieces and edge pieces independently by writing their permutations as products of 3-cycles. We implement this algorithm and analyze the run-time in terms of the number of basic moves (face-twists) and also in terms of 3-cycles required to return the puzzle to its unscrambled state. We also extend these results to puzzles constructed on regular prisms and present a related algorithm based on sorting algorithms from computer science, and perform a similar analysis in this case.

20. Sparsity, Fuzziness, and Synergy: Methods for Cyber Crime

Evgeni Dimitrov Princeton University
Kizza Nandyose Hood College
David Wen Polytechnic Institute of New York University
Sandra Rankovic University of Oxford
Advisor(s): Stacey Beggs, IPAM, UCLA

In recent years, Multi-Criteria Decision Analysis (MCDA) techniques have appeared in the literature as a way to address problems in attack attribution, in particular root cause analysis of cyber attacks. The application of MCDA methods involves the construction of aggregation functions, which model a set of user preferences. Our team proposes a novel method of building such aggregation functions using L1 optimization. In addition, we develop a new aggregation operator for modeling user preferences, based on the Ordered weighted averaging function and the Choquet integral. All proposed models have been tested on some toy examples we built and the real data provided by Symantec Research Labs, and have shown satisfactory results. This work is an initial step that could potentially provide security experts with a new tool for fighting against cyber crime.

21. Bounded Gradient Projection Methods for Sparse Video Recovery

David Jones University of California at Merced
Advisor(s): Roummel Marcia, University of California at Merced

The recovery of sparse images from noisy, blurry, and potentially low-dimensional observations can be accomplished by solving an optimization problem that minimizes the least-squares error in data fidelity with a sparsity-promoting regularization term (the so-called $\ell_2-\ell_1$ minimization problem). This paper focuses on the reconstruction of a video *sequence* of images where known pixel-intensity bounds exist at each video frame. It has been established that the $\ell_2-\ell_1$ minimization problem can be solved efficiently using gradient projection, which was recently extended to solve general bound-constrained $\ell_2-\ell_1$ minimization problems. Furthermore, the reconstruction of the video sequence can be made more efficient by exploiting the similarities between consecutive frames. In this paper, we propose a method for reconstructing a video sequence that takes advantage of the inter-frame correlations while constraining the solution to satisfy known a priori bounds, offering a higher potential for increasingly accurate reconstructions. To demonstrate the effectiveness of this approach, we have included the results of our numerical experiments.

22. Calculating Call Blocking, Preemption Probabilities and Bandwidth Utilization for Satellite Communication Systems

Yacoub Kureh Harvard College
Leah Rosenbaum Scripps College
Advisor(s): Nam Lee, Johns Hopkins University

Some satellite communication (SATCOM) systems employ a static resource allocation scheme in which high-priority users can reserve a channel according to their maximum foreseeable bandwidth requirements, regardless of their actual patterns of use. While ensuring instant service for high priority users, this practice often leaves lower priority users without service even while the high priority users are not fully utilizing their reserved bandwidth resources. Addressing this inefficiency could reduce expenditures by decreasing the need for more communication satellites. This report analyzes the effects of implementing a dynamic and more efficient resource allocation policy to SATCOM systems. In particular, we use the M/M/m model from classic queueing theory as a spring board and use Markov chains to extend the theoretical M/M/m framework to include competing priority and bandwidth classes. To validate and analyze our mathematical models, we make heavy use of discrete event system Monte Carlo simulations and the resulting estimates of the key performance measures. Properties under consideration include call blocking and preemption probabilities as well as server utilization.

23. An Investigation of Second Generation Wavelets

Karleigh Cameron Central Michigan University
Michael Gustin Central Michigan University
John Holden Central Michigan University
Stacy Siereveld Central Michigan University
Advisor(s): Yeonhyang Kim, Central Michigan University

It is well known that any element in L_2 has a basis expansion. Because of its localization property and fast transform algorithms, a wavelet basis expansion has many applications including speech, images, video, graphics, and engineering. For an infinite or periodic function a traditional wavelet basis works well. However, in many applications the domain of a function is not infinite and functions are not periodic. The need for improvements of wavelet bases introduces the second generation wavelets. We study properties of a lifting operator that serves as a tool to construct these second generation wavelets.

24. Wavelet-Based Recognition Algorithms

Pierce O'Donnell Western Connecticut State University
Sami Merhi Western Connecticut State University
Phanuel Mariano Western Connecticut State University
Advisor(s): Xiaodi Wang, Western Connecticut State University

The capability of positively recognizing individuals has become a necessity. In an attempt to solve identification issues, researchers have devised numerous algorithms. However, face recognition and fingerprints are arguably the

most effective methods of identification. Biomedical research has shown the uniqueness of each fingerprint. It has also shown that human beings often recognize one another through facial characteristics. In as early as the late nineteenth century, researchers have tried to identify dominant features through automatic methods of classification. Popular recognition algorithms include Principal Component Analysis, Linear Discriminate Analysis, Elastic Bunch Graph Matching, and the Hidden Markov Chain Model. In these types of recognition, data storage is an issue, especially when dealing with a large population. Wavelet Analysis will not only decompose data into approximation and detail components, but also compress data dramatically, thus saving memory. In this research, we apply new algorithms to face and fingerprint recognition and compare the results with those of others.

25. A Quantitative Analysis Of Mass In The Absence Of Gravitational Fields Without Resorting To The Equivalence Principle.

José Caraballo Universidad Metropolitana
Advisor(s): Luis De La Torre, Universidad Metropolitana

The main concept of this research project was to provide an alternative solution to the equivalence principle and to calculate the mass of an object in the absence of a gravitational field.The method consisted of using numerical analysis and multivariable calculus to construct a 3D model of solids.The numerical analysis was based on 'Lagrangian Interpolation' in polar coordinates and multivariable calculus to formulate scalar functions that model the object's surface.The approach of this research was to simulate operations in which a machine reads several points in the object's surface by manual digitalization of cross section images.These digitalized images were used to develop a methodology to create a model to determine the amount of mass and a complete oriented 3D description of an object.The method developed relies completely on direct measurements of the exterior objects with the condition that its compact, with limited concavity and has a constant density.From the functions constructed we proceeded to use integration to determine the approximate volume of the object and multiplying by the density.According to the results, this provides an alternative solution for quantifying mass without the necessity of gravitational fields or other external forces acting on the object.

26. Global Enforcement of Rigid Body Articulation

Mauricio Flores Institute of Pure and Applied Mathematics
Andreu Boada de Atela Institute of Pure and Applied Mathematics
Leah Fritter Institute of Pure and Applied Mathematics
Gina Ma Institute of Pure and Applied Mathematics
Advisor(s): Craig Schroeder, University of California in Los Angeles

PhysBAM (Physics Based Modeling) is a physics simulation tool currently used by Disney Animation Studios and Pixar Animation Studios. One of the uses of this software is for the simulation of articulated rigid bodies. These simulations require solving many large systems of equations. PhysBAM's default iterative solver, Gauss-Seidel, produces simulations of acceptable quality. However, the time spent in creating simulations can be improved. The objective of this project was to improve the simulation of articulated rigid bodies by reducing the computational cost involved in generating these simulations. This was accomplished through the use of a variety of Krylov iterative solvers and matrix preconditioners.

27. Trees and Motors in the Parallel Chip-Firing Game

Ziv Scully Brookline High School; MIT Program for Research in Mathematics, Science, and Engineering
Advisor(s): Yan Zhang, Massachusetts Institute of Technology

The *parallel chip-firing game* is an automaton on graphs in which vertices "fire" chips to their neighbors when they have enough chips to do so. We characterize positions that repeat every 2 turns. This can be used to determine the eventual periods of games on trees given only the total number of chips. We introduce the concepts of motorized parallel chip-firing games and motor vertices, study the effects of motors connected to a tree, and show that some motorized games can be simulated by ordinary games. We also prove the equivalence of two conjectures: one restricts the firing pattern of a single vertex over a period; the other restricts the set of vertices that fire at a single time step. These conjectures, if shown to be true, would greatly simplify the study of the parallel chip-firing game.

28. Lax-Wendroff Methods for the 2-D Shallow-Water Wave Equations

Sarah Sexton Coastal Carolina University
Advisor(s): Tessa Weinstein, Coastal Carolina University

The shallow-water wave equations, which are hyperbolic in nature, are used to model tidal and tsunami waves, as well as a myriad of other physical phenomena. In two spatial dimensions, there are several finite difference methods for solving hyperbolic equations called Lax-Wendroff methods. Here, we investigate several explicit two-step Lax-Wendroff methods for solving the two-dimensional shallow-water wave equations. Specifically, we explore the consistency, stability and convergence of these methods and weigh this against the computational cost of executing the methods by calculating the number of operations necessary to advance a given method to the next time step.

29. Adversary Deception in Planning Under Uncertainty

Katherine Todd Institute of Pure and Applied Mathematics
Leon Chang Institute of Pure and Applied Mathematics
Fanni Selley Institute of Pure and Applied Mathematics
Yasar Kocaga Yeshiva University
Advisor(s): Janusz Marecki, IBM Research

We consider the multistage optimization problem of an agent acting in a stochastic environment in the presence of an adversary. Within each stage, the idea is to optimize utility accounting for both immediate rewards and the uncertainty of the adversary. To find an optimal strategy, we employ a Markov Decision Process (MDP). This allows us to generate a set of all possible decision sequences, or histories. We assume that the adversary has a set of initial beliefs about the state of the MDP agent and continues to update his beliefs at each decision epoch. Based on this assumption, we are able to calculate the level of uncertainty of the adversary at any given time. Our initial results compute the exact solution for small data sets. For larger data sets, we formulate approximation methods to estimate the solution. Finally, we introduce ideas for future research which could improve the applicability of this model.

30. Homogeneous and Multi-State Markov Process Models for Melodic Sequences

Eric Buenger Butler University
Advisor(s): Christopher Wilson, Butler University

We compare melodic musical samples generated from both homogeneous and multi-state Markov processes to the source melodies on which the models were based. Because the multi-state Markov processes take into account only recent melodic history, one might predict that these samples will be more similar to the original melody than their homogeneous counterparts. We test this prediction using chi-squared goodness-of-fit tests. We also identify other mathematical methods for evaluating and comparing the abilities of homogenous and multi-state models to generate material resembling the original sources.

31. Effects of Non-Independent Behavior on a Macroeconomic Model

Nicholas Chaung George Mason University
Advisor(s): Harbir Lamba, George Mason University

The standard macroeconomic models in use today are Dynamic Stochastic General Equilibrium (DSGE) models. DSGE models attempt to calculate macroeconomic variables based on microeconomic principles. These variables are determined at each time step (dynamically) by calculating the equilibrium solution to a system of equations, that represents the behaviors and expectations of the involved agents. By assumption, these agents behave independently and rationally to maximize a given function. Taking for example inflation, the DSGE model studied here predicts that changes in inflation over time converge to a normal distribution. However in real economies, very large fluctuations occur much more frequently than a normal distribution would predict; i.e., frequency distributions of macroeconomic variables show ‘fat tails’. An explanation for these fat tails is that agents do not always act independently and rationally, but can instead exhibit non-independent and sometimes perverse behavior. One such non-independent behavior agents have been known to exhibit is ’herding’, where agents mimic the behavior of other agents. We will investigate the sensitivity of such DSGE models to herding behavior.

32. Quantitative Modeling of Financial Contagion

Jialu Chen Mount Holyoke College
Ya Le Peking University
Andre Pradhana California Institute of Technology
Yitzchak Solomon University of California, Los Angeles
Advisor(s): Erik Lewis, University of California, Los Angeles

Due to the high level of interconnectedness among sovereigns and banks, it is challenging to assess the risk of contagion in today's global financial system. Successful assessment of contagion risk would improve credit risk evaluation and help prevent financial stress from building up. In this project, we consider the effects of the interactions among sovereigns and banks and the evolution of sovereign-specific macroeconomic variables on the health of the global and regional economy. We propose the Financial Market Framework Model, an agent-based model that computationally simulates the actions of and interactions among autonomous sovereigns and banks in a Eurozone-like system. A sovereign's actions affect the ratings of its banks, and the recapitalization needs of banks may influence the sovereign's balance sheet and risk level. The simulation of the model gives valid results in predicting sovereign risk levels. We modified the deterministic model by adding mean-reverting stochastic processes to the macroeconomic input curves. The stochastic model gives consistent results with the deterministic model in most cases but alternative results in others. Our model captures some basic relationships in the global financial system that need to be included in a more complicated model.

33. Risk Aggregation

Bianca Cung UCLA (RIPS-HK and IPAM)
Advisor(s): Chi-Wai Yu, RIPS-HK and IPAM / HK Univ. of Science and Tech.

While there is a general set of economic aggregation methodologies known to banks and and similar companies, each company uses a methodology specific to its own needs. A reliable aggregate model capable of predicting the future path of the market is difficult to achieve due to the complex interdependency among risks. This project used the variance-covariance and copula aggregation methodology to examine economic capital under the life insurance content. We analyzed the S&P 500 historical data and effective federal funds rate for any relationships. We created potential scenarios and generated four major categories of risks for use in each method.

34. Searching for the Implied Market Utility Function

Aniket Panjwani George Mason University
Advisor(s): Tim Sauer, George Mason University

Modern portfolio theory tells us how to choose the optimal portfolio given the returns and variances of assets. We choose to modify modern portfolio theory, as created by Merton (1972), by introducing a stochastic element to the standard model. Then, after making an assumption on agents' valuation functions, an assumption on agents' pricing functions, and a 'no arbitrage' assumption on prices, we use parametric and nonparametric methods to estimate a 'market utility function'. This market utility function allows us to see how the market differentiates between portfolios of identical means and variances, but different higher moments and distributions.

35. Modern Portfolio Theory Enhanced by Manifold Learning

Esther Jackson George Mason University
Advisor(s): Tim Sauer, Geogre Mason University

In the realm of investment theory, Modern Portfolio Theory (MPT) remains the most important and influential mathematical model. Modern Portfolio Theory attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets. The theory has been widely used in the financial industry. We examine the assumptions underlying MPT, and investigate replacing some of the underlying linearity assumptions with nonlinear approaches from manifold learning theory.

36. Potential for Bio-oil Production in Kentucky

Amir Ahmadi Morehead State University
Advisor(s): Michael Dobranski, Morehead State University

Clean and sustainable fuel in the United States is limited at best. The United States government has combated this scenario over the past decade through investments in next generation biofuels. This newer form of biofuels includes the application of woody biomass. The produced energy potential determines the monetary value of such potential biomass. Unfortunately, this biofuel production process is variable and has yet to be fully commercialized.

This poster presents a two-model system that outlines the costs and benefits of investment in next generation biofuels for Kentucky sawmills. By applying a wood residual model in Mathematica, regular woodchip yields are estimated based on saw dimensions and wood species of a representative Kentucky sawmill. The biofuel potential of this woodchip output is computed by a Matlab model, based on wood flash pyrolysis in a fluidized bed (CFB) reactor model. This model incorporates the dynamics of pyrolysis in a CFB reactor with a finite element method using semi-discretization in time. The Mathematica model applies both model outputs in order to incorporate variable process parameters and multiple wood species parameters indigenous to Kentucky. Conclusions from this analysis shed light on possible applications for the US sawmill market.

37. Growth Patterns of Ethnic Groups in Bexar County

Judith Arriaza University of the Incarnate Word
Advisor(s): Zhanbo Yang, University of the Incarnate Word

The purpose of this study is to establish a mathematical model that can be used to predict the population growth pattern of Bexar County, Texas, with the emphasis on demographic structures such as age groups and ethnicities. Throughout the past decades, Bexar County's population has increased rapidly. As a result to the population growth in San Antonio, it has also changed the ratio of various ethnic groups, which will have consequences for the county's development. Using population data gathered from the US Census Bureau and the Texas Department of State Health Services from 1990 to 2010, a mathematical model known as the Leslie Model was established. This model allows to study the subgroups of a population by age groups. Population projections of Bexar County were made until the year of 2020. These projections include the total population, ethnic composition and age group composition.

38. SPASM: Stochastic Particle Approach to Simulating Morphogenesis

Jeffrey Moulton University of Pittsburgh
Aashish Gadani University of Maryland-College Park
Advisor(s): Sharon R. Lubkin, North Carolina State University

We construct a stochastic modeling tool, called SPASM, to analyze the dynamics of groups of moving cells. In our modeling framework, cells exist as groups of particles. Each particle represents a fixed volume, and particles interact through fluid and/or elastic forces. Stochastic noise represents dynamics of the cytoskeleton. Our tool includes options for a cell cortex and extracellular matrix, and allows the user to control cell stiffness and the viscosity of the environment. We ran many simulations to test our modeling framework. To analyze the biological process of cell sorting, we distributed cells of different types and varied adhesive strengths between cells. We found that a high ratio of homogeneous adhesion to heterogeneous adhesion caused some cell sorting to occur. We placed cells above an adhesive surface, representing a plate used in experiments, to observe the change in aspect ratio (height/width) of the cells. A higher adhesive ratio caused the cells to round out slightly on top. SPASM can simulate mitosis and cell growth. Although many dynamic cell models already exist, ours is one of the first to include both fine details of individual cells and adaption to large tissues.

39. A Mathematical Model for the Effects of Plaque Aggregation on the Neuronal Network

Thomas Howard Bridgewater State University
Advisor(s): Irina Seceleanu, Bridgewater State University

In this project we build a mathematical model to study the effects of plaque deposits on the neuronal pathways in the human brain. To simulate the complex biological system of the neuronal network, we use a computer algorithm to generate a fractal image that resembles the neuronal connections in the brain. Given that plaque deposits form in clusters, we employ a non-homogeneous Markov process to model the location in the brain where the plaque granules are deposited. Finally, to study the effects of the plaque granules on the neuronal network, we integrate the two models and use graph theoretical tools to measure the number of neuronal connections before and after the plaque

depositing. We also present the computer generated images from our simulations and draw a parallel to the actual image of plaque deposits in the human brain. Our mathematical model can be used in the field of medicine to study the influence of different treatments that slow the degeneration of neuronal pathways due to plaque aggregation.

40. Stem Detection of Strawberries Utilizing the Medial Axis Transform

Danika Lindsay CSU Channel Islands
Rebecca Strawbridge CSU Channel Islands
Eduardo Reynoso CSU Channel Islands
Advisor(s): Kathryn Leonard, CSU Channel Islands

This project addresses the problem of stem detection of strawberries for an automated strawberry-picking robot currently under construction in the CSU Channel Islands computer science department. We propose a geometric approach to automated strawberry stem detection using the medial axis transform, or the skeleton, of the strawberry to locate the berry stem. The medial axis is the collection of the centers of all maximal circles contained in the boundary curve and tangent to the curve at two or more points, together with their radii. The medial axis captures the global geometry of the berry shape and therefore encodes the key structures of the shape that allow for stem identification. We develop a MatLab algorithm to pre-process a berry image, find the medial axis of the berry, extract the salient branches of the medial axis, and output the berry stem location.

41. Two-dimensional simulation of a porous insect wing at low Reynold's numbers

Audrey Low University of North Carolina at Chapel Hill
Advisor(s): Laura Miller, University of North Carolina at Chapel Hill

We used the immersed boundary method to model a two-dimensional representation of a small porous insect wing, motivated by the fringed wing of the thrips, an insect of about a millimetre in size. The simulations were run on Reynolds numbers 1, 2 through 10 in increments of 2, and 10 through 100 in increments of 10, outputting for each a vector field representing the velocity of the fluid around and through the wing. For Reynold's numbers 10, with porosities of 0 and of 10^{-7}, the angle of attack was varied between 0° and 90° in increments of 9°. For the rest, angle of attack was kept at 45° while porosity was varied between 10^{-5} and 10^{-9}, as well as 0. We found that for a narrow range of values, lift and drag forces rapidly decrease as porosity increases. This suggests that the porous structure of the wing may reduce the forces required for wing clapping and rotation.

42. Modeling Immune Reconstitution of HIV-1 Patients on HAART

Leah DeCoste College of the Holy Cross
Advisor(s): David B. Damiano, College of the Holy Cross

In this project, we utilize a multi-compartment ordinary differential equation model of the adaptive immune system-HIV interaction to determine which parameters might affect the rate of immune reconstitution in some patients. In particular, we focus on data from a cohort of patients (N=40) that have undergone highly active anti-retroviral therapy for at least six years and have maintained undetectable viral loads. Although many patients' CD4+ and CD8+ counts return to normal values in this time period, for some patients the return to normal occurs more slowly or appears to plateau before reaching the normal range. Greenough, Damiano, *et al.* observed that for many patients the CD4+:CD8+ ratio increases in a linear manner. Using the eight compartment model of Banks and Davidian., we optimize model parameter values against this ratio data for individual patients and cohort averages and carry out sensitivity analyses to identify influential parameters. This project is part of an ongoing retrospective study of patient data from the HIV/AIDS Clinic at the University of Massachusetts Medical School.

43. Relating Steady States of Continuous and Discrete Models in Systems Biology

Joseph Arthur North Carolina State University
Laura Hochstetler Asbury University
Victoria Klomps Northwestern College
Erikka Korpi University of Wisconsin - River Falls
Advisor(s): Alan Veliz-Cuba, University of Nebraska - Lincoln

We present methods to transform discrete dynamical system models of biochemical networks into continuous ODE models that preserve the dynamical characteristics. Moreover, we provide theoretical conditions for a 1-1 correspondence between stable steady states in discrete and continuous models.

44. How does the effort a mother bird expends on her offspring depend on the attractiveness of her mate?

Dana-Adriana Botesteanu Mount Holyoke College
Advisor(s): Robert Gilman, NIMBioS

The Differential Allocation Hypothesis (DAH) proposes that selection would favor individuals in a population that invest more resources in their current reproductive attempt when paired with a high quality mate. Additionally, it is argued that differential allocation should take place to a greater extent in polygamous species, since these species are more likely to engage in extra-pair copulations or mate switching. A mathematical model was developed to illustrate the relationship between male attractiveness and female fitness, while taking into account viability and sexual selection, and also allowing varying levels of extra-pair paternity (EPP). The model provides a theoretical framework for determining whether DAH depends on EPP, assuming that male attractiveness only signals indirect fitness benefits. Moreover, meta-analytical techniques with correction for phylogeny were used to examine data from 31 empirical studies of 20 species of birds, using egg size and egg androgen content as response variables. A multiple regression model was formed using data collected from literature on avian species to determine the correlation between the male's attractiveness and EPP in the context of DAH. The goal was to verify the predictions of the theoretical model with empirical evidence.

45. Topological analysis of young and old human red blood cells

Allison Proffer College of William and Mary
Advisor(s): Sarah Day, College of William and Mary

Human red blood cells exhibit an oscillatory behavior akin to "flickering". Previous studies have shown a degradation of the complexity of this flickering pattern when measured with multiscale entropy and detrended fluctuation analysis. In this study, we approach the quantification of the dynamical complexity between young and senescent red blood cells from a topological perspective. As in previous work, we analyze images obtained from time lapse phase contrast microscopy recordings of young and senescent red blood cells. Using cubical homology software (CHomP), we studied the topological structure of the respective cells' membranes through timeseries of Betti numbers.

46. Inverse Modeling of Dynamical Systems

Ariel Setniker Western Oregon University
Stephanie Ger Boston College
Advisor(s): Daniel Dougherty, Michigan State University

Conditioning likelihoods are typically much simpler to model than the full joint distribution which may be difficult or impossible to find analytically. Conditioning has the potential to improve the identifiability of the estimation problem. We will quantify various features of dynamical systems - for example, frequency, peak amplitude, inter-peak intervals, phase synchrony, etc. Parameter estimates are often obtained as the minimizers of a loss function which measure departure between the model prediction and the data at each of N points. Estimators with improved prediction bias are obtainable by adding conditional penalties to the loss function. We define a cumulative power penalty and compare its performance to the derivative matching penalty (Ramsay et al, 2007), as well as an optimal weighted average of different methods. We provide case studies of stochastic switching, bistable oscillation (Terman and Wang, 1995), and predator-prey relationships. We will suggest applications of these methods to wildlife population management, neuroscience, and cryptography.

47. Modeling the Effect of Diversity in Host Plant-Herbivore-Predator Interactions

Karissa Smith Ursinus College
Allison Bugenis Ursinus College
Advisor(s): Mohammed Yahdi, Ursinus College

Predator and plant diversity can control Potato leafhopper (PLH) pest damage to the host-plant Alfalfa, used to feed cattle. New mathematical tools and computer simulations are used to construct deterministic models using systems of differential equations with particular emphasis of the damsel bug Nabis, a major predator of the PLH. The PLH damage done to alfalfa is costly, and pesticides are unsafe. Recent data and results on enemies and diversity hypotheses, in both enclosure and open-field experiments, were used to determine accurate parameter ranges and

validate the models. Effects of diversity are introduced using the Shannon Index. Two models are constructed using explicit and implicit age structures, and logistic, Beverton-Holt, and Allee effect modeling approaches. Simulations are analyzed to examine the roles of the parameters in reducing the alfalfa damage. Uncertain parameters are adjusted for the models to fit the experimental data and predict outcomes for scenarios not covered by both the enclosure and open-field experiments. In conclusion, the project provides a frame work for designing cost-effective and environmentally safe strategies to minimize alfalfa damage, determine critical parameters, and utilize enemies' hypothesis and polyculture diversity.

48. Bounds on the Minimum Semidefinite Rank of Circulant Graphs

Troy Klingler Central Michigan University
Anthony Pochini Central Michigan University
Chennah Heroor Massachusetts Institute of Technology
Gretchen Schillemat Brigham Young University
Advisor(s): Sivaram Narayan and Emily Krause, Central Michigan University

The minimum rank of a graph is the smallest possible rank of any real symmetric matrix associated to the given graph. The real (complex) minimum semidefinite rank of a graph is the minimum rank among symmetric (Hermitian) positive semi-definite matrices associated to the given graph. A circulant graph, $G = Circ(n, S)$, is defined as a graph with n vertices in which the i^{th} vertex is adjacent to the $(i + j)^{\text{th}}$ and $(i - j)^{\text{th}}$ vertices for each j in the given list S. We examine both the zero forcing number, $Z(G)$, and the semidefinite zero forcing number, $Z_+(G)$, for different circulant graphs to find lower bounds on the minimum rank and minimum semidefinite rank respectively. In particular we present some bounds on the minimum rank and minimum semidefinite rank for $Circ(n, 1, t)$ and $Circ(n, a, a + 1, \ldots, t)$.

49. Generating Functions and Wilf equivalence on Θ_k-embeddings

Russell Chamberlain University of Wisconsin Eau Claire
Chi Zhang University of Florida
Sam Ginsburg Allegheny College
Advisor(s): Amanda Riehl, University of Wisconsin Eau Claire

Langley, Liese, and Remmel, in *Rationality, Irrationality, and Wilf-equivalence in generalized factor order*, found generating functions for word embeddings, and several results on Wilf-equivalence. Earlier, Noonan and Zeilberger, in *The Goulden-Jackson Cluster Method: Extiensions, Applications and Implementations*, extended the Goulden-Jackson Cluster Method to find generating functions of word factors. We define a generalization of both ideas called Θ_k-embeddings, which includes word factors and word embeddings. A word w is said to Θ_k-embed u if there is some subword v of w such that for all $1 \leq i \leq length(u)$, $u_i \leq v_i \leq u_i + k$. We present weight generating functions for the number of w embedding u for some classes of words u. We will also present some results and conjectures on Wilf-equivalence for Θ_k-embeddings. We first present a redistribution-rearrangement theorem which gives all Wilf classes for non-overlapping words. We also present a conjecture for the Wilf-equivalences of k-regular overlapping words.

50. Squares in Binary Partial Words

Yang Jiao University of Pennsylvania
Advisor(s): Francine Blanchet-Sadri, University of North Carolina - Greensboro

Here we investigate the number of positions not starting a square and the number of square occurrences in binary partial words. Letting $\sigma^{(h)}(n)$ be the maximum of number of positions not starting a square for binary partial words with length n and h holes, we show that in the limit $\sigma^{(h)}(n)/n = 15/31$ if the limit of h/n goes to zero. Also letting $m(n, h)$ be the minimum number of square occurrences in a binary partial word of length n with h holes, we show that in the limit $m(n, h)/n = 103/187$ for any finite h. Both limits match with the limits for binary full words. We also investigate distinct squares in binary partial words. We bound the maximum difference between the number of distinct squares in a binary partial word of length n with h holes and that of a completion of the partial word by $\mathcal{D}_{n,h} = (2^h - 1)(n + 2)$. This allows us to improve the bound on the number of distinct squares in binary partial words with one hole from $3n$ to $3n - \Theta(\log n)$.

51. The Minimum Number of Givens in a Fair Sudoku Puzzle

Nicholas Smith University of South Carolina
Advisor(s): Joshua Cooper, University of South Carolina

Sudoku is a tremendously popular solitaire game in which, starting from a 9×9 matrix over $[9] \cup \{*\}$ (a "puzzle"), one replaces each $*$ with the values 1 through 9 so that each row and column, and each of nine 3×3 nonoverlapping submatrices contains no repeated values. A "Sudoku board" is a matrix over [9] satisfying these constraints; the non-$*$ entries of a puzzle are "givens"; and a puzzle is said to be "fair" if there is precisely one way to complete it to a valid Sudoku board. Perhaps the most important open problem in the mathematics of Sudoku is the determination of the fewest number $N_{\min}$ of givens in a fair puzzle. It is well known that $N_{\min} \leq 17$, as there are thousands of mutually inequivalent fair puzzles with 17 givens, but none known with 16. Surprisingly, there is essentially no literature providing a lower bound. It is easy to see that that $N_{\min} \geq 8$; J. Cooper, in a recent paper, showed that, in fact, at least 9 givens are necessary. Here we push the lower bound up to 10. The proof consists of a combinatorial classification of puzzles, a careful choice of "Sudoku group" action invariants, and a large computation.

52. A Metacalibration Proof of the Isoperimetric Inequality on Constant Gaussian Curvature Surfaces

Xin Chen Carleton College
Holly Arrowood Furman University
Abraham Frandsen Brigham Young University
Neil Steinburg Brigham Young University
Advisor(s): Michael Dorff, Brigham Young University

A new proof of the isoperimetric inequality on spheres and hyperbolic planes. We use a new method of optimization proof called metacalibration, in which competitors are compared directly to the proposed minimizer via vector fields and the divergence theorem. This approach paves the way to solve open problems such as multiple bubbles and isoperimetric problems with boundary on constant Gaussian curvature surfaces.

53. Tilings with nonconvex pentagons

Ping Ngai Chung Massachusetts Institute of Technology
Advisor(s): Frank Morgan, Williams College

In 2001, Thomas C. Hales proved the Honeycomb Conjecture, which says that regular hexagons provide a least-perimeter unit-area way to tile the plane. Squares and equilateral triangles provide least-perimeter unit-area tilings by quadrilaterals and triangles. It is interesting to ask about a least-perimeter unit-area *pentagonal* tiling, since regular pentagons do not tile the plane. A recent paper by Frank Morgan and his students proves that among all *convex* unit-area pentagonal tilings of the plane and of appropriate flat tori, the Cairo and Prismatic pentagons minimize perimeter. They also conjecture that the convexity assumption is not necessary. We attempt to eliminate nonconvex pentagons by restricting the ratio of convex and nonconvex pentagons, and prove the result for some small flat tori. In the process we prove some bounds on the perimeter of certain classes of pentagons.

54. Triangle Subgroups of Hyperbolic Tetrahedral Groups Redux

Thomas Crawford Williams College
Kelli Burkhardt University of Texas
Advisor(s): Shawn Rafalski, Fairfield University

Maclachlan and Rafalski have classified the triangle subgroups of hyperbolic tetrahedral reflection groups for large classes of hyperbolic tetrahedra. We contribute to this classification of these subgroups and provide further evidence for the conjectural classification of these subgroups through the use of a variety of three-dimensional hyperbolic geometric techniques.

55. On Chasles' Property of the Helicoid in the Tri-Twisted Real Ambient Space

Peter Ho California State University, Fullerton
Lucy Odom California State University, Fullerton
Advisor(s): Bogdan D. Suceava, California State University, Fullerton

An elementary property of the helicoid is that at every point of the surface the following condition holds: $\cot\theta = C \cdot d$, where d is the distance between an arbitrary point to the helicoid axis, and θ is the angle between the normal and the

helicoid's axis. This rigidity property was discovered by M. Chasles in the first half of the XIXth century. Starting from this property, we give a characterization of the so-called tri-twisted metrics on the real three dimensional space with the property that a given helicoid satisfies the classical invariance condition. Similar studies can be pursued in other geometric contexts.

56. Minimal Pentagonal Tilings

Niralee Shah Williams College
Advisor(s): Frank Morgan, Williams College

In 2001, Thomas Hales proved that regular hexagons provide a least-perimeter unit area tiling of the plane, better for example than squares and equilateral triangles, which are minimizing for polygons with three or four sides respectively, and no worse than a mixture of any other shapes. We seek the least-perimeter unit-area tiling of the plane by pentagons.

Work by Frank Morgan and students resulted in a proof that two other pentagons, called Cairo and Prismatic, yield least-perimeter unit-area tilings by convex pentagons. The original version of the paper asked whether there exist tilings by mixtures of these two pentagons. We have found uncountably many such mixtures and classified the doubly periodic ones by their wallpaper symmetry groups. We also consider tilings by mixtures of convex and nonconvex pentagons and perimeter-minimizing tilings on various flat tori.

57. Centers of Magnitude

Thunwa Theerakarn Brown University
Advisor(s): Thomas Banchoff, Brown University

It is a well-known fact in calculus that any group of points of masses, discrete or continuous, possesses a center of gravity. However, there are infinitely many of them which do not have a center of magnitude. A closed star-shaped planar region, for example, has a center of area if and only if it is centrally symmetric. (By center of area, we mean a point such that every line passing through this point cut the area in half.) This is not necessary in the case of its boundary. In his Disquisition on the Center of Magnitude (1751), Roger Joseph Boscovich showed that there exists a non-centrally symmetric planar closed curve that has a center of length - a point such that every line passing through the point divide the length of the boundary in half. In this article, we investigate generalizations in higher dimensions. We show that a closed star-shaped n-manifold embedded in n-dimensional Euclidean space has a center of n-volume if and only if it is centrally symmetric. On the other hand, there exist non-centrally symmetric closed star-shaped n-manifolds embedded in n-dimensional Euclidean space whose (n-1)-dimensional boundaries has a center of (n-1)-volume. We also find a family of examples that could be thought of as a generalization of Boscovich's example.

58. Isoperimetric Sets of Non-Negative Integers

We Ro Jung Gordon College
Advisor(s): Karl Crisman, Gordon College

We studied sets of non-negative integers and their perimeters and volumes. Following Miller et al., the volume of a set is defined to be the sum of all the elements of the set, and the perimeter of a set is defined to be the sum of all the elements whose preceding or succeeding integer is not an element of the set. We investigated the patterns of changes in minimal perimeters of sets as their volumes increased. In particular, we collected a large pool of data for minimal perimeter sets and their volumes, and we proved several lemmas on the way to what we call the Recursive Conjecture, which states that in a minimal perimeter set, the sum of the smallest elements of a set between 0 and s(n) is less than s(n) itself, where s(n) is the smallest element of the consecutive part of the set that contains the largest element of the set.

59. Numerical Methods for Poisson-Nerst-Planck Equations with Applications in Ion Channels

Michael Machen Illinois Institute of Technology
Advisor(s): Xiaofan Li, Illinois Institute of Technology

Poison-Nerst-Planck (PNP) equations are found in several areas of research such as: electrochemistry, electrophysiology, molecular biophysics, semiconductors. The PNP equations can be used to model the flows inside ion channels

which are driven by a combination of electrical potentials(ϕ) and charge concentrations(c_i). These potentials and charges within the channel can be modeled by this PNP equation:

$$\frac{\partial c_i}{\partial t} = \nabla \cdot \left\{ D_i \left[\nabla c_i + \frac{c_i}{k_B T} z_i e \nabla \phi \right] \right\} \nabla \cdot (\epsilon \nabla \phi) - (\rho_0 + \sum_i z_i e c_i),$$

where we have the diffusion coefficients (D_i),the Boltzmann constant($k_B T$), and the physical constants(z_i,e,ϵ).
The research plan is to develop efficient numerical methods for solving the PNP equations accurately while investigating the conservation properties of the existing numerical methods for the time integration.

60. Mean-reverting pricing models

Jody Shipp George Mason University
Advisor(s): Harbir Lamba, George Mason University

We examine changes to a standard pricing model used in finance and economics. This model is a mean-reverting (Ornstein-Uhlenbeck) stochastic process where fluctuations in supply and demand occur but a drift pushes the price back towards a mean value, giving rise to a Gaussian price distribution.
Hysteresis refers to memory-dependent or non-reversible effects. For example, an agent may switch their investment position due to a price change but, in the presence of non-zero transaction costs, they will not immediately switch back if the price change reverses. We add hysteretic economic agents to an Ornstein-Uhlenbeck process and numerically simulate the system using the Euler-Maruyama method. We then compare the statistics of Ornstein-Uhlenbeck processes with and without hysteresis-type effects.

61. Pebbling on a Graph with a High Degree of Symmetry

Herman Sameisky Cornell University
Advisor(s): Gene Fiorni, Rutgers University

Graph pebbling is a game played with pebbles placed in an arrangement on the vertices of a graph. A pebbling move is defined as the process of taking two pebbles from one vertex, removing one pebble from the graph, and placing the other pebble on an adjacent vertex. The ith pebbling number, $\pi_i(G)$, is defined to be the least number of pebbles such that for any arrangement of $\pi_i(G)$ pebbles and any vertex, v, there exists a sequence of pebbling moves that results in i pebbles being placed on v. We let $\pi(G) = \pi_1(G)$. A specific pebbling arrangement $P : V(G) \to \{x | x \text{ is an integer and } 0 \leq x \leq \pi(G)\}$, is said to be solvable if a pebble can be moved to any vertex. The problem of pebbling is at least as hard as the class of NP complete problems. [G. Hurlbert and H. Kierstead] Furthermore Graham's Conjecture, GC, which states that for any graphs G and H, $\pi(G \times H) \leq \pi(G)\pi(H)$ and has been verified in specific instances, has yet to be verified in general. We prove GC for a product of a specific graph with a high degree of symmetry with itself and then extend this result to a family of graphs that include the product of this graph with itself. Our method for proving that the specific graph product satisfies Graham's conjecture involves showing that a specific subgraph of the graph has $\pi_2(G) < 25$.

62. Discrete Models with Proportional Harvesting

James Cameron University of Texas at Austin
Luis Granera Vega Yale University
Peter Jaworski Central Michigan University
Loren Santana Skidmore College
Advisor(s): James Angelos, Central Michigan University

Difference equations used to model populations are analyzed, and we show how to control stability and induce chaos using proportional harvesting. In particular, we use the Beverton-Holt and Ricker models with proportional harvesting to demonstrate that a harvested system can be more stable than its unharvested counterpart. We also use digraphs to analyze the periodic structure of continuous functions, and we provide necessary and sufficient conditions for a digraph to support a continuous or piecewise monotone function.

63. Multivariate Differentiation Methods within the HOL Light System

Nicholas Volker University of Pittsburgh
Advisor(s): Thomas Hales, University of Pittsburgh

It is the focus of my work to develop differentiation tools for the HOL Light proof assistant, to be used in the formal proof of the Kepler Conjecture. I have finished formally proving the derivatives for the basic primitive functions needed for the proof, and am working on coding methods for differentiation of functions of multiple variables, and on methods for calculating second derivatives. All of the basic differentiation rules have been coded within our 'differentiate' method (originally developed by Dr. Hales and John Harrison), such as the chain rule. Using these methods and the known derivatives of certain primitive functions (sine, cosine, arctangent, etc.) any function necessary to the Kepler conjecture can be reduced to some product/sum/composition of these, possibly in multiple variables. The goal is to have all of the tools within the Flyspeck project to automatically accumulate all of the information necessary to produce second order Taylor expansions, which are used extensively in the computer calculations for the proof of the Kepler conjecture.
HOL Light code: http://code.google.com/p/hol-light/ Flyspeck Project code: http://code.google.com/p/flyspeck/ General materials on the Kepler conjecture: https://sites.google.com/site/thalespitt/kepler-conjecture

64. Independence polynomials of regular caterpillars

Michaela Stone Alfred University
Gregory Ferrin Western Carolina University
Advisor(s): Patrick Bahls, UNC-Asheville

We offer a basic background of independence polynomials and their importance in graph theory. Specifically, we look at the independence polynomials of regular caterpillars. We will introduce a closed-form of the standard recursive formula for the independence polynomials of regular caterpillars of any finite size. We will then apply our formula to identify the location of the modes of the independence polynomials of regular caterpillars.

65. An Introduction to Dessins d'Enfants: The Intersection of Graph Theory, Group Theory, and Differential Geometry

Anika Rounds Purdue University
Advisor(s): Edray Goins, Purdue University

Suppose there are three cottages, and each needs to be connected to the gas, water, and electric companies. Using a third dimension or sending any of the connections through another company or cottage are disallowed. Is there a way to make all nine connections without any of the lines crossing each other? To answer such a question, we explore the properties of planar graphs. It is natural to generalize to graphs which can be embedded into Riemann surfaces, such as the sphere and the torus. In this talk, we discuss how to draw such graphs using Grothendieck's concept of a Dessin d'Enfant. This is based on joint research with Edray Goins through the Zoltners Summer Undergraduate Research Fellowship.

66. Competitive Color Graphing

Blanche Ngo Mahop Howard University
Advisor(s): Leon Woodson, Morgan State University

Abstract: Imagine your best friend is getting married to your ex. It's probably safe to say you are no longer best friends anymore. Your best friend decides to invite you to the wedding and asks for your help. Your job is to place everyone around the table in a girl/boy order. Since you are upset that your 'best friend' is marrying your ex you don't do your job affectively so you may 'accidently'place two girls or guys by each other. In fact, Competitive Color Graphing is a way of coloring the vertices of a graph such that no two adjacent vertices share the same color; this is called a vertex coloring. Similarly, an edge coloring assigns a color to each edge so that no two adjacent edges share the same color, and a face coloring of a planar graph assigns a color to each face or region so that no two faces that share a boundary have the same color.

67. Analysis of Diatoms and Arcellacean death assemblages to determine inclusion or exclusion of rare event data in comparison with various missing data analyses techniques

Edgar Aquino University of Houston- Downtown
Cori Ali University of Houston- Downtown
Ilian Rojas University Houston-Downtown
Advisor(s): Edwin Tecarro, University of Houston- Downtown

In this project, diatom and arcellacean death assemblages were studied to determine if trends exist in the succession of a mitigation wetland ecosystem in Greens Bayou Wetland Mitigation Bank compared to a stable wetland ecosystem in the Anahuac National Wildlife Refuge. The statistical package EstimateS (Statistical Estimation of Species Richness and Shared Species from Samples) was used to calculate the various statistical indices used to determine alpha and beta diversity between GBWMB and ANWR. EstimateS was used to analyze the raw data and alternatively, the rare species in the diatom and arcellacean assemblage were removed using the theory of listwise deletion and then analyzed by EstimateS. The comparison of traditional missing data analyses such as listwise deletion, partial listwise deletion, imputation, and partial imputation via SAS to the inclusion and exclusion of rare genera results used by EstimateS were compared to analyze the impact of the rare genera of the diatom and arcellacean death assemblage. This missing data analysis shows that the inclusion of rare genera is important to the succession signal in the mitigation wetland.

68. Optimizing Algorithm for Reliability Assessment of Radial Lifeline Systems

Lynette Guzman University of Arizona
Advisor(s): Javier Rojo, Rice University

A prominent reason for finding efficient methods to quantify reliability of radial lifeline systems may be attributed to the susceptibility of the system to large scale failure when a single line segment in the system fails. Proposed methods include Monte Carlo simulation techniques and probabilistic recursive algorithms, which are traditionally limited in their computational efficiency, accuracy, and full analysis of the general case radial lifeline system. This study proposes an algorithm for calculating the complete probability distribution of customer service availability (CSA) for the general case for radial lifeline systems, and explores the sensitivity of components to large scale failure.

69. Genome-Wide Association Study of Inbreeding and Long Runs of Homozygosity

Yue Shao Bryn Mawr College
Advisor(s): Bruce S. Weir, University of Washington

Inbreeding is a key concept in biostatistics, as the usual SNP-disease associations tests are rendered invalid if the observed genotypes come from genetically related parents. This study explores the association between inbreeding and long runs of homozygosity (ROH). We created a model for ROH using the Markov chain and calculated the expected lengths. We found the first order Markov chain is a good model as the distribution of expected lengths of ROH resembles that of the observed lengths. We also tested the inbreeding coefficient against measures of homozygosity and found that the maximum LOD score is better correlated to an individual's inbreeding coefficient than the proportion of homozygosity and the length of the longest ROH.

70. Distribution of Eigenvalues of Weighted, Structured Matrix Ensembles.

Karen Shen Stanford University
Advisor(s): Steven J. Miller, Williams College

The study of the distribution of eigenvalues of large random matrices has many applications (nuclear physics, number theory). Previous work has determined the limiting spectral measures for many matrix ensembles, famously the semicircle for the real symmetric matrices, but also more structured ensembles such as the Toeplitz and circulant matrices. We introduce a parameter p to continuously interpolate between such structured ensembles and the real symmetric ensemble by multiplying each entry by $\epsilon_{ij} = \epsilon_{ji} = \pm 1$ where $p = \mathbb{P}(\epsilon_{ij} = 1)$. For $p = 1/2$, we prove the limiting measure is the semicircle. For all other p, we prove the measure has unbounded support. The proofs are by Markov's Method of Moments where the moment analysis involves analyzing pairings of vertices on a circle. We prove that the contribution of each pairing is weighted by a factor depending on p and m, the number of vertices in crossing pairs. The number of pairings with no crossings ($m = 0$) is well-known as the Catalan numbers; we

discover and prove similar formulas for $m = 4, 6, 8$ and 10 and find closed-form expressions for the expected value and variance. As the variance converges to 4, these results yield significant information about the limiting measure.

71. Invariant Subspaces and Group Representation of Matrices over $\mathbb{F}_p$

Chenxi Cai Brown University
Advisor(s): John McCleary, Vassar College

Our study focused on the existence of non-trivial invariant subspaces of invertible linear transformations from a vector space to itself over the finite field $\mathbb{F}_p$. In our research, we transformed the problem to one about partially ordered sets, here defined by the set of proper subspaces of the vector space with the order relation of inclusion. We studied the homology groups of the order complex of this poset. We employed methods in algebraic topology, such as the Hopf trace theorem and the Lefschetz fixed point theorem, which led us to methods in representation theory, such as the Steinberg character. By combining these tools, we came to our main result that, given an invertible matrix over $\mathbb{F}_p$, if p divides its order, or equivalently, if it is diagonalizable over an extension field of $\mathbb{F}_p$, there must be a non-trivial invariant subspace associated with the matrix.

72. Investigation of 7-edge Polygonal Chains, Stuck Unknots and Stuck Trivial Chains

Susannah Coates Metropolitan State College of Denver
Advisor(s): John Carter, Metropolitan State College of Denver

The space of all piecewise linear chains, Ch_n, without length restrictions is trivial, that is all knotted chains may be un-knotted and straightened. However, when length restrictions are included, the space $\mathrm{Ch}_n(l_1, ..., l_n)$ has a variety of interesting topological properties. Using physical modeling and topological methods, we investigate the equivalence classes and properties of the space of 7-edge chains, $\mathrm{Ch}_7(l_1, ...l_7)$. We also investigate "stuck" unknots, loops which contain no knots and yet are not isotopic to a convex polygon; and "stuck" trivial chains, chains which contain no knots and yet are not isotopic to a straight trivial chain.

73. Analyzing the Structure of the Coefficient Space for Polynomial Knot Families

Dayna Goehringer Adelphi University
Anthony Del Latto Adelphi University
Steven Roveto Adelphi University
Advisor(s): Lee Stemkoski, Adelphi University

A polynomial knot is the image of a parametric function whose components are polynomials. For a parameterization of fixed degree, the coefficient space is the set of values of coefficients of the polynomials that yield polynomial knots. In particular, we investigate polynomials of degrees 4,5, and 6 and classify the types of knots that are produced. We also discuss related families of higher-degree equations and present a functional interpretation of Reidemeister moves.

74. Reduced Colored Khovanov Homology of Alternating Knots

Lisa Piccirillo Boston College
Advisor(s): Eli Grigsby, Boston College

Khovanov homology associates to a knot a bigraded chain complex whose homology is a knot invariant (i.e., independent of the particular diagram used to describe the knot). Moreover, Khovanov homology categorifies the classical Jones polynomial in the sense that the graded Euler characteristic of the Khovanov homology of a knot yields the Jones polynomial of the knot. It was proved by Manolescu and Ozsvath that the Khovanov homology of alternating (and, more generally, quasi-alternating) knots is homologically 'thin', hence determined by the Jones polynomial and the signature. In *Categorifications of the Colored Jones Polynomial*, Khovanov describes a generalization of his construction categorifying the reduced colored Jones polynomial. A natural question to consider is whether Khovanov's reduced colored Khovanov homology is also homologically thin on alternating knots. We give a number of counterexamples to this question, obtained computationally using a Mathematica program we developed, building upon Dror Bar-Natan's FastKh algorithm contained in the open source KnotTheory package.

75. The Bollobas-Riordan-Whitney-Tutte Polynomial and Ribbon Graph Operations

Timothy Spencer UNC Asheville
Advisor(s): Neal Stoltzfus, LSU

Given a ribbon graph $\mathbb{D}$ and a collection of pointed ribbon graphs $\mathbb{M}_r$ for each edge of $\mathbb{D}$, Farmer has defined the generalized iterated parallel connection and iterated two sum. Essentially, this replaces each edge in $\mathbb{D}$ by the chosen ribbon graph (with the edge deleted for the two-sum). Brylawski developed these ideas of series and parallel connections in graph theory and found formulae for their Tutte polynomial (essential for computational complexity results on the Jones polynomial). We develop new techniques that lead to a formula for the topological rank polynomial of Bollobas-Riordan-Whitney-Tutte for ribbon graphs and corresponding formulae for other operations. The explicit formulae are expressed in terms of the three polynomial constituents of the pointed ribbon graph polynomials of Farmer and a decomposition of the base ribbon graph $\mathbb{D}$.

76. Computing the First Higher Order Alexander Polynomial of a Knot

Josh Tobin Columbia University
John Schultz Columbia University
Eden Prywes Columbia
Advisor(s): Peter Horn, Columbia University

Given a knot $K \subset S^3$, Cochran showed how to define the n^{th} higher-order Alexander module of K. These modules are obtained by studying successively deeper terms in the derived series of the fundamental group of K. In analogy with the usual Alexander module, one can define a sequence of higher-order Alexander polynomials $\Delta_n(K)$, the degrees of which yield successively better bounds on the Seifert genus of K. Building on existing work of Holum, we establish and implement an algorithm to compute the degree of the first higher-order Alexander polynomial for all knots of crossing number less than or equal to ten.

77. Linking Number of a Linear Embedding with Two Components

Julie Wasiuk California State University, San Bernardino
Advisor(s): Rolland Trapp, California State University, San Bernardino

The linking number, the crossing number, and stick number of a linear embedding with m-vertices is defined. The linking number of any graph with two components on m-vertices is shown to have a maximum related to the number of vertices on the graph. This allows for maximization of linking number and a solid base with which we can prove that the difference between the Ramsey number and the stick number goes to infinity as m goes to infinity.

78. Vertex-Transitive Directed Strongly Regular Graphs

Chetak Hossain University of California, Berkeley
Angelica Gonzalez Whittier College
George Shakan Worcester Polytechnic Institute
Charles Watts Morehouse College
Advisor(s): Sung-Yell Song, Iowa State University

Two infinite families of vertex-transitive directed strongly regular graphs are described. Each graph in the first family is a point-block combinatorial object constructed from primitive roots of $GF(p)$. Each graph in the second family is obtained from the flags of a symmetric design derived from the quadratic residues of $GF(p)$. A transitive automorphism group of each of these graphs and the rank of the transitive permutation group on its vertex set are determined. The structure of the automorphism groups is a semidirect product of cyclic groups. The relationship between these directed strongly regular graphs and the orbital graphs of their transitive permutation groups are discussed. In particular, the 2-orbits (or orbitals) of the vertex-transitive permutation group of the first family form a non-commutative association scheme of class 5. This yields an infinite family of 5-class nonsymmetric association schemes. From this family, we find a new way to construct Johnson graphs, and in particular the Petersen graph. For our second family, we find that, in most cases, the automorphism groups are sharply transitive.

79. The cubic formula in characteristic 3

Dubravka Bodiroga Hood College
Advisor(s): James Parson, Hood College

Cardano's formulas express solutions of a cubic equation $x^3 - a_1x^2 + a_2x - a_3 = 0$ in terms of its coefficients a_1, a_2, and a_3 by extracting square root and cube roots. However, Cardano's method does not work in characteristic 3, where $3 = 0$, because the formulas require dividing by 3 and because cube roots behave badly in characteristic 3. The proper replacements for cube roots in characteristic 3 are solutions y to "Artin-Schreier" equations $y^3 = y + b$. I will explain how to construct a characteristic 3 replacement for Cardano's formulas that solves cubics in terms of Artin-Scheier equations. I use the method of Euler and Bézout, and analyze the solution in the style of Lagrange.

80. Monoids Defined by Second Order Recurrence Relations

Trey Brock University of Central Missouri
Advisor(s): Nicholas Baeth, University of Central Missouri

We investigate certain properties of monoids defined by linear equations in three variables whose coefficients are consecutive terms of a sequence defined by a second order recursive relation. In particular, we classify all irreducible elements of these monoids and give a measure by way of computing certain invariants I of how they are from having unique factorization.

81. The Probability of Randomly Generating Finite Abelian Groups

Tyler Carrico Indiana Wesleyan University
Advisor(s): Melvin Royer, Indiana Wesleyan University

Let $G = \mathbb{Z}_{p^m} \oplus \mathbb{Z}_{p^n}$ where $m, n \in \mathbb{N}$ and p is prime, and let A be the event where two random elements are chosen from G without repetition that together generate G. Following an overview of earlier results in related research, we prove the formula for the probability of A occurring, and extend this result to groups of the form $\mathbb{Z}_{p^{n_1}} \oplus ... \oplus \mathbb{Z}_{p^{n_k}}$. Future research possibilities are considered.

82. The Product Field of Values

Daniel Corey University of Notre Dame
Ryan Kirk University of North Carolina at Chapel Hill
Advisor(s): Charles Johnson, The College of William and Mary

The classical field of values for a matrix $A \in \mathbb{C}^{n\times n}$ is the set

$$F(A) = \{x^*Ax | x \in \mathbb{C}^n, x^*x = 1\}$$

This is a compact convex subset of the complex plane. A natural generalization arises when considering two matrices $A, B \in \mathbb{C}^{n\times n}$, namely

$$Q_{k,l}(A, B) = \{(x^*Ax)^k(x^*Bx)^l | x \in \mathbb{C}^n, x^*x = 1\}$$

where k and l are integers. It is understood that $F(A)$ $(F(B))$ does not contain the origin when $k < 0$ $(l < 0)$. Unlike the field of values, $Q_{k,l}$ is rarely convex. For $|k| + |l| > 2$, $Q_{k,l}(A, B)$ may not even be simply connected. Recently, the set $Q_{1,-1}(A, B)$ (which we call the ratio field $R(A, B)$) was found to be simply connected. In fact, the ratio field satisfies a property called ray convexity that is stronger than simple connectivity. A set S in the plane is said to be ray convex if for each point p not in S there is a ray emanating from p not intersecting the set S. We will complete the description of $Q_{k,l}(A, B)$ with respect to simple connectivity by showing that the product field of values $P(A, B) = Q_{1,1}(A, B)$ is ray convex.

83. Four Dimensional Tops

Ryan Davis University of Wisconsin- Eau Claire
Advisor(s): Ursula Whitcher, University of Wisconsin- Eau Claire

The polar duality transformation takes any polytope with vertices in an integer lattice to its polar dual. We call a lattice polytope reflexive if its polar dual is also a lattice polytope. Reflexive polytopes have been classified in 3D

and 4D, with 4,319 and 473,800,776 classes of equivalent polytopes respectively. A related object is a top, which is a lattice polytope with one facet containing the origin. Bouchard and Skarke have classified the 3D tops corresponding to each class of reflexive 2D polytope shadow, and described the connections between tops and elliptic fibrations and string theory. An important result of the analysis was that they showed that there are infinitely many families of 3D tops. We identify infinite families of 4D tops generated from 3D reflexive polytopes.

84. The Rook Brauer Algebra

Elise delMas Macalester College
Advisor(s): Tom Halverson, Macalester College

We define an algebraic structure called the rook Brauer algebra $\mathbf{RB}_k(n)$, which has a basis of rook-Brauer diagrams and a multiplication given combinatorially by diagram concatenation. Rook-Brauer diagrams are in bijection with the partial matchings on a set of size $2k$, and thus the dimension of $\mathbf{RB}_k(n)$ is $\sum_{\ell=0}^{k} \binom{2k}{2\ell}(2\ell-1)!!$. Furthermore, $\mathbf{RB}_k(n)$ contains, as subalgebras, the symmetric group algebra, the Brauer algebra, and the rook monoid algebra. We construct a matrix representation of $\mathbf{RB}_k(n)$ on a tensor space $V^{\otimes k}$, where $\dim(V) = n + 1$, and show that it is faithful. We then show that $\mathbf{RB}_k(n)$ commutes with the group $O_n(\mathbb{C})$ of $n \times n$ complex orthogonal matrices acting on $V^{\otimes k}$. Using this duality with the orthogonal group, we build a graph which describes the structure of $\mathbf{RB}_k(n)$.

85. Modular representations of Cherednik algebras associated to symmetric groups

Sheela Devadas Lexington High School
Advisor(s): Steven Sam, MIT

We study lowest weight irreducible representations of rational Cherednik algebras attached to the symmetric group S_n in characteristic p, focusing specifically on the case $p \leq n$, which is more complicated than the case $p > n$ (since S_n-modules are not semisimple). The goal of our work is to calculate characters (and in particular Hilbert series) of these representations. By studying the kernel of the contravariant bilinear form on Verma modules, we proved formulas for Hilbert series of irreducible modules in a number of cases, and also obtained a lot of computer data which suggests a number of conjectures. Specifically, we find that the shape and form of the Hilbert series of the irreducible representations and the generators of the kernel tend to be determined by the value of n modulo p.

86. Infinitesimal Cherednik Algebras of $\mathfrak{gl}_n$

Fengning Ding Phillips Academy, PRIMES Research Program
Advisor(s): Oleksandr Tsymbaliuk, MIT

Infinitesimal Cherednik algebras $H_c(\mathfrak{gl}_n)$ are deformations of $U(\mathfrak{sl}_{n+1})$ and continuous analogues of rational Cherednik algebras. Despite these connections with other areas of representation theory, infinitesimal Cherednik algebras are not widely-studied, and basic questions of intrinsic algebraic and representation theoretical nature remain open. In this study, we construct the complete center of $H_c(\mathfrak{gl}_n)$ for the case of $n = 2$ and give one particular generator of the center, the Casimir operator, for general n. We find the action of this Casimir operator on the highest weight modules to compute the Shapovalov determinant, which gives a criterion for the irreducibility of Verma modules. From the reducible Verma modules, we construct finite dimensional representations and compute their characters. We classify all finite dimensional representations and prove the existence of all finite-dimensional irreducible representations in the classification, showing that the representation theory of infinitesimal Cherednik algebras has a much richer combinatorial structure than that of undeformed Lie algebras.

88. The number of nonzero coefficients of powers of a polynomial over a finite field

Caroline Ellison MIT
Advisor(s): Giorgia Fortuna, MIT

Coefficients of polynomials over finite fields often encode information that can be applied in various areas of science; for instance, computer science and representation theory. The purpose of this project is to investigate these coefficients over the finite field F_p. We use Stanley's matrices to find what we conjecture to be an approximation for the sum over n of the number of nonzero coefficients of $P(x)^n$ over F_p. This leads to questions in representation theory and combinatorics. We hope for further research in this area to find a relationship between the number of nonzero coefficients in the expansion of a polynomial to the n^{th} power and the digits of n base p.

89. Classification of the T-avoiding permutations and generalizations to other Coxeter groups

Zach Goldenberg Plymouth State University
Joseph Cormier Plymouth State University
Jessica Kelly Plymouth State University
Advisor(s): Dana C. Ernst, Plymouth State University

We say that a permutation w has property T if there exists i such that either $w(i) > w(i+1), w(i+2)$ or $w(i+2) < w(i), w(i+1)$. A permutation w is T-avoiding if neither w or w^{-1} have property T. We will present a classification of the T-avoiding permutations in the symmetric group, which is a Coxeter group of type A. In addition, we will discuss generalizations to other Coxeter groups and classify the T-avoiding elements in Coxeter groups of types B.

90. Presentation of the Motzkin Algebra

Kristofer Hatch University of California at Santa Barbara
Megan Ly Loyola Marymount University
Eliezer Posner CUNY City College of New York
Advisor(s): Stephen Bigelo, University of California at Santa Barbara

In 2011, Halverson introduced the Motzkin algebra, a fascinating generalization of the Temperley-Lieb algebra, whose elements are diagrams that can be multiplied by stacking one on top of the other. Halverson gave a diagrammatic algorithm for decomposing any Motzkin diagram into diagrams of three subalgebras: the right planar rook algebra, the Temperley-Lieb algebra, and the left planar rook algebra. We first explored the right and left planar rook subalgebras, proving that their cardinalities are Catalan numbers. We found presentations for these algebras by generators and relations, using a counting argument to prove that our relations suffice. We then turned to the newly developed Motzkin algebra, where we described Halverson's decomposition algorithm algebraically and found a presentation by generators and relations. We justified our presentation using a counting argument, but with a more sophisticated algorithm.

91. Classifying Patterns for Commutative Matrix Completions

Angela Kraft Bethany Lutheran College
Jennifer Aguayo UC-Santa Barbara
Advisor(s): Geoffrey Buhl, CSU-Channel Islands

Matrix Completion Problems explore whether partial matrices having some unspecified entries can be completed in a strategic way so that the partial matrix has a certain property. We focused on the question: under what circumstances can the unspecified entries of a partial matrix X be chosen so that X commutes with a fully specified matrix A? Or, when can X be completed so that AX=XA? Using a Polynomial Approach and a Matrix Equation Approach used in earlier research as well as a less developed Graph Theory Approach, our research group created and proved a Classification Theorem. The theorem gives all the admissible patterns of specified entries for a partial matrix X that allow it to be completed to commute with a matrix A when A is a Jordan Block. Using the resulting admissible patterns from the Classification Theorem, admissible patterns for partial matrices that commute with a permutation similar matrix to a Jordan Block can be constructed. Using some previous research done by our advisor, we also created admissible patterns for matrices that are the direct sum of partial matrices. This research on commutative matrix completion was conducted over the summer of 2011.

92. The Effects of Varying Conditions on a Fibonacci-Type Polynomial Sequence

Rebecca Miller University of Central Oklahoma
Advisor(s): Kristi Karber, University of Central Oklahoma

Consider the following Fibonacci-type polynomial sequence given by the recurrence $G_0(x) = -a$, $G_1(x) = x - a$, $G_n(x) = x^k G_{n-1}(x) + G_{n-2}(x)$ where $n \geq 2$ and $a > 0$. Let $g_{a,n}$ represent the maximum real root of $G_n(x)$. In this poster, we will display asymptotic results for $g_{a,n}$ numerically, as well as, analytically for the case $k = 2$. We will also explore $g_{a,n}$ for positive integers k where $k > 2$.

94. Galois Groups of Iterates of Cubics

Matej Penciak University of Rochester
Advisor(s): Thomas Tucker, University of Rochester

Fix a polynomial of the form $f(x) = g(x) - t$ in $\overline{\mathbb{F}_p}(t)[x]$ where $g(x) \in \overline{\mathbb{F}_p}[x]$ We are interested in the Galois group of the splitting field of $f^2(x)$ over $\overline{\mathbb{F}_p}(t)$. Because the roots of $f^n(x)$ have a natural tree structure associated to them, we build up the Galois group for $f^2(x)$ from $f(x)$ and relate it to the orbits of critical points of $f(x)$ via ramification of prime ideals in intermediate field extensions.

95. Growth of Groups Defined by Automata

Aaron Reaves Morehouse College
Ashley Dougherty Kutztown University
Lydia Kindelin University of Dayton
Andrew Walker York College
Advisor(s): Dan Farley, Miami University

Nekrashevych conjectured that iterated monodromy groups of quadratic polynomials with a pre-periodic kneading sequence have intermediate growth. In this paper we prove intermediate growth for two such groups. We also provide a new proof of intermediate growth for a class of groups containing the Grigorchuk group, previously known to have intermediate growth.

96. Square Roots and Model Characters in the Symmetric Group

Michael Reeks Macalester College
Advisor(s): Tom Halverson, Macalester College

Let σ be a permutation of $\{1, 2, \ldots, n\}$. A square root of σ is a permutation π such that $\pi^2 = \pi \circ \pi = \sigma$. We give a formula for the number of square roots of a given $\sigma \in S_n$ as a function of the cycle type of σ. This result has surprising connections to the representation theory of the symmetric group S_n; namely, it gives the trace of a model matrix representation of σ. This representation has a basis of involutions, which are the $t \in S_n$ such that $t^2 = 1_{S_n}$, the identity permutation in S_n. Using this connection, we rewrite our formula as the sum of two statistics on involutions, the Saxl and Roichman weights, which come from the representation theory of S_n. In particular, we give combinatorial proofs that the sums of these statistics also give the model character of S_n: the sum of the irreducible characters of S_n, each appearing exactly once, evaluated at σ.

97. Invariant Polynomials under Symmetric Group Action

Aaron Rodriguez University of Saint Thomas
Cole Stiegler St. Olaf College
Eric Chen Princeton University
Advisor(s): R. Michael Howe, University of Wisconsin - Eau Claire

Let V be a vector space and let $\mathcal{P}(V)$ be the polynomials on V. The symmetric group, S_n acts on V and so also on $\mathcal{PD}(V)$, the polynomial coefficient differential operators on $\mathcal{P}(V)$. We are seeking to understand the operators in $\mathcal{PD}(V)$ that commute with the action of S_n, which leads us to investigate polynomials that are invariant under action by the Symmetric Group, S_n. Such polynomials are generated by a set of elementary symmetric polynomials. Given the set of elementary symmetrics from the space of polynomials with variables from one vector space, V, we explore the nature of an analogous set of elementary symmetric polynomials from the space of polynomials with variables from both the vector space V and its dual space V^*. In particular, we explain various techniques for counting the number of invariant polynomials in a given space. Such techniques result from the relationship between properties of young diagrams and the group structure of S_n.

98. Bounding the Degree of Regularity of a Hidden Fields Equation System

Jacob Schlather University of Cincinnati
Advisor(s): Timothy J Hodges, University of Cincinnati

We find a bound on the degree of regularity of a Hidden Fields Equation (HFE) system that proves to be strict in the quadratic case when the field is of prime order. Our study of the degree of regularity is motivated by work of Dubois and Gamma which gives strong evidence that Faugere's F5 algorithm terminates near the degree of regularity when

breaking HFE. Assuming this result to be true our bound shows that the complexity of a generalized HFE system is quasi-polynomial.

99. Zero Divisor Graphs

Alonza Terry Xavier University of Louisiana
Advisor(s): Mark Davidson, Louisiana State University

We define and study the zero divisor graphs associated to a ring R, consisting of those by I.Beck [B] and D. Anderson & P. Livingston [AL]. In particular, the Anderson and Livingston [AL] zero divisor graphs were of primary interests. We constructed the [AL] graph $\Gamma(R)$ for $R = (\mathbb{Z}_n)$ up to order n, for $n \leq 100$.

100. Eventual Positivity in Classes of Graphs

Charles Watts Morehouse College
Advisor(s): Ulrica Wilson, Morehouse College

A square matrix $A = (a_{ij})$ is positive, denoted $A > 0$ if $a_{ij} > 0$ and eventually positive if there exists k_0 such that for all $k \geq k_0$, $A^k > 0$. Eventually positive matrices were introduced in 1978 by S. Freidland. We present some results about the eventual positivity of adjacency matrices of several classes of graphs including cycles and zero divisor graphs.

101. Zero Divisor Graphs of Inverse Systems of Abelian Groups

Cynthia Wu Gonzaga University
Advisor(s): Thomas McKenzie, Gonzaga University

Let S be a commutative semigroup with zero ($0x = 0$ for all $x \in S$). F.R. DeMeyer, T. McKenzie, and K. Schneider (2002) associated a graph to S. The vertices are the nonzero elements of S, with two vertices a, b joined by an edge in case $ab = 0$. We associate a commutative semigroup with zero to any inverse system of abelian groups over a poset with least element. Then we use the associated graph to construct a number of classical graphs.

102. Trees Fixed by Permutations and their Applications to Tree Isomorphism Classes and Other Combinatorial Objects

George Arzeno MIT
Advisor(s): Alexander Postnikov, MIT

We present an explicit closed formula for the number of trees fixed by a permutation of vertices. This formula generalizes Cayley's famous formula n^{n-2} for the number of trees on n labeled vertices. Another special case is the formula $(2n+1)^{n-1}$ for the number of symmetric trees on $2n+1$ vertices. We derive a closed formula for the number of unlabeled trees by applying Burnside's lemma. We also discuss related combinatorial objects, such as parking functions and their generalizations. The proofs are based on bijective methods.

103. A measure of variance between the distribution of first digits from samples of Hill's mixtures and of Benford's Law

Marta Calitoiu Lisgar Collegiate Institute
Annie-Claude Beland SCM School of Competitive Math
Advisor(s): Dragos Calitoiu, Carleton University

First, we explore the distributions of numbers from two different areas, namely from Physics and from Journalism, in order to verify if they confirm the behaviour of Benford's Law. Then, as suggested by Hill's proof of Benford's Law, we generate a mixture of three distributions of numbers that can be found in nature, namely uniform, normal, and exponential. We also intend to explore numbers that cannot be found in nature and thus, we generate a second mixture by adding a triangular distribution to the normal and exponential ones. Next, by taking random samples with different sizes from these two mixtures, we produce distributions of the first digit from these numbers. Finally, we propose an entropy-like measure in order to quantify the variance between these distributions and the theoretical distribution of Benford's Law obtained with $lg[1 + (1/D)]$ from each digit D.

104. On an Inverse Problem of Parameter Identification in Compressible and Incompressible Linear Elasticity

Erin Crossen Rochester Institute of Technology
Selin Sariaydin Rochester Institute of Technology
Advisor(s): Akhtar A. Khan, Rochester Institute of Technology

This poster will present optimization based approaches for solving the inverse problem of identifying the variable elasticity parameters in the system of linear elasticity. Both compressible as well as incompressible linear elasticity model will be considered. The main emphasis will be on the use of a new convex objective functional to identify the variable parameters. Finite element based numerical simulations will be presented.

106. Eigenvalues and Eigenfunctions of the Laplacian on Isotropic Quantum Graphs

Patrick King College of William and Mary
Advisor(s): Junping Shi, College of William and Mary

We present solutions of the Laplacian eigenvalue problem on several simple quantum graphs. Three basic graph structures are defined; we primarily consider the case when all edges of the graph in question are of the same length, or isotropic. Isotropic graphs provide a symmetry which can be exploited to calculate the solutions. Eigenvalues and eigenfunctions for these isotropic structures are characterized; anisotropic structures are also examined. Additional properties of eigenvalues and eigenfunctions on these graphs are demonstrated.

107. Characteristics of Non-Symmetric Edge Flames in Micro-Channels

Casey McGrath University of Redlands
Advisor(s): Joanna Bieri, University of Redlands

Research on the understanding and application of micro-combustors is growing due to the potential scientific and industrial uses. The enticements of smaller engines, better batteries, and more efficient heat sources are drawing scientists toward seeking a better understanding of the fundamental physics and mathematics behind the flames themselves. Experimentally, edge-flames are found in the narrow channels of non-premixed micro-combustors. Finite difference and Euler time step methods are used to solve a system of coupled non-linear partial differential equations that model the flame under a constant density assumption. Specifically, I am studying the characteristics of non-symmetric edge-flames, those whose ratio of fuel to oxidizer doesn't meet perfect stoichiometric proportions, and the effects of heat losses on these flames. Thus far, my research has given me extinction points for these non-symmetric edge flames, and has shown that flame position and shape are significantly changed due to the effects of heat loss and mixture stoichiometry. A long term goal of this research is the creation and understanding of a complete and accurate model that both experimentalists and theorists can use to accurately predict the behavior of flames under specific conditions.

108. The Alignment of Arbitrary Contours Using Area Difference Distance Measurement

Karen Murata CSU Stanislaus
Jessica DeSilva CSU Stanislaus
Advisor(s): Jung-Ha An, CSU Stanislaus

Innovation in the process of medical imaging relies heavily on the metric function that outputs the distance between a fixed and arbitrary set of anatomical contours. The distance function presented is the area difference distance measurement, taking into account two sets containing data points which represent their respective contours. We have produced a formal proof which demonstrates the validity of the presented area difference distance measurement. In order to accurately measure distance using this function, Procrustes method is applied by aligning the arbitrary contour to a fixed contour. This optimal alignment requires minimizing the distance function in terms of rotating, scaling, and translating the arbitrary contour. The succeeding proof presented validates that these values, which optimize the distance between two contours, can be found with the sets of data points representing each contour. Accurate area difference distance measurement between two contours can then be achieved after this optimization. Through the use of Matlab, synthetic data is applied to test the effectiveness of area difference distance measurement. The application of this metric function to medical imaging will be our future work.

109. Absorption probabilities for quantum walks on the half line with projective measurements

Monique Ogburn Bowie State University
Forrest Ingram-Johnson Bowie State University
Stephen Pegram Bowie State University
Advisor(s): Chaobin Liu, Bowie State University

We explore the behavior of a discrete-time quantum walk on the half-line subject to a disturbance known as "decoherence." The decoherence effect is modeled by the condition that at every time step of the walk, the option persists, with the decoherence rate q, of exercising a projective measurement both on the coin and the position degrees of freedom. After a given number of time steps t, the "absorption probability" refers to the likelihood that the position of the walker coincides with the boundary of the half-line. Through numerical simulation, we compute the absorption probability at each time step of the walk, as well as the total absorption probability as t becomes large. Our findings suggest that the total absorption probability tends to unity regardless of the value of q. However, the rate of convergence to unity of the absorption probability depends on q. As expected, the rate of convergence is faster when q is close to 1. On the other hand, our findings suggest that even when q is very close to 0, it forces the QW to behave ultimately like a classical random walk, where the total absorption probability is known always to converge to unity. Our ultimate aim is to provide an analytic explanation of these findings.

110. Limiting Behavior of Single Qubit Quantum Operations

Monique Ogburn Bowie State University
Forrest Ingram-Johnson Bowie State University
Stephen Pegram Bowie State University
Advisor(s): Chaobin Liu, Bowie State University

In this project, we explored the limiting behavior of single qubit quantum operations as reflected by their density matrices. A variety of quantum channels were considered, including depolarizing, amplitude, damping, phase damping, phase-flip, bit-flip, and bit-phase-flip channels. With the Pauli matrices serving as a basis, we modeled the quantum operators and showed that they possess no eigenvalues on the unit circle other than $\lambda = 1$. As expected, in all cases studied, we observed that successive iterations of these quantum operations converge to a stationary state.

111. Counterexamples to a Conjecture of Noy and Ribó

Aaron Ostrander Berry College
Advisor(s): Christina Graves, Univeristy of Texas at Tyler

In a publication by Noy and Ribo it was shown that recursively constructible families of graphs are recursive. The authors also conjecture that the other direction holds; that is, recursive families are also recursively constructible. We provide two specific counterexamples to this conjecture, which we extend to an infinite family of counterexamples. We then adjust the conjecture accordingly.

112. Bijective proofs of colored partition identities

Colin Sandon MIT
Advisor(s): Fabrizio Zanello, Michigan Technological University

We provide a general combinatorial framework for a number of colored partition identities, which include the five identities corresponding to the exceptional modular equations of prime degree due to H. Schroter, R. Russell and S. Ramanujan, which were recently proved analytically by B. Berndt. Our approach generalizes that of S. Kim, who has given a bijective proof for two of these five identities, namely the ones modulo 7 (also known as the Farkas-Kra identity) and modulo 3. As a consequence of our method, we determine bijective proofs also for the two highly nontrivial identities modulo 5 and 11, thus leaving open combinatorially only the one modulo 23. We also prove bijectively several other interesting partition identities.

113. Real Time Boundary Element Node Location Optimization

Samuel Smith United States Military Academy
Advisor(s): Theodore Hromadka II, United States Military Academy

Boundary Element Method (BEM) computer models typically involve the use of nodal points as the locations of singular potential functions such as the logarithm or reciprocal of the Euclidean distance function. Recent research on

the types of basis functions used in a BEM approximation has shown that considerable improvement in computational accuracy and efficiency can be achieved by optimizing the location of the singular basis functions with respect to possible locations on the problem boundary and also locations exterior to the problem boundary. To develop such optimum locations for the modeling nodes (and associated singular basis functions), the approach presented here is to develop a Real Time Boundary Element Node Location module that enables the program user to click and drag nodes throughout the exterior of the problem domain. The provided module interfaces with the Complex Variable Boundary Element Method (CVBEM) program, built within the computer program Mathematica, so that various types of information can flow to the display module as the node is moved, enabling the user to determine optimal nodal locations in real time.

114. Achievable Radio Numbers and The Radio Number Formula

Ben Sokolowsky Bucknell University
Advisor(s): Cynthia Wyels, CSU Channel Islands

The span of a radio labeling c is the largest integer in the image of c. The radio number of a graph G is the smallest integer M such that $\text{span}(c) = M$ for some radio labeling c. It is known that a graph of n vertices has a radio number of at least n and at most $\frac{(n-1)^2}{2} + r$, where r is determined by the parity of n. This paper defines and examines three-parameter graphs known as Sok graphs. We show that for all but one integer between the known minimum and maximum, there exists a Sok graph whose radio number is that integer. The results of this work entirely settle the question of what the possible radio numbers are for graphs of order n. We also present additional results relating to algorithms that can be used to simplify the process of determining radio numbers.

115. Analyzing a Chemical Reaction System Using Phase Plane

Wenke Sun UMass Boston
Zihuan Liu UMass Boston
Advisor(s): Xiaodi Wang, Western Connecticut State University

Phase plane analysis is a very useful tool to understand the qualitative behavior of solutions of systems of first order autonomous systems. Such systems frequently arise in physics, chemistry and mathematical biology. In chemistry, we have the following reaction model: $A \to^{k_1} X \to^{k_2} B$, $C \to^{k_3} Y$, $X + Y + X \to^{k_4} 3X$. In this project we'll show that $x(t)$ and $y(t)$ in above reaction model satisfy the following system of first order ODEs:

$$\begin{cases} \dfrac{dx}{dt} = k_1 a - k_2 x + k_4 x^2 y \\ \dfrac{dy}{dt} = k_3 c - k_4 x^2 y \end{cases}$$

We'll analyze the nullclines and the equilibrium point, and then study the behavior of orbits near the equilibrium point. We will then establish some theorem about equilibrium point. Finally, we can tackle the stability question of the equilibrium point and run computer simulations using Matlab to draw the phase portraits in different cases.

116. Propagation Time for Zero Forcing of a Graph

Shanise Walker University of Georgia
Advisor(s): Leslie Hogben, Iowa State University

Zero forcing (also called graph infection) on a simple, undirected graph G is based on the color-change rule. The color-change rule states that if each vertex of G is colored either white or black, and vertex v is a black vertex with only one white neighbor u, then v forces u to become black. A zero forcing set is a set of black vertices that can force all vertices of G black using the color change rule. A minimum zero forcing set of G is a zero forcing set of minimum cardinality. The propagation time of a graph G is the minimum number of steps that it takes to force all the vertices of G black. The study of propagation times of graphs is related to the study of control quantum systems. Examples that demonstrate various features of the propagation time of a graph are introduced; in particular, minimum zero forcing sets having different propagation times are given. Bounds on the propagation time in terms of various graph parameters are presented, and it is shown that the propagation time and diameter of a graph are not comparable. Graphs having extreme propagation times $|G| - 1$, $|G| - 2$, and 0 are characterized, and results regarding graphs having propagation time 1 are discussed.

117. Rubbling Number and t-Rubbling Number for the Root of a Complete Binary Tree

Yi Ming Yu New York City College of Technology
Advisor(s): Brian Nakamura, Rutgers, The State University of New Jersey

A pebbling move on a graph removes two pebbles from one vertex of a graph and adds one pebble to an adjacent vertex. A rubbling move is a generalization of the pebbling move — a pebble is moved to vertex v after removing two pebbles from the neighborhood of v (either two pebbles are removed from the same neighbor, as in a pebbling move, or one pebble is removed from two different neighbors). The rubbling number of a graph is the minimum number of pebbles needed such that, regardless of the starting distribution of pebbles on the graph, any vertex is reachable by a sequence of rubbling moves. The rubbling number of a specific vertex v in a graph is the minimum number of pebbles needed such that vertex v is reachable by a sequence of rubbling moves given any starting distribution of pebbles. More generally, the t-rubbling number of a vertex v in a graph is the minimum number of pebbles needed such that at least t pebbles can be moved to vertex v by a sequence of rubbling moves given any starting distribution. In this work, we find and prove a closed form for the rubbling number and t-rubbling number of the root of a complete binary tree.

[This work was done during the 2011 DIMACS REU program at Rutgers University.]

118. Semi-Dynamic Hedging with Transaction Costs

Jason Bello University of California, Los Angeles
Andrea Arauza California State University, Stanislaus
Advisor(s): Marcel Blais, Worcester Polytechnic Institute

The process of forming a replicating trading strategy that mimics the value of an option is of great interest in the financial world. One such process involves the use of the European call option solution to the Black-Scholes partial differential equation. This process does a fair job of providing direction as to how to rebalance a replicating trading strategy, but it assumes that readjustments can be made in continuous time. This is not realistic, because it is physically impossible to adjust continuously and also because of transaction costs. Thus, the idea of using a self-financing strategy, or a strategy in which a portfolio is formed and no funds are required after the initial investment, is no longer realistic. Methods for discretizing this continuous process will be studied in order to find an optimal strategy. Uniform partitioning will be examined but will most likely not be the sole basis of our optimal strategy. We show that, with the proper set of conditions, a strategy for readjusting a replicating portfolio can be found such that our goals are met. Once this strategy has been developed, the assumption of the absence of transaction costs will be eliminated, and a more realistic way of forming a replicating trading strategy that hedges risk will have been formed.

119. Pricing American Options using the Longstaff-Schwartz Algorithm

Daniel Eckhardt Illinois Institute of Technology
Michael Osorio Duke University
Adrian Ochoa University of Arizona
Advisor(s): Marcel Blais, Worcester Polytechnic Institute

Unlike European options, American options can be exercised at any time up to maturity. As a result of the early exercise feature of American options, they are at least as valuable as their European counterparts. This, however, makes them harder to price as the analytical closed form equations used for pricing vanilla European options do not apply. In order to price an American option, each time t prior to maturity T must be considered to determine whether it is optimal for the option holder to exercise the option immediately or to hold on to the option until a more advantageous future time before it expires. We implemented the Longstaff-Schwartz algorithm, which incorporates Monte Carlo methods and regression to price American options. We also used variance reduction techniques and quasi-Monte Carlo methods to improve the convergence and computational speed of the algorithm. We were able to significantly reduce the width of the 95% confidence interval of our estimated price of the option by using control variates, and we determined the exercise boundary that results from applying the stopping rule.

120. Investigating the Use of Volatility Derivatives to Hedge Portfolios

Jasmine Osorio CUNY York
Kerisha Burke Howard University
Nathan Lopez UC Santa Barbra
Advisor(s): Susan Weekes, Worcester Polytechnic Institute

The trading of volatility derivatives was introduced less than two decades ago; a fairly new practice in mathematical finance. When markets crash, stocks are often traded frantically, and volatility imminently increases. Hence, adding volatility derivatives to a portfolio should balance losses in the event of bearish market behavior. We quantify/minimize the risk using Markowitz's optimization theory. One assumption of this model is that past performance of a portfolio is indicative of its future behavior. The goal is to minimize the portfolio's risk given a fixed rate of expected return. We use MATLAB to plot the minimized portfolio risk versus portfolio returns, producing a curve known as the efficient frontier. We collect market data on stocks from various indices and sectors, as well as data on volatility derivatives. From this data, we construct portfolios with and without volatility derivatives. By comparing the efficient frontier generated by the portfolio with volatility derivatives against the efficient frontier generated without the derivatives, we can quantify the risk reduction induced. Preliminary results confirm our hypothesis, which suggests that volatility derivatives can serve as effective instruments of insurance in an investor's portfolio.

122. Conditioning the Capital Asset Pricing Model (CAPM) with Implied Volatility

Alejandro Samaniego San Francisco State University
Elisa Rosales University of Kansas
Advisor(s): Marcel Blais, Worcester Polytechnic Institute

Financial economists are always interested in improving models to predict expected returns. Previous research indicates that variations in time of expected returns are connected with business cycles. Particularly, investors are less likely to hold risky assets during economic recessions, so expected returns during those times are higher than expected returns in times of economic expansions. Thus it is implied that variables of the business cycles affect time variations in equity premiums. Supporting this assumption and expanding on previous research, we develop a conditional macroeconomic variable by the Johansen cointegration method, a method used to determine a stationary relationship between multiple non-stationary time series, to measure time variations in risk premiums and incorporate it into the Capital Asset Pricing Model. The conditional term includes the following macroeconomic variables: dividend yield, default spread, term spread, short-term interest rate, and implied volatility. Using linear regression our analysis of the models show that the multi-factor models generally predict better than CAPM, but there is room to improve upon the accuracy of all of the models.

123. Quantitative Financial Risk Analysis

Hoang Tran Georgia Institute of Technology
Sidafa Conde University of Massachusetts Dartmouth
Weici Hu Smith College
Jessica Myles Claflin University
Susan Clark University of Pittsburgh
Advisor(s): Tao Pang, North Carolina State University

Today Value at Risk (VaR) is the most common measure of risk used by many financial institutions and regulators. Conditional Value at Risk (CVaR) is another risk measure that finds the expected loss beyond the threshold given by VaR. We began by calculating these risk measures for a single asset under the assumption that financial data is normally distributed. However, we found that extreme losses are larger and occur much more often than the normal distribution would predict. Hence, VaR and CVaR tend to underestimate risk under the assumption of the normal distribution. Several distributions with heavier tails such as Student's t and alpha stable have been proposed as an alternative to the normal. We also implemented the GARCH model to update volatility calculations. We found that heavier tail distributions such as Student's t are generally more suitable for modeling financial data in the single asset case. In addition, we investigated the VaR measure for a portfolio of stocks. Here, we need to consider not only the volatility of individual stocks, but also their dependence on each other. We began by considering the correlation coefficient; then, we investigated the more advanced copula model.

124. Black-Scholes Option Pricing Model: Analysis, Approximations, and Applications

Kathryn Dillinger University of Mary Washington
Rebecca Presor University of Mary Washington
Advisor(s): Jangwoon (Leo) Lee, University of Mary Washington

In our interdisciplinary study, we examined option pricing using the Black-Scholes model, a partial differential equation which shows the current fair price of an option. Using finite difference equations, we derived three numerical methods, Crank-Nicolson, explicit, and implicit, which gave us the finite difference approximations needed to price the options. We then created original MATLAB codes of our analytical and numerical methods. By comparing the outputs we obtained from our programming, we discerned how precise our numerical solutions were relative to the analytical solution. Finally, we applied our result to real-life European options on the market. Looking at the numerical outputs, we were able to determine whether or not to buy a specific option, and at maturity, our would-be investments were profitable demonstrating the applicability of our research.

125. Pricing and Hedging Variable Annuities using Monte Carlo Methods

Dimin Xu Bard College
John Collins University of Tennessee Knoxville
Daniel Gitelman Brown University
Kevin McDermott Johns Hopkins University
Xiao Xu Harvard University
Advisor(s): Matthew Willyard, Penn State University

Variable annuities (VA) are becoming a very popular retirement option because of the various guarantees and protections they offer, but they tend to charge an overly complicated set of fees due to systematic market and mortality risks. Insurance companies are seeking more efficient fee structures while still hedging against the aforementioned risks. This report mainly uses Monte Carlo methods to determine the VA price under different stock path models: Lognormal, Stochastic Volatility and Stochastic Volatility with Jumps, and provides three hedging strategies: Greek hedging, Stop-loss hedging and auto-transfer. We show that all hedging strategies effectively reduce market risk, even when they are implemented sub-optimally. Furthermore, under the assumption that interest rate and long-term life expectancy do not vary, annuitization of VA will not affect insurance companies' payroll as long as a large number of the products are sold.

126. Measuring Seat Value and Quantifying Viewing Angle Quality in the Rose Bowl Stadium

Leah Whitaker Pepperdine University
Advisor(s): David Strong, Pepperdine University

In the Rose Bowl Stadium, the "best" seats are also the most expensive. In our analysis, we find a measure of value for each seat location, which is a function of ticket price and the distance to the action on the field, which was modeled by a relative frequency distribution, based on data from dozens of college football games. We then used various relations between seat value to ticket price and distance to the action to determine the relative values of viewing angles throughout the stadium.

127. Optimal Allocation and Scheduling of Inspection Operations under Multiple Risk Categories

Renee Clarke New York City College of Technology
Advisor(s): Elsayed A. Elsayed, Rutgers University

Ninety-five percent of the non-North American foreign trade arrives into US ports by ship. Cargo containers can be used as a means of smuggling various substances, such as nuclear materials, drugs, biological agents and other illegal substances. Slowing the flow long enough to inspect either all or a statistically significant random selection of imports would be economically intolerable. Moreover, containers have associated destination delivery due date. Therefore, a portion of the containers, based on a risk assigned attribute, are screened in an optimal sequence in order to minimize accepting a container containing contraband and any penalties incurred for not meeting the due date of the containers. Each container is considered to be in multiple risk categories. The two parts of the problem that are taken into account are the allocation of the containers to inspection stations and the schedule at which they go to the allocated stations. Since the solution space of this type of problem can be very large a genetic algorithm is used to move through this space in an efficient procedure. Using the genetic algorithm and an appropriate scheduling algorithm it is possible that this problem can be optimized simultaneously in a reasonable time period.

128. Rationality with Cooperation: A New Prisoner's Dilemma

Rahul Pillay California State University - Monterey Bay
Advisor(s): Rachel Esselstein, California State University - Monterey Bay

It is well understood that strategic play in the non-iterated Prisoner's Dilemma will always lead both players towards defection. The strategies for the iterated Prisoner's Dilemma however are much richer. Certain models show that the ideal strategies for the iterated game lead both players towards ultimately defecting. We know intuitively that this is not how populations behave in certain applications of this model. We will challenge the assumptions made by this model and demonstrate that certain environments may require cooperation after a period of defection. We will utilize Lee Stemkoski's paper, A Tale of Two Wolves, in order to predict different scenarios where cooperation becomes advantageous when deterioration constant is added to the value of the "profit". We expand on the ideas in Stemkoski's paper to look at other models of deterioration as well as applications of these models to evolution, economics, insurance, and contract negotiations.

129. The Optimal Snowboarding Half-Pipe

Kerry Gannon Nazareth College of Rochester
Advisor(s): Yousuf C. George, Nazareth College of Rochester

Over the years, snowboarding has become one of the more extreme sports in existence. Snowboarders have been pushing the laws of physics from year to year. In 2010, snowboarder Shaun White won the Olympic gold medal in the Men's Half-Pipe, and in doing so, managed to reach an astonishing 25ft, 7.62m, above the half-pipe. In order for snowboarders, like Shaun White, to be able to reach these staggering heights, half-pipes must be meticulously created and produced. By examining and studying the conservation of energy and computing the amount of energy lost due to friction, an optimal, record-breaking half-pipe was created. This half-pipe features an incline, called the "Mega Ramp," which gives the snowboarder a direct runway into the half-pipe. This Mega Ramp offers a faster approach to the first jump, where the most height is attained, than traditional half-pipes. Overall, a mathematical model of all jumps has been created, allowing for the half-pipe to be altered to meet the needs and skill levels of individual mountains' snowboarders.

130. Resource Scheduling in Heterogeneous Networks

Christy Duan University of Michigan
Jeffrey Jacobs University of Maryland
Advisor(s): Stacey Beggs, UCLA - Institute for Pure and Applied Mathematics

An increasing demand on cellular wireless networks has targeted the heterogeneous network (HetNet) as an encouraging solution, despite unresolved issues. HetNet deploys small picocells in the coverage of a single macro base station to provide additional wireless access. Because picocells and macrocells transmit and receive signals at the same frequency, co-channel interference becomes an issue. However, a focus on minimizing interference starves certain users of resources and decreases fairness. We provide a user selection algorithm for zero-forcing beamforming (ZFBF) that maximizes number of users given constraints for power and signal-to-interference-plus-noise ratio (SINR). We construct the user selection criteria, norm and orthogonality (NO) and pathloss reciprocal (PR). Using a hill climbing algorithm, we find that the optimal weights for each criteria is 0.5. Performance analysis shows that, given several SINR thresholds, our user selection algorithm outperforms norm-based, angle-based and random user selection algorithms. Finally, we propose a weighting and priority selection method to prevent fairness violations.

131. Structure of n-qubit Werner states

Abigail Skelton Lebanon Valley College
Advisor(s): David Lyons, Lebanon Valley College

Quantum computation is an exciting field of research with potential applications to the fields of mathematics, computer science, cryptography, and physics. The field gained widespread interest in 1994, when Peter Shor discovered an efficient algorithm for integer factorization. The basic unit of information in quantum computation is the qubit, modeled mathematically by a vector in $\mathbb{C}^2$, and states of n-qubit systems are elements of a 2^n-dimensional complex vector space. The focus of our research is the Werner states, which are invariant when acted upon by matrices representing certain types of background noise and interference. A basis for the pure Werner states is known, and we achieve results toward a basis for the more general mixed Werner states. We use the representation theory of Lie groups to determine the dimension of the space of n-qubit Werner states and construct a basis for Werner states

for systems of up to 5 qubits. Finally, we use a generalization of the pure Werner states to construct mixed Werner states for systems of any number of qubits, and we hypothesize that these states form a basis for the Werner states.

132. Co-orbital Dynamics in Early Solar System Formation

Joseph Thomas University of North Carolina Asheville
Advisor(s): Sam Kaplan, University of North Carolina Asheville

In an attempt to explain highly eccentric orbital motion in some solar systems we decided to examine the possibilities of co-orbital systems existing in the formation phase of the planets. We consider the coorbital problem previously established by Josep M. Cors and Glen R. Hall in a paper titled "Coorbital Periodic Orbits in the Three Body Problem". From there we reconstructed a model in mathematica in order to measure what happens to a co orbital systems with orbiting bodies above or below the stable mass range found by Cors and Hall.

133. Central Configurations in the Planar 7-Body Problem

Rebecca Moran College of the Holy Cross
Advisor(s): Gareth Roberts, College of the Holy Cross

The Newtonian n-body problem assumes the gravitational force on n bodies depends solely on their positions and masses. It involves challenging differential equations and for $n \geq 3$, it is difficult to find analytic solutions. One way to find solutions is by looking for *central configurations*. A set of positions forms a central configuration if it satisfies the algebraic equation $\sum_{j \neq k}^{n} \frac{G m_j m_k (x_j - x_k)}{{r_{jk}}^3} + \omega^2 m_k x_k = 0$, where x_j and m_j are the position and mass respectively of the jth body, r_{jk} is the distance between the jth and kth bodies, and ω is a parameter. An equilateral triangle formation of any three masses is a central configuration in the 3-body problem. This example occurs with the recently discovered asteroid which forms an equilateral triangle between the Earth, itself, and the Sun. We looked for central configurations in the 7-body problem with bodies set up in a double rhombus formation. We found examples of central configurations that had a subset which formed an equilateral triangle. Because of the relatively large number of bodies, there has not been much study of the 7-body problem. However, we were able to use symmetry to simplify the problem and make substantial progress.

134. Volumetric Mode Sorter based on Phase Holography

Amanda Howard Stanford University
Stuart Harrell University of Texas at Dallas
Advisor(s): Jorge Balbas, California State University, Northridge

In this project, we develop a robust and scalable algorithm for sorting Orbital Angular Momentum (OAM) modes with a volumetric Computer Generated Hologram (CGH) using the Born Approximation, for application in highly efficient, low-power communication. The primary algorithm is implemented and tested in Matlab, based on previous results for sorting plane waves. Starting with the one-dimensional formulation of the problem, implemented so as to be able to compare the computational solution with the analytical solution that exists for the one-dimensional case and establish criteria for accuracy, the algorithm is extended to handle problems in higher dimensions. Code has been implemented in three dimensions to successfully separate an arbitrary number of OAM modes. A numerical solver based on finite differences has also been developed for the three-dimensional Helmholtz equation. Along with a detailed description of the algorithm and the numerical scheme for the Helmholtz equation, we present results that illustrate the validity of our approach, and suggest additional techniques that we believe would make the sorting of OAM modes more robust.

135. Stability Analysis of Plate Deformation Equations Derived using the Hamiltonian Principle

Charles Daly CSUMS / George Mason University
Advisor(s): Padmanabhan Seshaiyer, George Mason University

Airplanes, helicopters, and the majority air-crafts rely heavily on wings, which can be modeled by very thin malleable plates. Through the use of the Kirchoff-Love plate theory model we intend to derive the necessary and sufficient conditions for stability of a uniformly dense three dimensional plate. Working with this model, the Green's stress and strain tensor, and the Hamiltonian Principle we derived a set of differential equations whose stability will be examined in thorough detail through the use of an energy norm and stability analysis.

136. Nondestructive Electrothermal Detection of Corrosion

Brittany Ambeau Rochester Institute of Technology
Harris Enniss Harvey Mudd College
Stefan Schnake Murray State University
Advisor(s): Kurt Bryan, Rose-Hulman Institute of Technology

Nondestructive testing and imaging plays an important role in many industries, e.g., the monitoring and maintenance of corrosion in aircraft. The general technique is to input energy in some form into an object, observe the object's response, and from this input-output information determine the internal structure. New techniques are always being explored, and recently there has been much interest in methods that use multiple forms of energy. In this vein, we examine a new technique for imaging corrosion or material loss in an object by combining electrical and thermal measurements on some accessible portion of the object's outer boundary. The flow of electrical and thermal energy through the object is modeled using partial differential equations, and imaging the corrosion leads to a mathematical "inverse problem." We examine limits and stability of this type of imaging, and develop an effective numerical algorithm for solving these types of problems.

137. M-band Wavelet Based Authentication Algorithm Using Principal Component Analysis

Yunhan Jing Affiliated High School to Jilin University
Tong Liu Affiliated High School to Jilin University
Xuan Affiliated High School to Jilin University
Advisor(s): Qingjin Xu, Affiliated High School to Jilin University

Computational techniques derived from digital image processing are playing a significant role in the study of the visual arts. This technology has the effect of placing the plastic arts (painting, drawing, and objects) within the domain of the computer. As a result scientists are beginning to play an important role in the investigation of the plastic arts, for art historical research as well as the authentication. This project presents a 4-band wavelet based authentication algorithm by incorporating the principal component analysis (PCA). The proposed algorithm is expected to achieve higher perceptual transparency. Specifically, the developed watermarking scheme can successfully resist common signal processing such as compression and geometric distortions. In addition, the proposed algorithm can be parameterized, thus resulting in more security. To meet these requirements, the image is first transformed to YIQ to decrease the correlation between different bands. Then 4-band wavelet transform is applied to each channel separately to obtain one approximation sub-band and fifteen detail sub-bands. PCA is then applied to the coefficients in all detail sub-bands. Finally, last component band represents an excellent domain for inserting the water mark.

138. Automatic Camera Geo-Location from Solar Shadows

Sabrina Gordon Institute for Pure and Applied Mathematics
Advisor(s): Stacey Beggs, Institute for Pure and Applied Mathematics

Determining the latitude and longitude of a particular object from photographs is called *geo-location*. Although manual scene analysis is possible to determine location, a more successful and less time-consuming automatic approach is possible. We show that given five or more photographs taken outdoors over the course of a day with a stationary camera, each depicting two vertical objects casting shadows on a horizontal surface, we can calculate the latitude and longitude of the camera, assuming in addition that the date of the photos and the Coordinated Universal Time of each photo are stored with the images. We describe our implementation of an algorithm for geo-location (not fully automatic), give a performance analysis, and suggest future research.

139. Derivations of Euler-Lagrange Equations in Imaging

Jason Barnett CSU Stanislaus
Advisor(s): Jung-Ha An, CSU Stanislaus

In imaging, it is necessary to minimize functionals to construct a smoothed image close to the original. This is best accomplished by using calculus of variations, specifically the Gateaux variation and Euler-Lagrange equations. In this poster, I will present and compare two methods of deriving Euler-Lagrange equations, one by directly computing when the Gateaux variation is equal to zero for all test functions on the necessary set, and the other by using the general Euler-Lagrange equation. Using these methods, I will demonstrate an elementary example and then apply this concept to a specific imaging model.

140. Applying Gradient Descent And Finite Differences schemes In Contour Extraction

William Bishop CSU Stanislaus
Advisor(s): Jung-Ha An, CSU Stanislaus

In some cases, the Euler-Lagrange equation can be solved directly in closed form. In extracting contours from images, we use numerical techniques. I propose to compare various methods of gradient descent as tools for computing solutions to the Euler Lagrange equation, and explore different applications of finite differences as they are used in image manipulation.

141. Discrete signal processing with fractal wavelets

Taylor Sibbett Westminster College
Advisor(s): Jonas D'Andrea, Westminster College

Digital signal processing is used to represent functions (signals, data) efficiently using some optimal combination of basic functions. By taking advantage of the structure of information inherent in typical signal data, complicated functions that represent images, sounds, and other signals can be represented with less information and a simpler function. However, it is difficult to understand what sort of underlying structure exists with a given signal, so different discrete wavelet transforms are used on different data sets to obtain better results. In my research, I have implemented numerous fractal wavelet transforms, which uses the self-similarity of certain fractals to match the self-similarity used in wavelets. My research advisor and I are continuing to experiment with new data sets and comparing the efficiency of each of the algorithms to each other.

142. Mapping MoS_2-Co Catalytic Nanostructures Using HRTEM and TEM Simulations

Eduan Martinez-Soto Universidad Metropolitana
Advisor(s): Manuel Ramos, University of Texas at El Paso

Understanding the morphology of catalytically active materials has been approached in past decades with very good results when using field electron microscopy in scanning and transmission modes. In the past some simulated TEM measurements for alumina supported molybdenum sulfide AlO_2/MoS_2 provided some insights about molecular structure in those catalytic layered transition metal sulfides (LTMS). However, due to resolution, color enhancement, tomography and other factors, sections of those materials observed under TEM do not resolve the structure by itself; in particular for localization of cobalt atoms for MoS_2 unsupported catalyst. This work concludes a lattice distance of 0.62 nm and 0.299 nm for Mo-S and Co respectively; results presented here were obtained using experimental HRTEM and molecular modeling to produced TEM simulations, which performs a full dynamical calculation by multi-slice method with a slice thickness of 0.1 Ã… and using projected potential, $f(U) = \sum_{i=1}^{n} a_i e^{(-b_i U^2)}$ where a_i and b_i are coefficients to be determined. The variable $U = (u, v, w)$ is used to represent coordinates in reciprocal space (Fourier space) quantities (spatial frequencies). $f(U)$ is the atomic dispersion factor.

143. Cloaking Against Thermal Imaging

Maple So Arizona State University
Advisor(s): Kurt M. Bryan, Rose-Hulman Institute of Technology

There has been a lot of recent interest in cloaking and invisibility in the mathematics and science communities, and in fact physically plausible mechanisms have been proposed (some built) for cloaking an object against detection using a variety of electromagnetic methods. The ideas are very general, however, and should allow one to design cloaks that work against other forms of imaging. We examine the possibility of cloaking an object to make it invisible to an observer using thermal energy (heat) as the imaging tool. Specifically, we desire to cloak an object inside a 2-dimensional disk by cutting a small hole in the center of the disk to place the particular object. Mathematically, we want to make a large cavity in the unit disk to appear small to outside observers by analyzing the norm of the boundary data and completing a change-of-variables argument.

144. Smartphone Sensors and Detecting Road Anomalies

SaraJane Parsons Indiana University of Pennsylvania
Nathan Marculis Grand Valley State University
Advisor(s): Ed Aboufadel, Grand Valley State University

Data logs from accelerometers in iPads and smartphones are a source of noisy, spiky data. This data can be collected while driving on bumpy roads or from other dynamic activities. This poster will show analyses of this data using wavelets and other mathematical tools. The research was done at the 2011 REU program at Grand Valley State University with Prof. Edward Aboufadel.

145. Micro-Combustion: Modeling Reignition and Hot spots on Channel Walls

Duncan Ashby University of Redlands
Advisor(s): Joanna Bieri, University of Redlands

Edge-flames have been observed experimentally in the narrow channels of micro-combustors and researchers have observed extinction reignition patterns in these systems. These patterns are currently not well understood. In order to better understand this behavior, a series of non-linear partial differential equations are used to model an edge-flame in a micro-combustor. These equations are solved numerically. This research focused on the phenomenon of edge-flame reignition and the effect of hot spots, or heated areas, along the channel walls. Thus far, numerical simulations have shown reignition when sections of the walls have high temperatures. These results imply that hot spots can cause reignition, however this behavior has only been found under extreme conditions that do not occur naturally in the channel. Future goals include modifying the code so that it will better simulate pseudo heat conduction in the walls rather than using forced hot spots. Applications of micro-combustor research include smaller and more powerful heat sources, engines and batteries. The demand for miniaturized products is increasing rapidly and micro-combustors are an essential key to the growth of these technologies.

146. Optimizing Plasmonic Effects for a More Efficient Nanoscale Bio-Photovoltaic Device

Jason Pina George Mason University
Advisor(s): Igor Griva, George Mason University

Biophotovoltaics utilize biological components from photosynthetic systems for the normal charge separation process in a photovoltaic device. The promise of such devices is great, as they are highly efficient in their biological environment at converting photons into free charge. However when utilized in artificial photovoltaic devices, their efficiency is quite low, even when compared with conventional solar cells. One way to increase their efficiency is to use plasmonic effects to augment the intensity of incident light in the charge separation area. Plasmonic effects occur when electromagnetic waves at a conductor-dielectric interface causes the oscillation of free elections relative to the atomic lattice, allowing the electrons to act as a plasma. In our current computational study, we are investigating the use of plasmonic effects with silver and gold to optimize the delivery of light to a film of bacterial photosynthetic reaction centers in an attempt to increase the efficiency of a biophotovoltaic nanodevice.

147. Cost-Efficient Roof Renovations

Shannon Samples University of the Incarnate Word
Advisor(s): Zhanbo Yang, University of the Incarnate Word

The purpose of this study is to help find affordable green roof renovations for San Antonio families by introducing incentives, tax credits, and rebates. The main question of this study is “How can I show families that if they renovate now they can save in the future?” The hypothesis is that if tax incentives and rebates for green roof renovations are introduced then citizens of San Antonio will be more aware and inclined to renovate their roofs in the near future. The approach to this study was finding the costs of roofs without tax and energy savings and then comparing these costs to the costs of the same roofs with tax and energy savings. The costs of the asphalt, cool, and reflective cool roofs were determined by finding the price per square foot and the cost of the solar roof was determined by finding the price per watt. After finding the cost per square foot and the cost per watt, the roofs were compared in price by finding the cost of each type of roof for one variable which in this study was a 3,000 square foot roof. Through statistical analysis it was found that green roofs cost less and yield higher savings than asphalt roofs when tax savings and energy savings are applied. Investments in green roofs are cost-efficient options for renovating a house in San Antonio.

148. CSUN Green Initiative: Power Consumption Optimization

William Sherman California State University Northridge
Andrew Duhancioglu California State University Northridge
Advisor(s): Ramin Vakilian, California State University Northridge

Sustainability is "development that meets the needs of the present without compromising the ability of future generations to meet their own needs." Six undergraduate students investigated the nature of power consumption at CSUN and developed specific proposals to optimize power consumption on campus. This is a mathematical optimization problem. The focus of this project is on outdoor lighting and parking structures on campus. All aspects of power consumption at CSUN were thoroughly investigated. We accurately evaluate the total power consumption and the magnitude of wasted energy throughout the university. Extensive research was done on the state of the art technologies available today. We developed detailed strategies that may be used to optimize power consumption at CSUN and minimize our carbon footprint. For each considered strategy the group computed the initial required capital investment, total savings, timelines for the break-even points with the initial capital investments, and carbon footprint savings. These results are summarized in a hierarchical list of proposals, ordered from the one requiring the smallest capital investment to the most ambitious and costly proposal. A thorough and detailed cost analysis of each proposal is then provided.

149. An Analytical Approach to Solving Green Oxidation Processes

Diego Torrejon George Mason University
Advisor(s): Maria Emelianenko, George Mason University

Oxidation, a process in which oxygen is added to break pollutants or organic wastes, is important in many industries. This process often uses chemicals that can result in the production of hazardous substances, so it is imperative to be able to control the process to make it environmentally safe. In this poster, we study the problem of suicidal inactivation of enzymes and man-made oxidation catalysts. Based on experimental data obtained from our colleagues at Carnegie Mellon University, we formulate a system of differential equations that models chemical reactions and analyze its numerical and analytical properties. The main goal is to estimate the rates of the reactions based on limited experimental observations. The nonlinear 3-dimensional ODE system under investigation does not allow for an exact solution. However, noticing its similarity with Michaelis-Menten system, we have been able to develop quasi-state approximation of the model that together with perturbation techniques has allowed us to derive a highly accurate approximate solution. Analytical results developed using this approach, generalized upon previously known relations between the rate constants, allow for a much deeper understanding and control of the oxidation processes.

150. Modeling the Effects of the Fukushima Daiichi Nuclear Meltdown on the Marine Environment

Bernard Lipat New Jersey City University
Rusty Laracuenti New Jersey City University
Advisor(s): Zhixiong Chen, New Jersey City University

With the increasing frequency of nuclear power as a source of energy, understanding the effects of radioactivity on the marine environment is a problem whose solution is vital for a comprehensive approach to nuclear safety. In particular, the recent meltdown of the Fukushima Daiici Nuclear Power Plant provides a case study out of which we develop a mathematical model using numerical approximations of the diffusion equation with advection and decay to study the spread of radionuclides and their effects on the populations of the Pacific Bluefin Tuna (Thunnus orientalis) and the Pacific Pink Salmon (oncorhynchus gorbuscha).

151. Modeling Genetic and Environmental Influences on Resource Allocation Traits of Arabidopsis lyrata

Erika Helgeson Gonzaga
Advisor(s): David Remingtom, University of North Carolina, Greensboro

As climate change becomes a concern more needs to be understood about how plants evolve in changing environmental conditions and how this affects both the ecosystem and food crops. We hypothesize that resource allocation tradeoffs are generated by cause-effect relationships between traits expressed during development. We simulate resource allocation traits expressed in the life history of Arabidopsis lyrata as a causal network in order to better understand how phenotypic evolution is shaped by environmental and genetic variation. In addition to generating values for specific phenotypic traits, the model was also used to generate survival predictions and reproductive output

values. To validate the model we compared the correlations generated from the model to those found in field data and found them to be similar. This indicates that the model could be a useful tool for understanding how tradeoffs in plants arise. The model also gave novel insights into how alleles associated with vegetative growth and alleles associated with reproductive growth could have opposite relationships with survival in certain environments. This finding provides a mechanism to explain adaptive evolution in resource allocation strategies.

152. Ecological Stability: Mono vs. Poly

Olivia Romeo Bard College Summer REU
Andrew Ma Bard College Summer REU
Advisor(s): Gidon Eshel, Bard College

It is known that when planting crops, a monoculture produces more biomass per unit area than a polyculture under ideal conditions. We investigate if it is possible for a mixed garden, a polyculture, to have transgressive over yielding when faced with natural disasters due to it having a greater resiliency. In order to compare the resiliency of a mixed garden to a monoculture, we develop mathematical models based on the Lotka-Volterra predator-prey model. We include terms to account for diffusion of plants, pests, and nitrogen resources, and then use Matlab to evaluate and solve these equations. Further, we analyze the results by using Singular Value Decomposition at specific critical points in the system, where the mixed garden and monoculture, is vulnerable. This provides a better understanding of how the systems we are modeling will respond to a disturbance. Based on our trails, we discover that polycultures most often yield the average of the range of the total biomass produced by a monoculture. We conclude that it would make mathematical sense for farmers to continue growing polycultures.

153. A Mathematical Model of the Emission and Optimal Control of Photochemical Smog

Arturo Vargas University of California Irvine
Daniel Burkow Arizona State University
Kathryn Heal University of California Los Angeles
Advisor(s): Luis Melara, Shippensburg University

This study models traffic regulations as well as vehicle modifications as tools for reducing the emission of NO, NO2, CO, and CO2 from vehicles on a subset of highways in the Los Angeles basin. We make changes to an established chemical network creating a simplified, non-autonomous, coupled set of six ordinary differential equations. We demonstrate, and prove when able, the existence of periodic pollutant levels.

154. Refining the Contour Advection with Surgery Method for 3 Dimensional Flow Analysis

Peter Bradshaw Arizona State University
Advisor(s): Eric Kostelich, ASU

Contour Advection with Surgery (CAS), is a computational method in which a contour is represented by a collection of nodes. These nodes are transported along a velocity field. Within advection, new nodes are redistributed along the contour using an interpolation scheme based upon continuous curvature, with greater density along areas experiencing greater stretching or folding. The CAS method is less computationally expensive than full-Lagrangian representation. In addition, it allows for analysis of fine scale structures that develop throughout the advection process. We have developed a new computational method where CAS is generalized to 3-dimensional space. In this method, the nodes representing the contours are advected by the three components of the velocity field. At each time step, the contours are refined using a new interpolation method. I will present our method as it is applied to resolve the fine-scaled transport of contour particles by velocity fields produced by Hurricane Katrina. I have conducted simulations of tracer transport over various layers of atmospheric flow and will provide illustrations in 3-dimensional analysis and compare them to their 2-dimensional counterparts in order to demonstrate the robustness of our approach.

155. Lagrangian Transport of Inertial Particles in Hurricane Katrina

Angelica Deibel Arizona State University
Advisor(s): Wenbo Tang, Arizona State University

Using data from the Weather Research and Forecasting Model (WRF), we analyze Lagrangian transport of inertial particles in Hurricane Katrina. We use the reduced version of the Maxey-Riley equations, which captures the particle displacement on a globally attracting slow manifold, and studied the organizing patterns (Lagrangian Coherent Structures) for different particle sizes. We discuss the dependence of the transport patterns on different sizes.

156. Weather Research and Forecasting Model and Lagrangian Analysis

Hershey Kelly Arizona State University
Advisor(s): Wenbo Tang, Arizona State University

We analyze Lagrangian transport of different inertial particles on real extreme events such as Hurricane Katrina using the Weather Research and Forecasting Model (WRF). We use the WRF Processing System to generate multiple nests to get high-resolution results from global forecasting system to regional weather forecasts. We find that even though individual trajectories of different sized particles differ significantly, the structure is similar.

157. Trend Analysis of Northeast US Gridded Historical Climate Data

Micah Broehm University of Wisconsin - Stevens Point
Advisor(s): Aaron Luttman, Clarkson University

Gridded historical climate records are widely used sources of high spatial resolution ($\sim$10 km per grid cell) climate information. These spatial grids are derived from weather station measurements by using varying interpolation algorithms. The PRISM Climate Group at Oregon State University and the Northeast Regional Climate Center at Cornell compile separate data sets containing the Northeast United States. Here, each spatial cell of these two data sets is systematically analyzed using three techniques. Linear regression estimates the magnitude and direction of the trend in the cell, though it is sensitive to stray data points near ends of the set. We also use two statistical techniques to determine a trend. The Sen slope is not altered by outliers near the ends of the data, but may not return a slope that represents the trend well. Mann-Kendall reveals the direction and significance of a trend, but not the magnitude. The results of these computations are then searched for statistically significant agreement and disagreement cell by cell. We look for this between different methods within a set, and between similar methods in both sets. While these sets show similar large scale trends, there are significant inconsistencies that should be seriously considered and addressed.

158. The Generalized Pareto Distribution and Threshold Analysis of Normalized Hurricane Damage in the United States Gulf Coast

Anthony Daspit Lamar University
Advisor(s): Kumer Pial Das, Lamar University

Hurricane damage from 1926 to 2009 is normalized using the Pilke-Landsea procedure. The Generalized Pareto Distribution is then fitted for the distribution of losses at different thresholds via the method of maximum likelihood. Application of the Anderson-Darling and Kolmogorov-Smirnov tests determine goodness of fit for each threshold. The change in the parameters of the distribution are then analyzed as the threshold is adjusted. A Mean excess plot is used to define the possible range for the threshold and to assess the soundness of the Generalized Pareto Distribution as a model choice.

159. A Potential Solution to the Separability Problem in Referendum Elections

Ada Yu University of Puget Sound
Clark Bowman University of Rochester
Advisor(s): Jonathan K. Hodge, Grand Valley State University

In referendum elections, voters are often required to cast votes on multiple proposals simultaneously. This leads to a separability problem when a voter's preference on one proposal depends on the known or predicted outcomes of other proposals. In this poster, we propose an iterative voting scheme that allows voters to revise their voting strategies based on the outcomes of previous iterations. Using a robust computer simulation, we investigate the potential of this approach to solve the separability problem.

160. A Population and Path-Based Measure of Compactness

Carl Corcoran Macalester College
Advisor(s): Karen Saxe, Macalester College

The Supreme Court of the United States has held that bizarrely shaped congressional districts threaten the equal voting rights of U.S. citizens. Attempting to quantify this bizarreness, mathematicians have produced myriad measures of compactness. Some consider only the geometric shape of the district, while others attempt to integrate the population

of the district into the calculation. In this project, we develop a new measure of compactness that takes into account the geometric shape of the district, the geometric shape of the state, as well as the spread of the population throughout that district. This measure is intended to be a hybrid of path-based and population measures, which will use the census blocks of a given district as basic unit for calculation. By hybridizing path-based and population measures, we are able to overcome some deficiencies in population dispersion measures, namely the bias toward smaller districts and high scoring of districts that meander within a compact area. Finally, the measure is then used to evaluate various plans currently being considered for Minnesota's eight congressional districts.

161. Geographic Profiling through Six-Dimensional Nonparametric Density Estimation

Austin Alleman Santa Clara University
Advisor(s): George Mohler, Santa Clara University

Geographic profiling is the problem of identifying the location of the offender anchor point (offender residence, place of work, etc.) of a linked crime series using the spatial coordinates of the crimes or other information. A standard approach to the problem is 2D kernel density estimation, which relies on the assumption that the anchor point is located in close proximity to the crimes. Recently introduced Bayesian methods allow for a wider range of criminal behaviors, as well as the incorporation of geographic and demographic information. The complexity of these methods, however, make them computationally expensive when implemented. We have developed a nonparametric method for geographic profiling that allows for more complex criminal behaviors than 2D kernel density estimation, but is fast and easy to implement. For this purpose, crime locations and anchor point are considered as one data point in an infinite dimensional space. Dimension reduction is then used to construct a 6D probability density estimate of offender behavior using historical solved crime series data, from which an anchor point density corresponding to an unsolved series can be computed. We discuss the advantages and disadvantages of the method, as well as possible real-world implementation.

162. Novel Crime Prediction Algorithms

Emmanuel Tsukerman Stanford University
Olivier Mercier McGill University
Advisor(s): Laura Smith, UCLA

Our project is part of a broader initiative to utilize data to inform and direct police work. An increased understanding of the distribution of crime will serve the LAPD in reducing the occurrence of crime and focusing valuable resources on problem areas. In our work we aimed to improve hot-spot mapping, which is mapping that produces relative probability density estimates of crimes occurring in a given region. We develop original algorithm modifications that improve upon Maximum Penalized Likelihood Estimation algorithms and Kernel Density Estimation by taking advantage of the temporal progression of crime and its periodicity. Tests on real data show that our algorithms perform significantly better than their unmodified version.

163. Geo-Semantic Data Mining and Knowledge Discovery for a Large Collection of Historical Narratives

Margo Smith Kenyon College
Peter Sugihara Columbia University
Rodrigo Mendoza Smith Instituto Tecnológico Autónomo de México
Advisor(s): Peter Broadwell, UCLA

The USC Shoah Foundation Institute has digitized nearly 52,000 testimonies (over 100,000 hours) of survivors of the Holocaust. These testimonies are now preserved in the online Visual History Archive (VHA). Each one-minute segment of testimony was tagged with appropriate keywords drawn from an aggregated 50,000-term thesaurus. A great amount of these keywords refer to specific geographic locations. The searchability of the VHA could be greatly improved if this geographic information was incorporated in the search. The objective of our project was to find relationships between geographic keywords and topical keywords using the sole assumption that keywords that occur in nearby segments are related to each other. We applied unsupervised learning techniques, including various clustering algorithms, to our data in order to group locations based on the topical keywords they are associated with. We also produced multiple algorithms to rank topical keywords related to a single or multiple

locations in order to emphasize 'unique' stories. Our results enable the user of the VHA to explore geographic localization of Holocaust topics. The user can also be directed to topically similar locations in geographically disparate areas.

164. Vera Sanford

Jessica Bartalsky State University of New York College at Oneonta
Christine Baxter State University of New York College at Oneonta
Advisor(s): Toke Knudsen, State University of New York College at Oneonta

Vera Sanford was an accomplished mathematics professor, who finished her teaching career at the State University of New York College at Oneonta. For this conference we would like to present information on her life and on her impact as a woman in mathematics not only locally in Oneonta, NY but also across the country. We would like to shed light on her accomplishments, and have her recognized for them in a time when math was majorly dominated by men. She had a major impact on the College at Oneonta where she formed the Mathematics department when it was included as one branch with science in the early 1930s. Sanford was on the editorial board of the Scripta Mathematics. She was the vice president of the N.E.A. council of teachers of mathematics from 1948-1950. Sanford has written numerous articles in various math journals. These journals include the mathematics teacher, and the arithmetic teacher. Sanford wrote the book, A Short History of Mathematics in 1930, which was one of the first ever American Books written about Mathematics. Sanford had numerous accomplishments and this is why a poster is merited about her and her life in the world of mathematics, as a way of recognizing her for what she did, when she has not been recognized before.

165. Mathematics for Nonmath Majors and Mathematics Anxiety.

Moira Devlin Saint Joseph's University
Advisor(s): Agnes Rash, Saint Joseph's University

Abstract: A new curriculum for undergraduates took effect in 2010 at Saint Joseph's University. The change included "mathematics beauty courses," satisfying the mathematics requirement. One of the new courses was designed to introduce number theory and proofs pertaining to elementary concepts. Through implementing a pretest-posttest design in this course on math anxiety and knowledge, we determined if there is an increase in knowledge and an attitudinal change as a result of taking this course. From the literature research, the principal author explored the different relationships between math anxiety and achievement among middle school and high school students, but we focused on that relationship at the college level. Another issue we explored was intervention programs to help decrease math anxiety and increase achievement. This idea is encouraged but has not yet been researched. The presenter will discuss the research and games introduced to mitigate the problems of math anxiety and content knowledge.

166. Computer Monitored Problem Solving Dialogues

Nicole Rutt Concordia College, Moorhead MN
Lisa Dion Providence College
Jeremy Jank Concordia University, NE
Advisor(s): Michael Glass, Valparaiso University

This project "looks over the shoulder" at students collaboratively engaged in a math problem-solving activity. In order to mechanically classify student sentences, we produced a classifier that examines these sentences and is 55% accurate at identifying utterances as containing certain bits of knowledge. Classification was achieved by comparing new, unknown, sentences with pre-built bundles of manually tagged sentences, one bundle for each classification. We also characterized the dialogues with problem-independent categories: a math collaborative dimension and a problem-solving dimension. This will enable us to classify utterances with regard to in what ways students are participating in the dialogue and the problem-solving process. The context of this work is a quantitative problem-solving course in which students work in small groups. Our goal is for the computer to notice some of the same aspects of the activity that a teacher walking around the classroom might observe, such as what realizations a group has achieved and how students are collaborating.

167. The Mathematical Relationship between Musical Phrases

Joseph Mileski Ohio Northern University
Advisor(s): William Fuller, Ohio Northern University

We will be looking at the angles between musical phrases. We will be showing how to convert a musical line into a sequence of vectors, how to find the circular path for that sequence, and how to associate a graph with that musical line.

168. Hierarchical Analysis of Music and Spider Silk using Category Theory

Edgardo Vazquez Rodriguez Universidad Metropolitana
Advisor(s): Markus J. Buehler, Massachusetts Institute of Technology

Complex hierarchical structures composed of simple basic building blocks form the basis of most biological materials. The comprehension of these materials is mostly done by understanding its structure in a top-down approach. Here we show that with a simple analogy between seemingly different fields it is possible to gain insights in general principles and properties. Specifically, we showcase that material science exhibits comparable properties as classical music in terms of hierarchical structures, functionality and mutability. As a comparative tool we utilize an application of mathematical category theory called ontology logs (olog). Ologs provide an illuminative system representation by expressing knowledge in a conceptual map that follows a rigorous mathematical formulation based on category theory. We exemplify these similarities with a case study of classical music and spider silk which intrigued scientists for a long time. We draw connections on several levels of hierarchy and identify similar patterns that govern the structure of these two hierarchical systems. The analogy developed will enable further understanding of natural and novel protein based materials and show that seemingly disparate fields such as material science and classical music are closely related.

169. Game Theory and Vaccination with a Network Epidemiology Approach

Jordan Angel East Tennessee State University
Samuel Peters East Tennessee State University
Manuel Gonzalez East Tennessee State University
Advisor(s): Ariel Cintron-Arias, East Tennessee State University

When parents are faced up with the dilemma of whether to vaccinate their children or decline, many issues come to their mind. Are the side effects more detrimental than not being immunized? What are the risks of vaccination and opting out of it? Game theory analysis suggests that populations with voluntary vaccination are unlikely to reach optimal coverage, while the perceived risk of a vaccine can drastically affect vaccine uptake. Recent studies propose that when individuals in a population can choose from either protecting themselves or engaging in risky behavior, the latter is more likely when the decision is based on perceived disease prevalence. Here we implement an epidemic model with social network structure and age-structure with two main age groups: parents and children. Parents are players of a 'vaccination game', choosing between timely vaccination, delayer or non-vaccinator. Simulations of the proposed model are carried out using real data of social networks collected by The National Longitudinal Study of Adolescent Health, a longitudinal study of a nationally representative sample of adolescents in grades 7–12 in the United States during the 1994–95 school year. Our results characterize an interesting interplay between network modeling and game theory.

170. My BIG Fat Model: Beta Cell Compensation in Type 2 Diabetes

Javier Baez Arizona State University
Rosalia Zarate
Advisor(s): Erika Camacho, Arizona State University

In this work we explore the biological consequences of the effect of over nutrition, and beta-cell function in a model of the progression of Type 2 diabetes (T2D). More specifically, we focus on the effects of fat mass in the liver and the mechanism underlying the initiation and progression of beta-cell failure. We incorporated fat, the direct effect of insulin sensitivity, and the effect of beta-cell sensitivity in our model. We assumed that beta-cell sensitivity embodies a logistic response by initially increasing as fat accumulates due to the compensatory response triggered by increased glucose levels. As fat continues to accumulate, beta-cell sensitivity decreases due to beta-cell failure and eventually beta-cells begin to die. The threshold at which beta-cell compensation fails marks the clinical onset of T2D, which with time can progress to the stage where it is no longer reversible due to severe loss of beta-cell mass.

Using the theory of dynamical systems we analyze the various stages of T2D, investigate whether weight loss in the pre-diabetic and diabetic stages would reverse T2D, and study when this treatment strategy is no longer effective.

171. Synchronous Oscillatory Solutions in a Rosenzweig-MacArthur Multi-Patch Model

Matthew Becker College of William & Mary
Advisor(s): Junping Shi, College of William & Mary

Predation in nature can lead to oscillations in populations of both predator and prey, as seen best in the example of the Canadian Lynx and Snowshoe Hare. When this occurs, distinct coupled patches of species may synchronize such that populations in each patch oscillate together equivalently. Oscillatory solutions to a predator-prey model are studied to understand what leads to this synchrony. Here we use Alan Hastings' version of the Rosenzweig-MacArthur model. A correlation coefficient similar to Pearson's Correlation is used as a statistical method to quantify synchrony and we use this as a numerical tool to analyze our data and results.

172. A Model for the Population of the Blue Crab in the Chesapeake Bay

Timothy Becker College of William & Mary
Advisor(s): Junping Shi, College of William & Mary

The Blue Crab is a cannibalistic predator and partakes in intraguild predation. With this in mind, we model the population of the Blue Crab in the Chesapeake Bay by using differential equations. The differential equations describe the overall biomass per square meter of the adult blue crab, juvenile blue crab, and the resource. We study the effects of cannibalism, as well as the effect of the fisheries on the population dynamics of the system.

173. Sensitivity Analysis of the Wnt Pathway

Kelly Blake Wofford College
Brandon Groth University of Wisconsin-La Crosse
Kyle Dunn University of Massachusetts-Amherst
Jacob Schuurman Grand Valley State University
Advisor(s): Hien Tran, N.C. State University

Signal transduction begins with a signal binding to receptors on the membrane of the cell. This causes a cascade of protein interactions within the cell, ending in transcription of targeted genes. The Wnt pathway is a well studied protein system that plays an important role in the proliferation and adhesion of cancerous cells. When signaled, the Wnt pathway results in a buildup of β-catenin, which leads to transcription of certain genes. An excess of β-catenin can cause cells to divide unnecessarily resulting in tumors. Other factors in the pathway, such as Axin and APC, help regulate β-catenin buildup. In order to sift through the various protein interactions to determine the most influential, we analyze the response of a state variable to a change in the parameter, also known as sensitivity functions. Solving for sensitivity functions analytically in large models such as the Wnt Pathway is not practical because it requires computing a large number of partial derivatives. In this project, we use automatic differentiation to compute the partial derivatives related to sensitivity functions. Through our sensitivity analysis of the Wnt pathway, we were able to determine the most influential protein interactions.

174. Modeling Bacterial Contamination at an Urban Beach on Lake Michigan

Anne Bruckner University of Wisconsin-Milwaukee
Morgan Schroeder University of Wisconsin-Milwaukee
Advisor(s): Gabriella Pinter, University of Wisconsin-Milwaukee

E. coli and Enterococci are indicators of sewage pollution, and understanding their transfer and life cycle is important to maintaining a clean and healthy urban beach. Heavy rainfall can cause runoff and sewage overflow from urban areas, contributing to bacterial contamination on the beach and near-shore lake waters. A comparison of experimental data collected on Bradford Beach on Lake Michigan in Milwaukee from 2006 to 2010 shows a decrease in E. coli concentrations after altering the pathway of surface runoff by the installation of the rain gardens in 2008. Creating and implementing a mathematical model will improve our understanding of the most influential factors affecting bacteria levels on the beach. We present a model that represents both the growth dynamics of the bacteria population, resident in the beach sand, and the hydrology of the beach. A coupling between the two equations is necessary since rain events impact the nutrient levels in the sand and also the washout of bacterial contamination in the lake waters. The systems of equations will be solved numerically and compared with historical and collected data.

175. A Mathematical Model for Hyperparasitic Regulation of an Infectious Pathogen

Christopher Dottino The College of Saint Rose
Advisor(s): Amina Eladdadi, The College of Saint Rose

Infections within individual hosts often encompass multiple interactions, involving competing pathogen strains, symbiotic mutualists, hyperparasites, and (for human and agricultural hosts) antibiotics. Here, we present a mathematical model linking within-host dynamics to both time-dependent host survival and between-host disease transmission. We develop predictions for both acute and chronic infection. The former are being tested in a three-species experimental system.

176. Optimal Control Theory for a VRE Model

Michael Dunlea Ursinus College
Clinton Watton Kutztown University
Emily Dougherty
Advisor(s): Mohammed Yahdi, Ursinus College

Antibiotic resistance Vancomycin Resistant Enterococci (VRE) is one of CDC's top ten health concerns. Optimal Control Theory (OCT) is used to determine the most efficient and economically favorable strategies to prevent outbreaks and to control the emergence of VRE in hospital intensive care units. OCT provides mathematical optimization methods for a dynamic system, with certain controls and constraints, in order to optimize a certain output. The project focused on merging key OCT methods to provide necessary conditions for the existence and construction of optimal controls for VRE models with appropriate objective and control functions. Key controls included combinations of the levels of special preventive care, healthcare workers' compliance rates, and health and economical costs. In particular, it is shown that, rather than constant levels, variable and lower levels of special preventive care produced by OCT, are best for controlling VRE and preventing outbreaks.

177. Improved RNA Secondary Structure Prediction Using Stochastic Context Free Grammars

David Esposito Georgia Institute of Technology
Advisor(s): Christine Heitsch, Georgia Institute of Technology

Accurate RNA secondary structure prediction is an important problem in computational biology. Different RNA nucleotide sequences often fold to similar structures causing current prediction algorithms to range widely in accuracy for RNA strands with similar structures. To understand the origins of these inaccuracies we trained a stochastic context free grammar on a hard-to-predict training set and an easy-to-predict training set which corresponds to a set of sequences with low and high prediction accuracy respectively. We found interesting statistical differences in the nucleotide composition of the sequence as well as the distribution of nucleotide base pairs between the two training sets. Stochastic context free grammars provide a means to quantify subtle difference in the composition of native secondary structures. The discovery of these differences could potentially lead to the improvement of current prediction algorithms. We are currently performing a parametric analysis of several prediction methods.

178. Determination of Tucson, Arizona as an Ecological Trap for Cooper's Hawks (Accipiter cooperii)

Andrea Feiler Arizona State University
Advisor(s): Stephen Wirkus, ASU

The term "ecological trap" has been used to describe a habitat in which its attractiveness has been disassociated with its level of suitability. To date, fewer than ten clearly delineated examples of them have been found; they are either rare in nature, hard to detect, or a combination of both. It has been hypothesized that the city of Tucson, Arizona is an ecological trap for Cooper's Hawks (*Accipiter cooperii*) due to the abundance of prey species, namely columbids, which make up over 80% of the hawk's diet. Overall, more than 40% of these columbid populations are carriers of the protozoan *Trichomonas gallinae*, which directly contributes to a nestling mortality rate of more than 50% in the hawks. Using an epidemiological framework, we create two SIR-type models, one stochastic and one deterministic, utilizing parameter estimates from more than ten years of data from the dove (columbid) and hawk populations in the city. Through mathematical modeling and bifurcation theory, we found that the proportion of infected columbids, does not have an effect on classifying Tucson as an ecological trap for Cooper's Hawks, but by increasing the disease death rate, it can be considered an ecological trap.

179. Spatial Spread of Wolbachia: A Strategy to Control Dengue Fever

Alyson Fox Loyola University Maryland
Jared Catenacci University of Wisconsin-Milwaukee
Advisor(s): Alun Lloyd, North Carolina State University

Current strategies for controlling dengue, a mosquito transmitted virus, rely upon reducing mosquito populations. An alternative method involves releasing mosquitoes infected with the bacterium *Wolbachia*, which reduces mosquito lifespan and ability to acquire and transmit dengue virus. We developed both stochastic and deterministic discrete time metapopulation models to study the spread of *Wolbachia*. Using numerical simulations we studied different release strategies and effects of fitness costs and movement rates on the speed at which *Wolbachia* spreads though subpopulations (wave speed). We found that releasing *Wolbachia* into multiple, neighboring subpopulations increases its ability to spread for a wider range of fitness and migration rates. In addition, we studied the effects of varying subpopulation size and found that the wave speed decreased and was more sensitive to fitness costs and movement rates. Differences in wave speed between stochastic and deterministic models were also discovered. The fitness parameter determines which model has the faster wave speed for a given parameter set. Overall, we found that under a variety of conditions, *Wolbachia* can spread and *Wolbachia*-infected mosquitoes can replace an existing population.

180. Clustering Leukemia Patients

Katelyn Gao MIT
Heather Hardeman University of Montevallo
Cristian Potter East Carolina University
Advisor(s): Carl Meyer, North Carolina State University

In this technological age, vast amounts of data are generated. Various statistical methods are used to find patterns in this data, including clustering. Many common methods for cluster analysis, such as k-means and Non-negative Matrix Factorization (NMF), require input of the number of clusters. However, usually the number of clusters is unknown and must be estimated. The recent Stochastic Clustering Algorithm (2010) provides a way to do so by examining the eigenvalues of a consensus matrix for the data set. In this poster, we test the Stochastic Clustering Algorithm on the Broad Institute leukemia data set. We obtain accurate estimates of the number of clusters, which, when used in conjunction with the k-means or NMF methods, lead to excellent clustering of the data.

181. Application of a Hill Climbing Algorithm to Parallelize Graph-based Genome Assembly

Alexandra Gendreau Wellesley College
August Guang Harvey Mudd College
Advisor(s): Albert Ku, Hong Kong University of Science and Technology

Shotgun whole genome sequencing is a technique that has been developed to address the costs associated with sequencing an entire genome base by base. This technique outputs short fragments of DNA, which subsequently require assembly algorithms. ABySS (Assembly By Short Sequences) is a popular choice for assembly due to its highly parallel nature. We improve upon the runtime of ABySS using a hill climbing algorithm designed to minimize communication costs between processors. It executes in less than a minute and cuts down on communication between processors by 6 to 10 %, which in our preliminary trials has cut the run time of ABySS in half when running on four processors.

182. Metapopulation Modeling and Analysis with Demographic Stochasticity

Elise Hellwig Colorado College
Mary Hebert Northwestern State University
Francisco Hernandez-Cruz Occidental College
Bryan Wigianto Rice University
Advisor(s): Richard Rebarber, University of Nebraska, Lincoln

As human expansion claims more land, previously connected habitats become fragmented, converting larger populations into smaller subpopulations connected through migration. Demographic stochasticity has been shown to have a significant effect in small populations. We explore how taking demographic stochasticity into account affects metapopulations which are typically made up of small subpopulations. We develop a density dependent model using Markov matrices to simulate a metapopulation and compare the predictions with those of a deterministic model. We analyze the asymptotic population size predicted by the deterministic model, and the mean time to extinction

and the quasi stationary distributions predicted by the Markov model. We show that there is a significant difference between the two models under certain conditions and that the deterministic model often overestimates metapopulation persistence.

183. A Logical Model Approach to Iron Metabolism in Breast Epithelial Cells

Emily Hendryx Angelo State University
James Brunner University of Michigan
Andrew Reagan Virginia Tech
Paul Vines Roanoke College
Advisor(s): Julia Chifman, Wake Forest University School of Medicine

Iron metabolism is a tightly regulated intracellular network consisting of numerous regulatory feedback loops. Changes in this network have been observed in connection with cancers in breast and other tissues. Because of the network's complexity and apparent importance in cancer biology, we simulated intracellular iron metabolism using a discrete logical mathematical model, from which we created a polynomial dynamical system for analysis. This model includes the main regulatory elements of the network as well as the proteins involved in heme synthesis. By adjusting the logical model, we simulated many different experiments with little extra computational effort. This use of mathematics allows us to gain insight as to which species are crucial to cancer cell survival without the expense of laboratory time and materials. The state space of the model and model perturbations successfully replicated experimental results, and simulations suggest that misregulation of the iron regulatory network is necessary to meet the increased iron demand of a neoplastic cell.

184. Investigating the role of host competition in the transmission of waterfowl disease in the upper Mississippi River

Maria Jansen University of Wisconsin-La Crosse
Matthew Rittenhouse University of Wisconsin-La Crosse
Advisor(s): James Peirce, University of Wisconsin-La Crosse

Introduction of the aquatic snail, Bithynia tentaculata, into the upper Mississippi River (UMR) has had negative impacts on native wildlife in the region. One of the key reasons for this is that B. tentaculata transmits two parasite species to migrating waterfowl which leads to thousands of bird deaths annually. Although the invader and its parasites are adversely affecting the biota and economics of the UMR, little is actually known about the dynamics of disease in this system. We have utilized a combination of theoretical and empirical approaches to gain a better understanding of parasite transmission in the UMR. From a theoretical standpoint, we expanded the classic SIR model to include parameters which take into account intraspecific competition between infected and uninfected B. tentaculata. Experiments were then developed to specifically ascertain values for the parameters introduced into our model. Results from this expanded model suggest that intraspecific interactions between infected and uninfected B. tentaculata can influence the persistence and spread of waterfowl disease in the UMR. The consequences of these results for disease management in the UMR will be discussed.

185. Chaparral Population Models in Response to Wildfires

Garrett Johns Pepperdine University
Wancen Jiang Pepperdine University
Yuezhao Yang Cal Poly Pomona
Advisor(s): Timothy Lucas, Pepperdine University

In recent years, the Santa Monica Mountains have experienced an increase in wildfire frequency which has been a substantial burden on the surrounding plant life. The chaparral shrubs, which account for over half the vegetation, can be divided into three life history types according to their response to wildfires; non-sprouters are completely killed by fire and reproduce by seeds that germinate in response to fire, obligate sprouters resprout after fire, but their seeds are destroyed by fire, and facultative sprouters both reproduce by seeds and resprout. Based on these assumptions we developed a set of nonlinear difference equations to model different species of chaparral. These models can be used to predict species survivorship under varying fire frequencies such as the localized extinction of *Ceanothus megacarpus* behind Pepperdine University. Drawing from 25 years of data, we have estimated appropriate parameter values for several chaparral species. We also explored particular parameter relationships that lead to equilibrium populations.

Finally, we present a model that describes the growth and mortality of individual plants and their influence on the surrounding space due to seed dispersal, shading of young plants and competition for space.

186. How are Sequences Learned in a Zebra Finch Brain?

Jennifer Kile Marist College
Buck Fisk Louis and Clark College
Advisor(s): Sven Anderson, Bard College

Sequences of neural spiking underlie song production in the brain of the zebra finch. A recent computational model of the HVC, a part of the zebra finch brain that represents pre-motor activity, uses a binary neural network to implement biologically based neural response and learning. As the zebra finch learns songs, an initially random matrix of synaptic weights is transformed into a permutation matrix. This talk studies a model that predicts the distribution of lengths of firing sequences in the binary neural network. By studying these sequences of firing neurons, and their distribution of lengths, a hypothesis can be formed about the way in which songs are learned in the zebra finch brain.

187. Development of an Ozone Inhalation Model

Andrew Kirby University of Wisconsin-Madison
Analise Rodenberg Lewis & Clark College
Andrew Bernstein University of Maryland-College Park
Adrian McLean Fayetteville State University
Advisor(s): William LeFew, United States Environmental Protection Agency

The goal of this project that was conducted at North Carolina State University's Research Experience for Undergraduates in Mathematics was to develop a mathematical dosimetry model that simulates the inhalation and deposition of ozone through the respiratory tract. The model is based upon a transport diffusion partial differential equation which describes the flow of ozone through the respiratory tract and the diffusion of ozone in the air. The dosimetry model also encompasses the flux of ozone into the tissue giving total concentrations of ozone deposited in the lung. The model was solved using the Crank-Nicolson implicit scheme. Within our model, we split the domain into multiple compartments which mimic the different generations of the lungs. This required conservation of mass to be incorporated. The results generated by the dosimetry model were then linked with data regarding neurons called C-fibers, which are located in the bottom of the lungs. When exposed to ozone, C-fibers react and cause physiological changes such as frequency and depth of breathing. These are results of the body trying to counter the harmful effects of ozone inhalation. Incorporating this aspect to the inhalation model helps depict a more realistic cycle of ozone uptake.

188. Heuristic Optimal Control on Polynomial Dynamical Systems Expedited by Use of Algebriac Geometry

Atsuya Kumano Reed College
Hussein Al-Asadi University of Michigan, Ann Anbor
Advisor(s): Franziska Hinkelmann, Ohio State University

Polynomial Dynamical Systems (PDS), finite dynamical systems in which the transition of each variable is described by a polynomial, form a mathematical basis for many discrete models used in systems biology. Our novel mathematical contribution is combining heuristic search methods and algebraic geometry to conduct efficient optimal control on PDS. Specifically, we provide an implementation of an adaptive genetic algorithm to find optimal control for gene regulatory networks. If the control objective depends on the long term behavior of the system, we solve a system of polynomial equations instead of calculation by enumeration, which is computationally infeasible on large networks. Solving a system of polynomial equations is a long studied problem in algebraic geometry for which we use Gröbner bases. We demonstrate the feasibility of our algorithm by applying it to a network of proteins involved in the cell cycle transition, our algorithm confirms the optimum found in the published results and was computed in a matter of seconds. Our implementation is available through a web-based software tool (http://adam.vbi.vt.edu/).

189. Modeling feral cat population dynamics in Knox County, TN

Lindsay Lee University of Tennessee
An Nguyen University of Tennessee
Advisor(s): Suzanne Lenhart, University of Tennessee

We present a discrete mathematical model to predict population shifts of feral cats (Felis catus) in selected managed colonies under the current trap-neuter-release (TNR) program in Knox County, TN. We predict feral cat population changes for closed colonies over a period of five years in one-month time steps for three age classes. We tested different TNR parameters to assess how targeting spay/neuter programs seasonally might better address the feral cat problem. Current TNR efforts lack specific targeting and possess limited ability to curb population growth. Targeting TNR intervention at females during the time prior to mating season (December - February) is shown by our model to be more effective than non-targeted intervention, and it requires a fewer number of spay procedures to be preformed during each year. These results suggest a more efficacious and economical strategy than non-targeted TNR programs, and provide a humane and cost-effective alternative to trap-euthanasia (TE).

190. A Symmetric Intraguild Predation Model for the Invasive Lionfish and Native Grouper

Margaret-Rose Leung Oregon State University
Advisor(s): Baojun Song, Montclair State University

Lionfish are top-level venomous predators native to the Indo-Pacific Ocean. Over the past decade, the species *Pterois volitans* and *P. miles* have become established throughout most of the western Atlantic Ocean, where they drastically impact coral reef communities. Overfishing of native species, such as grouper, who share their niche with lionfish may be the reason for the lionfish's success; research has suggested that at high density, groupers can act as a lionfish biocontrol. To determine if competition or predation is the mechanism behind lionfish suppression, we construct a symmetric intraguild predation model of lionfish, grouper, and prey. Thus, we assume lionfish and grouper compete for prey in addition to consuming juveniles of the other species. Holling type I functional responses are used to represent fecundity and predation. We conduct an equilibrium stability analysis and bifurcation analysis of the general model, and find that the system is able to coexist in an equilibrium or sustainable oscillations. After estimating parameter ranges, simulations and a sensitivity analysis indicate the parameters most influential to lionfish growth rate. The implied control strategies are then tested by varying harvesting and predation rates.

191. Computational Efficiences of Stochastic Algorithms

Kathryn Link Bryn Mawr College
Kaitlyn Gayvert State University of New York at Geneseo
Anna Broido Boston College
Advisor(s): H.T. Banks, North Carolina State University

There are a variety of stochastic computational algorithms in use, but for a given model it is unclear which method is most advantageous. Deterministic approaches involving ordinary differential equations to approximate large sample sizes with a continuum have proven less descriptive when applied to small sample sizes. To address this issue, stochastic simulation methods are used when dealing with low species count or a large number of transitions. However, stochastic methods can prove to be computationally expensive. We compared one hybrid method and various stochastic methods, with respect to time, to determine appropriateness for use with a large HIV model.
We investigate five particular algorithms. To compare these methods, we used them to analyze two similar compartmental, stiff infection models (Vancomycin-resistant enterococcus (VRE) and HIV within host infection models). The relative efficiency of each algorithm is determined based on computational time and degree of precision required. We have found that with the more complex HIV model, Tau-Leaping methods are preferred. We aim to illustrate how widely performances vary between two infection models and demonstrate how one might perform computational studies to aid in selection of appropriate algorithms.

192. Short and Long Range Population Dynamics of the Monarch Butterfly (Danaus plexippus)

Komi Messan North Carolina A&T State University
Advisor(s): Sergei Suslov, Arizona State University

The monarch butterfly annually migrates from central Mexico to southern Canada. During recent decades, its population has been reduced due to human interaction with their habitat. We examine the effect of herbicide usage on the monarch butterfly's population by creating a system of linear and non-linear ordinary differential equations that

describe the interaction between the monarch's population and its environment at various stages of migration: spring migration, summer loitering, and fall migration.

193. Exploring phylogenetic relationships in Drosophila using ciliate operations

Anna Nelson Boise State University
Jacob Herlin University of Northern Colorado
Advisor(s): Marion Scheepers, Boise State University

Phylogenetics is the study of evolutionary relationships among groups of organisms. We studied phylogenetic relationships among fruit-fly species using the DNA-editing operations of ciliates. The relative order of orthologous genes in one species is a scrambled version of the corresponding genes in a canonical reference species. Ciliates are capable of permuting DNA segments using merge, swap, and reverse operations. We created an algorithm that simulates permuting DNA sequences using the three ciliate operations. We implemented the algorithm in Python. It determines in polynomial time evolutionary distances among scrambled genomes. Using the algorithm, we found a correlation between the published evolutionary distances of the fly species, found by other means, and the number of reverse operations used. This correlation also held for the total number of operations used for all species but one.

194. Methods for Reducing and Transforming Agent-Based Models into Polynomial Dynamical Systems

Laurel Ohm St. Olaf College
Advisor(s): Reinhard Laubenbacher, Virginia Tech

Discrete models, including agent-based models, are important tools for modeling biological systems, but model complexity may complicate meaningful analysis of the system. Representing a discrete model as a polynomial dynamical system (PDS) provides a framework for efficient analysis using theory from abstract algebra. To lay the foundation for a PDS translation of agent-based models, we came up with a set of general polynomials to describe common agent interactions as well as methods to reduce the complexity of the model while preserving key system dynamics. Algebraic tools are used in the construction of polynomials as well as in the reduction of the model. We demonstrate the feasibility of our methods by translating a complex agent-based model of the human innate immune response system (approximately 11,000 agents) into PDS of lesser complexity. The resulting PDS successfully simulated the system dynamics described by the agent-based model. Using abstract algebra theory, we hope to analyze and eventually apply optimal control to the resulting PDS.

195. Next Generation Sequencing

Matthew Rieck University of Sioux Falls
Advisor(s): Chad Birger, University of Sioux Falls

Ever since Mendel's experiments with pea plants from 1856 through 1863 initiated the area of science known as genetics, this science has grown dramatically, culminating in the decoding of the human genome in 2003. With the rise in cancer and other genetic abnormalities, the study of mutations within the genome has risen to the forefront. The research done last summer examined several different portions of DNA in order to determine whether one substitution mutation has a greater probability of occurring than other mutations, or if base pairs are substituted with the same frequency. This data was also used to statistically examine the genome alignment program, BLAT, to determine if there are variables that can be used to predict whether a portion of DNA will be mapped back to the genome that it was obtained from.

196. New Models for Aligned Swarm Behavior

Louis Ryan Harvey Mudd College
Dylan Marriner Harvey Mudd College
Advisor(s): Andrew Bernoff, Harvey Mudd College

Collective social behavior, otherwise known as swarming, has been demonstrated by many different types of organisms, including bacteria, insects, fish, birds, and mammals. Many attempts have been made to develop swarming models that exhibit traits found in natural swarms. In particular, many models have tried to recreate the aligned motion found in fish schools and mills, bird flocks, and other swarming groups. The most common method is the inclusion of an explicit alignment term in the model. We have investigated two biologically realistic models that demonstrate emergent aligned behavior without the presence of an arbitrary alignment turn. Our two models incorporate biological heterogeneity and asymmetric sensing, respectively, into standard attractive-repulsive swarming

models. Both approaches were capable of producing milling and migrating behaviors. In the asymmetric sensing model, we were able to obtain a migrating solution without incorporating a minimum velocity requirement. In the heterogeneity model, we observed a behavior transition between milling and migrating solutions which depended on the variability in the population. We also observed a biologically relevant result in which smaller particles were located at the center of the swarm.

197. Assessing the levels of unreporting in the 2009 A-H1N1 influenza epidemic in Lima, Peru

Michelle Salas University of Portland
Advisor(s): Karen Rios-Soto, University of Puerto Rico at Mayaguez

Mathematical models can provide insights on how future epidemics may behave. Evaluation and implementation of public health strategies can be more accurate when reliable data is used to estimate parameters. However, not all cases are reported, and the levels of uncertainty generated by the gap between the number of reported cases and the actual number of cases has not been studied in detail. We evaluate the impact of non-reported cases in the calculation of the final epidemic size and the effect of different control measures on reducing the attack rate.

198. Tear-film dehydration of a soft contact lens

Mihail Sharov George Mason University
Advisor(s): Daniel Anderson, George Mason University

The main focus of this research is to explore the factors that cause evaporative dehydration of tear film when a soft contact lens is present. Evaporation of the tear film is affected by various environmental conditions, such as relative humidity and wind speed. It also depends significantly on the time period of blink cycles, since the eyelid provides a new tear film to the surface of the eye during blinks. Every blink cycle has different duration and some blinks are not complete. I plan to improve a previously created model in order to incorporate this blink cycle variability and thus mimic more realistic blinking conditions.

199. A Novel Unsupervised Clustering Algorithm for Binning Metagenomic Sequences

Kyler Siegel Stanford University
Kristen Altenburger Ohio University
Advisor(s): Chenlong Yu, Chinese University of Hong Kong

Current DNA sequencing technology is incapable of directly sequencing most microbial communities. As an alternative, metagenomics captures short, random DNA fragments from all the species present, in a process known as "shotgun sequencing". The "binning problem" is to then determine how many species are present in the sample and which DNA fragments come from which species. This computational problem can be especially difficult since there could be sequencing errors and two or more species may be very closely related. We present a new binning algorithm which clusters data points by searching the feature space for spherical regions of high density. Our algorithm uses a mixed Gaussian distribution model and performs the Expectation-Maximization algorithm to estimate optimal clustering parameters. We also incorporate word-overlap information between fragments for improved accuracy. Tested on known datasets, our algorithm outperforms the best known unsupervised binning algorithms.

200. Modeling Blood Pressure Dynamics

Alberto Soto California Polytechnic University, Pomona
Christiana Sabett St. Mary's College of Maryland
Advisor(s): Mette Olufsen, North Carolina State University

This project discusses a patient-specific model predicting baroreflex regulation of blood pressure and heart rate during head-up tilt (HUT). The model contains five compartments organized analogous to an RC-circuit. The model uses heart rate as an input and predicts blood flow, volume, and pressure in the heart, arteries, and veins of the upper and lower body. Gravitational pooling of blood in the lower body in response to HUT is modeled via a modified Ohm's law. The regulatory response allowing the body to maintain homeostasis is modeled by defining cardiac contractility, peripheral resistance, and vascular compliance as piecewise linear functions in time. The objective of our study is to estimate model parameters that allow prediction of patient specific arterial blood pressure dynamics. Results showed that the model is able to predict physiologically reasonable dynamics of measured blood pressure as well as other pressures and volumes both during steady state (in supine position) and during HUT. During steady

state five upper body parameters were identifiable: ventricular elastance, peripheral resistance, arterial and venous compliance. Using optimized values for these parameters, the response to HUT was predicted by redefining four of the five parameters as linear functions.

201. Social Dynamics of Gang Involvement: A Mathematical Approach

Sowmya Srinivasan Bryn Mawr College
Joshua Austin University of Maryland, Baltimore County
Emma Smith Linfield College
Advisor(s): Fabio Sanchez, Arizona State University

Gangs have played a significant role in Chicago's social and political history, and continue to impact the city today, as gang violence rates continue to grow despite drops in overall crime. In this paper, we explore the dynamics of gang involvement between at-risk individuals, gang members, and reformed (temporarily removed) gang members. We focus on the effect that reformed gang members have on the at-risk population via a general function, which takes into account cost of gang membership and a threatening factor.

202. Using a combination of empirical and theoretical approaches to assess a host-parasite interaction of conservation concern

Kacie Van Calster University of Wisconsin-La Crosse
Kari Soltau University of Wisconsin-La Crosse
Advisor(s): James Peirce, University of Wisconsin-La Crosse

Bithynia tentaculata is an invasive snail that was introduced into the upper Mississippi River (UMR) in 2002. The snail harbors a parasite, Sphaeridiotrema pseudoglobulus, which causes high annual waterfowl mortality in the UMR. As part of our study, we utilized theoretical and empirical approaches to determine the effects that S. pseudoglobulus has on B. tentaculata life-history expression. Differential equations were developed to represent snail energy budgets in the presence and absence of infection. Parameters in the model were gleaned from an empirical study where snails were infected with known numbers of S. pseudoglobulus larvae. Experimental results showed that life-history responses of infected B. tentaculata differed from those of uninfected snails. Incorporating these data into our model revealed that S. pseudoglobulus can influence the energy budgets of infected B. tentaculata in the UMR which has implications for snail and parasite success. This work emphasizes the importance of host energy budgets in systems involving invasive species and disease, and furthers our understanding of interactions between B. tentaculata and S. pseudoglobulus in the UMR.

203. Physiologically-based pharmacokinetic (PBPK) modeling of metabolic pathways of bromochloromethane in rats

Melissa Venecek The College of Wooster
Tyler Janes University of Colorado Boulder
William Cuello University of California Berkeley
Jill Jessee Simpson College
Advisor(s): Marina Evans, U.S. Environmental Protection Agency

Bromochloromethane (BCM) is a volatile compound and a by-product of disinfection of water by chlorination. We developed a physiologically-based pharmacokinetic model and explored three hypotheses describing metabolic pathways of BCM in rats. The hypotheses are: 1) Michalis-Menten kinetics with one CYP2E1 binding site, 2) a two-pathway model using both CYP2E1 and glutathione transferase enzymes, and 3) a two-binding site model where metabolism can occur on one enzyme, CYP2E1. One goal of this project is to demonstrate the utility of PBPK modeling for hypothesis testing with BCM's kinetics. Our computer simulations show that all three hypotheses generally describe the experimental data. Of the three kinetic models tested, the two-binding site model provided better fits to the data, producing smaller differences between the data and computer simulations. Finally, we explore the sensitivity of different parameters for each model using our obtained optimized values. (This abstract does not represent EPA policy).

204. Mathematical Models of Infectious Diseases: Two-Strain Infections in Metapopulations

Rachel Von Arb Olivet Nazarene University
Sydney Philipps Carroll College
Daniel Rossi SUNY Geneseo
Advisor(s): Alex Capaldi, Valparaiso University

Viruses and bacteria responsible for infectious diseases often mutate and are carried between geographical regions. We consider a mathematical model which begins to account for these factors. We assume two disjoint populations that only occasionally comingle, and two strains of a disease present in these populations. Of interest are the equations describing the dynamics of this system, the conditions under which epidemics will occur, and the long term behavior of the system under various initial conditions. We find general conditions under which a state of disease-free equilibrium is stable; we examine the sensitivity of our system to changes in modeling parameters; and we find evidence that two disease strains of unequal strength may coexist in a two population system.

205. Non-Consecutive Pattern Avoidance in Binary Trees

Michael Dairyko Pomona College
Casey Wynn Hendrix College
Samantha Tyner Augustana College
Advisor(s): Lara Pudwell, Valarapsio University

In this paper we consider the enumeration of full binary trees avoiding non-consecutive binary tree patterns. We begin by modifying a known algorithm that counts binary trees avoiding a single consecutive tree pattern. Next, we use our algorithm to prove several theorems about the generating function whose nth coefficient gives the number of n-leaf trees avoiding a pattern. In addition, we investigate and structurally explain the recurrences that arise from these generating functions. Finally, we examine the enumeration of binary trees avoiding multiple tree patterns.

206. Hitting Set Size for Random Set Systems: The Quarter Pounder Approach

Jessie Deering East Tennessee State University
Advisor(s): Anant Godbole, East Tennessee State University

Let Λ be a random set system of $[n] = \{1, 2, ...n\}$, where $\Lambda = \{A_j | A_j \in p([n])$, and A_j selected with probability $p\}$. A set $H \subseteq [n]$ is a hitting set of Λ if $|H \bigcap A_j| \geq 1$ for all $A_j \in \Lambda$. We explore the cardinality of H with respect to p and n using probabilistic methods.

207. New Connections between the Abelian Sandpile Model and Domino Tilings

Laura Florescu Reed College
Advisor(s): David Perkinson, Reed College

We examine the connections between the abelian sandpile model and domino tilling. Several original theorems on grid graphs with Klein Four group symmetry are presented, relating the number of symmetric recurrent configurations on grid graphs to the number of domino tilings on different checkerboards. A new proof for the number of tilings on a checkerboard is presented, as well as a partial new proof for the number of tilings on a Möbius checkerboard. We also present a number of other theorems concerning specific graphs, as well as recurrent configurations on grid graphs without symmetry. In exploring grid graphs with dihedral symmetry we find a relation between the number of symmetric configurations and weighted domino tilings on a class of graphs studied in Pachter (1997). Future work involves finding the complete new proof for the number of tilings on a Möbius checkerboard, as well as investigating a group law for the tilings arising from different configurations, inspired by the rotor-router model proposed by Holroyd et al.

208. A Combinatorial Approach to r-Fibonacci Numbers

Curtis Heberle Harvey Mudd College
Advisor(s): Arthur Benjamin, Harvey Mudd College

In a recent publication of the Fibonacci Quarterly, Curtis Cooper and F. T. Howard prove a number of interesting identities involving generalized r-Fibonacci numbers using algebraic techniques. We present combinatorial proofs of many of their results using a tiling approach. We also investigate identities involving both Fibonacci numbers and binomial coefficients, and their appropriate generalizations.

209. Pattern Avoidance and Sorting Algorithms

Jennifer Herdan East Tennessee State University
Advisor(s): Anant Godbole, East Tennessee State University

This poster consists of recent work done in the area of permutation patterns. One direction is in the area of pattern avoidance over set partitions, particularly 123-avoiding set partitions. We will discuss a method for counting these partitions, particularly for the 3-partition problem. Another area of research is comprised of sorting random permutations with different sorting mechanisms. One of the sorting mechanisms studied was homing, which involves selecting an element and sorting it to its 'home'. Our results include homing in n-dimensions, and homing on multiset permutations, specifically 2-1 permutations.

210. The Abelian Sandpile Model

Jesse Herring Sam Houston State University
Christina Nieuwoudt Sam Houston State University
Everett Meza Sam Houston State University
Advisor(s): Luis Garcia, Sam Houston State University

In 1987 physicists Bak, Tang, and Westfield introduced the sandpile model to study the dynamics of sandpile avalanches. The sandpile model is represented by a combinatorial graph G. Surprisingly, this model has the structure of a finite abelian group which arises as the cokernel of the Laplacian of the graph G. One of the main problems consists in finding the group associated to a given graph, which is done by computing the Smith Normal Form of the Laplacian. It is a more challenging combinatorial problem to find the structure of the family of groups arising from a given family of graphs. Only a few results are known in this direction. Families of graphs whose sandpile groups have been characterized include cycle graphs, wheel graphs, complete graphs, and complete multipartite graphs. In this presentation we will describe the sandpile group of a family of graphs known as book graphs. We will also make connections between the elements in the group and the sandpile configurations in the model. Our proofs are based on a careful study of the Laplacian of these graphs and rely on linear algebra techniques.

211. Commutative Ideals of Upper Triangular Matrices Bijected to Well-Known Catalan Objects

Heather Kitada Lewis & Clark College
Advisor(s): Naiomi Cameron, Lewis & Clark College

In this talk, we will demonstrate a counting argument for the number of ideals in the ring of upper triangular matrices by right justified Ferrers diagrams. It is known that the n-th Catalan number is the number of ideals in the ring of $(n-1)\times(n-1)$ upper triangular matrices. In addition, we will focus on the subset of ideals that are commutative. It turns out that there are 2^{n-2} commutative ideals in the ring of $(n-1)\times(n-1)$ upper triangular matrices. We will show through a partition of Young's lattice how one can count these commutative ideals using binomial coefficients, which are manifested in Pascal's triangle. Furthermore, we hope to illuminate how commutative ideals translate to other Catalan objects, including Dyck paths, binary trees and ordered plane trees by creating binary operations on the objects and illustrating analogous cover relations.

212. The Well-Covered Dimension of Products of Graphs

Megan Kuneli California State University, Fresno
Robyn McDonald California State University, Fresno
Advisor(s): Oscar Vega, California State University, Fresno

Our project focuses on the well-covered dimension of (cartesian) products of graphs, specifically paths, cycles and wheels. We have proved that the well-covered dimension of these products is always zero when the paths and/or cycles are sufficiently large. We have also studied products of graphs in other families. By doing this we have found an example of a graph that has well-covered dimension depending on the characteristic of the field used to define the vector space of weights (on the vertices of the graph).
We are currently searching for a formula that relates the well-covered dimension of G X H with the well-covered dimension of G and H.

213. Universal Cycles under Equivalence Relations

Andre Kuney Oberlin
Melinda Lanius Wellesley College
Advisor(s): Anant Godbole, Eastern Tennessee State University

Consider all possible length-k words taken from a size-n alphabet. It is classical that we can create a string such that the set of all length-k consecutive substrings of this string consists of each of our words exactly once; this string is called a universal cycle. In 1992, Chung, Diaconis, and Graham introduced the concept of a universal cycle for other combinatorial structures. It turns out that often the machinery needed to deal with these combinatorial structures is a way of analyzing our length-k words under particular equivalence relations. Here, we develop techniques for dealing with equivalence relations under certain conditions, and consider some ramifications.

214. Zero Forcing Number, Maximum Nullity and Path Cover Number of Edge Subdivision Graphs

Kirill Lazebnik State University of New York at Geneseo
Anna Cepek Bethany Lutheran College
Advisor(s): Leslie Hogben, Iowa State University

For a simple graph G the zero forcing number $Z(G)$ is the minimum number of black vertices initially needed to force all vertices in G black according to the color change rule. The color change rule states that for G with all vertices colored either black or white, if a vertex v is black and an adjacent white vertex w is the only white neighbor of v, then v can force w to be colored black. The maximum nullity $M(G)$ is defined to be the largest possible nullity over all symmetric real matrices described by G. It is known that $M(G) \leq Z(G)$ for all G. The path cover number of G denoted $P(G)$ is the minimum number of induced paths needed to cover all the vertices of G. To subdivide the edge e between vertices u and v, delete e and add a new vertex w adjacent to exactly u and v. An edge subdivision graph of G is obtained from G by subdividing one or more edges of G. We present results on $Z(H)$, $M(H)$, and $P(H)$ for an edge subdivision graph H of G. An open question in the literature about $M(H)$ is answered in the negative.

215. A Family of Multidimensional Continued Fraction Stern Sequences

Sarah Peluse The University of Chicago
Chansoo Lee Williams College
Advisor(s): Thomas Garrity, Williams College

The Stern-Brocot tree is a tree of fractions in lowest terms that contains all rationals in the interval $[0, 1]$. The denominators of the Stern-Brocot tree form the sequence $1, 1, 2, 1, 3, 2, 3, 1, 4, \ldots$, known as Stern's diatomic sequence, which satisfies the recurrence relation $a_2 = a_n$ and $a_{2n+1} = a_n + a_{n+1}$. Each level of the tree describes a method of dividing up the unit interval and is intimately linked to the continued fraction algorithm. We have constructed analogous sequences from multidimensional continued fraction algorithms, which approximate pairs of real numbers by subdividing the triangle $\{(x, y) \in \mathbb{R}^2 | 0 < y \leq x \leq 1\}$, to create higher dimensional generalizations of the Stern-Brocot tree and Stern's diatomic sequence. These sequences give rise to the Fibonacci numbers and other well-known sequences and posses several remarkable combinatorial properties. In particular, we've determined which triples of integers appear in these sequences, the sums at each level of the resulting tree, and the sequence of maximum entries at each level of the tree and their locations.

216. An Anti-Waring Theorem and Proof

Nathan Saritzky UC Santa Barbara
Nicole Looper Dartmouth University
Advisor(s): Peter Johnson, Auburn University

In 1770, Edward Waring conjectured that for each positive integer k, there exists an integer $g(k)$ such that every positive integer is a sum of at most $g(k)$ k^{th} powers. Hilbert proved this conjecture in 1909, giving rise to the problem of finding the minimal $g(k)$ for each k. In 2010, Johnson and Laughlin posed the "anti-Waring" conjecture: That for all positive integers k and r, every sufficiently large natural number is a sum of at least r distinct k^{th} powers. The $k = 0$ case is a consequence of a theorem of Roth and Szekeres regarding complete sequences, and $k = 1$ was proved by Johnson and Laughlin. We give a general proof for all k and r.

217. Binomial Type Polynomial Sequences Related to Tiling Problems

Jonathan Schneider MIT
Advisor(s): Richard Stanley, MIT

In this paper, we show that the solution to a large class of "tiling" problems is given by a polynomial sequence of binomial type. More specifically, we show that the number of ways to place a fixed set of polyominos on an n-by-n toroidal chessboard such that no two polyominos overlap is eventually a polynomial in n, and that certain sets of these polynomials satisfy binomial-type recurrences. We exhibit generalizations of this theorem to higher dimensions and other lattices. Finally, we apply the techniques developed in this paper to resolve an open question about the structure of coefficients of chromatic polynomials of certain grid graphs (namely that they also satisfy a binomial-type recurrence).

218. Higher Rectification and Polytope Numbers: The Simplex Case

Christina Scurlock Grove City College
Chelsea Snyder Grove City College
Advisor(s): Michael Jackson, Grove City College

A polytope number is a term in the integer sequence which is determined by the arrangement of points in a polytope. Although the polygonal numbers are easily determined, our understanding of polytope numbers in higher dimensions is not yet complete. H.K. Kim has recently established a method for constructing the number sequence of any uniform polytope in any dimension. We will discuss Kim's inductive process as it applies to birectified simplices. In addition, we will explore higher rectification in relation to polytope numbers to propose conjectures for further research.

219. Permutation Patterns and Statistics

Kimberly Selsor University of South Carolina
Advisor(s): Bruce Sagan, Michigan State University

Let S_n be the symmetric group of all permutations of $\{1, 2, \ldots, n\}$ and Π be a set of permutations. We use $Av_n(\Pi)$ to denote the set of $\sigma \in S_n$ that avoid all permutations $\pi \in \Pi$. We say σ avoids π if σ does not contain a subsequence whose elements are in the same relative order as π. For example, 24513 contains 132, because of the subsequence 253, but avoids 321. Π and Π' are said to be Wilf-equivalent if $\#Av_n(\Pi) = \#Av_n(\Pi')$ for all $n \geq 0$. A permutation statistic is a function $st : S_n \to \mathbb{N}$ and the corresponding generating function is

$$F_n^{st}(\Pi; q) = \sum_{\sigma \in Av_n(\Pi)} q^{st\sigma}.$$

We define Π, Π' to be st-Wilf equivalent if $F_n^{st}(\Pi; q) = F_n^{st}(\Pi'; q)$ for all $n \geq 0$. We will focus on the inv and maj statistics, where $inv(\sigma)$ counts the number of pairs of out-of-order elements in σ and $maj(\sigma)$ counts the descents of σ weighted by position. In particular, we find all inv- and maj-Wilf equivalences for any $\Pi \in S_3$. We investigate the properties of the generating function, some relating to the Fibonacci numbers, integer partitions, and lattice paths.

220. Perfect Partitioning of Permutations

Jeff Soosiah College of William and Mary
Advisor(s): Gexin Yu, College of William and Mary

Given sets of permutations as block permutation matrices of fixed dimensions, we conjecture that the set of all such permutations can be perfectly partitioned, for any choice of dimensions and block size. We demonstrate that the conjecture holds for specific choices of the dimension with block size 2 x 2.

221. The structure of the Tutte--Grothendieck ring of ribbon graphs

Daniel Thompson Yale University
Advisor(s): Neal W. Stoltzfus, Louisiana State University

W. H. Tutte's 1947 paper on a ring generated by graphs satisfying a contraction-deletion relation is extended to ribbon graphs. This ring of ribbon graphs is shown to be a polynomial ring on an infinite set of one-vertex ribbon

graphs. First we introduce ribbon graphs, which correspond to embeddings of graphs in oriented surfaces such that the complement of the graph is a disjoint union of disks. These are intimately related to Grothendieck's theory of *dessins d'enfants*. We summarize the construction and structure of the Tutte–Grothendieck ring of graphs, then we extend the algebraic construction to ribbon graphs. Using ideas of Bollobas and Riordan, we construct relations in this ring and develop the technique of R–operations to show that certain elementary ribbon graphs generate the ring. We then prove that the elementary ribbon graphs are algebraically independent.

222. The Weak Bruhat Order and Separable Permutations

Fan Wei MIT
Advisor(s): Richard Stanley, MIT

A *separable permutation* is a permutation of $1, 2, \ldots, n$ that avoids the two patterns 2413 and 3142. For a separable permutation π we consider the rank generating function of the two intervals $[\mathrm{id}, \pi]$ and $[\pi, w_0]$ in weak Bruhat order, where $w_0 = n, n-1, \ldots, 1$. We show the surprising result that the product of these two generating functions is the generating function for the entire symmetric group with the weak order. We then obtain explicit formulas for the rank generating functions, which leads to the rank-symmetry and unimodality of the two graded posets.

223. The Periodicity of 3-Element Subtraction Games

Jordan White CSU Monterey Bay
Advisor(s): Rachel Esselstein, CSU Monterey Bay

The periods of 2-element subtraction games are already thoroughly understood. The book Winning Ways for Your Mathematical Plays states that proving the behavior of periods of n-element subtraction games is an important open problem. In this project we will seek to predict the period lengths of all 3-element subtraction games, but more specifically, we will evaluate the period lengths of games with subtraction sets 1, b, c and 2, b, c. To prove our results, we will consider each game's period using a form of induction. We will present conjectures as well as original theorems.

224. Subtraction Games on Finite Graphs

Karen Willis California State University Fresno
Jamie Peabody California State University Fresno
Advisor(s): Oscar Vega, California State University Fresno

We study winning strategies of a game played on a finite graph. A legal move in this game consists of a player removing a vertex and consequently all edges attached to the vertex. A player wins if they can leave the other player with no legal moves (when there are no more vertices left, or all remaining vertices are disconnected). This game seems to have not been studied before. Our focus has been on winning strategies on several different families of graphs. We have already found some fascinating strategies for complete graphs, complete bipartite graphs, some products of graphs, and any symmetric graph. We also have a myriad of results for paths, cycles, and wheels. We are currently investigating winning strategies in less symmetric graphs and more products of graphs, and also exploring the behavior in even paths.

225. Pattern Avoidance in α and β-permutations

Sophia Yakoubov MIT
Advisor(s): Richard Stanley, MIT

Define an α-permutation to be a permutation $a_1 \cdots a_{2n}$ in the symmetric group S_{2n} for which $1, 2, \ldots, n$ appear in increasing order and, for $1 \leq i \leq n$, $i + n$ appears to the right of i. Similarly, define a β-permutation to be a permutation $a_1 \cdots a_{2n} \in S_{2n}$ for which $1, 3, 5, \ldots, 2n-1$ appear in increasing order and, for $1 \leq i \leq n$, $2i$ appears to the right of $2i-1$. If $w = b_1 \cdots b_k \in S_k$, then we define a permutation $v \in S_m$ to be *w-avoiding* if no subsequence of v has its elements in the same relative order as w. We will consider the enumeration and structure of α and β-permutations that avoid one or two patterns of length three, a natural extension of earlier work on pattern avoidance in the set of *all* permutations in S_n. Of particular interest is the number of 312 (and, similarly, 321) avoiding β-permutations in S_{2n}, which we show is the same as the number of ternary trees on n nodes. Compare this with the well-known result that the number of permutations in S_n avoiding any fixed permutation in S_3 is the number of *binary* trees on n nodes.

226. The Insertion-Deletion Model Applied to the Genome Rearrangement Problem

Noah Williams University of Wisconsin-Eau Claire
Advisor(s): Manda Riehl, University of Wisconsin-Eau Claire

Many mathematical models have been developed to help solve the genome rearrangement problem, whose goal is to find the optimal sequence of mutations for the transformation of one genome into another. However, few of these representations consider small segments of DNA in which insertions and deletions are the primary mutations that occur. We created the Insertion-Deletion model in order to provide bioinformaticians with a tool for studying diseases, like Neurofibromatosis, which can develop as the result of a single insertion or deletion. In this research, we use combinatorics to develop a distance formula, and we analyze and apply it to simulate genome rearrangement by insertions and deletions. Our results contribute to the understanding of diseases and of the evolutionary relationships that exist between organisms and their biological ancestors.

227. Inequalities and Isomorphisms in L_p Spaces, $p > 2$, with the Alspach Norm

Sean Vanden Avond University of Wisconsin-Eau Claire
Noah Williams University of Wisconsin-Eau Claire
Chunyang Tang University of Wisconsin-Eau Claire
Advisor(s): Simei Tong, University of Wisconsin-Eau Claire

Understanding L_p spaces has important applications in mathematics, physics, and engineering. Mathematicians use inequalities to establish isomorphisms between subspaces of L_p and spaces of sequences, which are better known. In 1999, Alspach proposed a norm defined by partitions and weights to further classify the complemented subspaces of L_p. Tong, Phillipson, and Defrain have explored this norm for pairings of one and two partitions with weight functions. In our research, we expand upon their work by considering Alspach's norm for two or more partitions and weights.

228. A New Technique for Studying the Rationals

Nathan Bishop St. Olaf College
Advisor(s): Adam McDougall, St. Olaf College

Although using Lebesgue Measure is ideal when studying the Reals, the very properties that give it strength render it inapplicable to the Rationals. Hence, if we wish to study properties of $\mathbb{Q}$, we must develop different tools. This poster will show how, by embedding the Rationals into a $\mathbb{N} \times \mathbb{N}$ grid, we can use the existing techniques of $\mathbb{N} \times \mathbb{N}$ Asymptotic Density to define and utilize Rational Asymptotic Density. This poster will also highlight important Theorems and results in the theory.

229. Descriptive Ergodic Theory

Ke Cai Bard College
Wei Sun Williams College
Advisor(s): Cesar Silva, Williams College

We study dynamical systems from a descriptive point of view. A descriptive dynamical system consists of a Polish space X equipped with a Borel σ-algebra $\mathcal{B}$ and a Borel measurable automorphism $T : X \to X$. We explore ideas that come from measurable dynamics and formulate notions of descriptive ergodicity using σ-ideals, especially the Shelah-Weiss Ideal and the Hopf Ideal. We prove that descriptive ergodicity is preserved under the processes of induction and exduction of a transformation.

230. Nevanlinna-Pick problem with boundary conditions: minimal norm rational solutions of low degree

Stephen Cameron The College of William and Mary
Advisor(s): Vladimir Bolotnikov, The College of William and Mary

We study the following Nevanlinna-Pick type interpolation problem: given n distinct points $z_1, \ldots, z_n$ in the closed unit disk $\overline{\mathbb{D}}$ and n complex numbers $w_1, \ldots, w_n \in \overline{\mathbb{D}}$, find a rational function f which is bounded by one in modulus on $\overline{\mathbb{D}}$ and satisfies $f(z_i) = w_i$ for $i = 1, \ldots, n$. We say that f is of low degree if $\deg f < n$.

We show that there are infinitely many low degree solutions of the problem whenever the interior subproblem (containing all interpolation conditions at the points $z_i \in \mathbb{D}$) is indeterminate. Furthermore, we find the explicit formula for $\inf \|f\|_\infty = \sup_{z\in\mathbb{D}} |f(z)|$ where the infimum is taken over all solutions to the problem. We then present necessary and sufficient conditions for the existence and for the uniqueness of a solution with the minimally possible H^∞-norm. Finally, we construct efficient algorithms to construct infinite families of low degree solutions and (where applicative) minimal norm solutions.

231. Constructible Sets and Partitions

Miranda Polin Northeastern University
Advisor(s): Jorge Garcia, California State University, Channel Islands

Consider an initial family of subsets of some universe X. We define a 1-constructible family to be the family obtained from an initial family by adding all pairwise unions, pairwise intersections, and complements. We then iterate the operations to obtain a k-constructible family for each integer k. Under some conditions, there exists a k such that all families beyond the k-constructible family are identical. In this case, the final family has an algebraic structure. When the universe, X, is finite, we characterize initial families that generate the power set of X. In addition, given any universe and any finite initial family, we describe the elements of any finite algebra constructible from the initial family. We use analysis to prove that any finite algebra, A, has a unique maximum partition that generates A. We calculate the size of a smallest generating family based on the order of the algebra and provide a construction for a generating family of that size. We prove that there are multiple generating families of the smallest size and count the number of such generating families for algebras of size up to 2^8.

232. On Legendre Multiplier Sequences

Katherine Urabe California State University, Fresno
Advisor(s): Tamas Forgacs, California State University, Fresno

In this paper we give a complete characterization of linear, quadratic and geometric Legendre multiplier sequences. We also prove that all Legendre multiplier sequences must be Hermite multiplier sequences, and describe the relationship between the Legendre and generalized Laguerre multiplier sequences. We conclude with a list of open questions for further research.

233. Operations on Solid Angles

Michelle Bodnar University of Michigan
Aaron Mayerson Macalester College
Advisor(s): Tyrrell McAllister, University of Wyoming

We investigate how certain valuations, which arise from solid angle functions, interact with the operations of Cartesian product and Minkowski sum. For a polyhedron $\mathcal{P} \subseteq \mathbb{R}^m$, we prove the following relations about the solid angle $\omega_\mathcal{P}(x)$ of a point x, the solid angle generating function $\alpha_\mathcal{P}(x)$, and the solid angle polynomial $A_\mathcal{P}(t)$:

$$\begin{aligned}\omega_{\mathcal{P}\times\mathcal{Q}}(x,y) &= \omega_\mathcal{P}(x)\cdot\omega_\mathcal{Q}(y) \quad (*)\\ \alpha_{\mathcal{P}\times\mathcal{Q}}(x,y) &= \alpha_\mathcal{P}(x)\cdot\alpha_\mathcal{Q}(y)\ \ A_{\mathcal{P}\times\mathcal{Q}}(t)\\ &= A_\mathcal{P}(t)\cdot A_\mathcal{Q}(t).\end{aligned}$$

We generalize the preceding results by interpreting solid angles at a point as a measure, and classifying all measures for which the Cartesian product relation $(*)$ holds. These are precisely the measures which are locally spherically symmetric. Finally, we prove the following relation to show how solid angles behave under Minkowski sums:

$$\omega_{\mathcal{P}+\mathcal{Q}}(x+y) = \omega_\mathcal{P}(x) + \omega_\mathcal{Q}(y) + \omega_{(\mathcal{P}+\mathcal{Q}-y)\cap[\mathcal{P}\cup(\mathcal{Q}-y+x)]^C}(x) - \omega_{\mathcal{P}\cap(\mathcal{Q}-y+x)}(x).$$

234. Guarding a Koch Fractal Art Gallery

Lauren Cassell Ohio Northern University
Advisor(s): William Fuller, Ohio Northern University

We present an adaptation of Victor Klee's art gallery problem to a Koch fractal art gallery. We develop and solve a system of difference equations for the number of watchmen necessary to guard the nth stage Koch approximant art gallery and use this solution to show that the density of watchmen per side of our Koch fractal art gallery satisfies Chvatal's inequality.

235. Maximizing Volume Ratios for Shadow Covering by Tetrahedra

Christina Chen PRIMES
Advisor(s): Tanya Khovanova, PRIMES

Define a body A to be able to hide behind a body B if the orthogonal projection of B contains a translation of the corresponding orthogonal projection of A in every direction. In three dimensions, it is not easy to observe that there exist two objects such that one can hide behind another and have a bigger area than the other. We calculated two three-dimensional examples, one of which is a Minkowski interpolation of a tetrahedron and a ball that can hide behind the tetrahedron and has 12% more volume than the tetrahedron and the other of which is a Minkowski interpolation of a tetrahedron and an inverted tetrahedron that can hide behind the tetrahedron and has 16% more volume than the tetrahedron. The second example has the highest volume ratio calculated up to this date and is conjectured to be the best possible.

236. Geometry of Surfaces with Density

Miguel Fernandez Truman State University
Advisor(s): Frank Morgan, Williams College

Perelman's stunning proof of the million-dollar Poincaré conjecture needed to consider not only manifolds, but "manifolds with density" (like the density used in physics to compute the mass). We explore the basic geometry of such spaces by tackling one of the oldest problems in mathematics: the isoperimetric problem. That is, we seek curves that minimize weighted perimeter for a given weighted area. The classical (unit-density) isoperimetric theorem states that, for the plane, circles anywhere are isoperimetric. This usually changes when we introduce a non-unit density. The Log Convex Density Conjecture says that for radial, log-convex densities, circles about the origin are isoperimetric. We present our results so far regarding the borderline case of the plane with density e^r, and offer numerical evidence suggesting that circles about the origin are indeed isoperimetric. We conclude with possible ways to make our numerical study rigorous.

237. Coordinate Transformations Preserving Direct Sums of Covariant Tensors

William Franks University of South Carolina
Advisor(s): Corey Dunn, California State University, San Bernardino

The poster investigates the coordinate transformations that preserve direct sums of covariant tensors. Given a vector space W that is a direct sum of several subspaces, and a covariant tensor of rank greater than two on each subspace, the tensor direct sum is a natural way of building a new tensor T on W so that any tuple of vectors containing one vector in one of the subspaces that sums to W and another vector in another one of said subspaces is always mapped to zero by T. The main result of the paper is that an invertible matrix A that preserves a direct sum of covariant tensors of rank three or higher can only permute the subspaces on which the constituent tensors are defined. Restrictions are found for when A can map one such subspace to another; for one, the dimensions of the subspaces must match. Restrictions are also found on possible submatrices of A.

238. A Geometric Extremal Result for Cubic Arrays

James Gossell University of Central Missouri
Advisor(s): Peter Johnson, Auburn University

Imagine a game in which your goal is to select as many points as you can from an $n \times n$ square lattice in $\mathbb{Z}^2$. There is just one rule: No three points in your selected set may form a right triangle. For $n = 2$ you will find that you can pick up to $2n - 2$ points from the lattice without forming any right triangles. But try as you may, it is impossible to avoid forming a right triangle if you pick at least $2n - 1$ points.

We will examine a 3-dimensional variation to this game: How many points can be selected from an $n \times n \times n$ cubic lattice in $\mathbb{Z}^3$ without forming a right triangle in which 2 of the points are in the same coordinate line and 2 of the points are in a different coordinate line? We will give a tight bound on the maximum number of points one can pick without forming such a right triangle. Several similar, but unsolved problems will also be on display.

239. Heesch Numbers and Periodicity in Polyforms with Generalized Edge Matching Rules

Owen Hawkins Pomona College
Chris Ball University of Texas at Tyler
Jacobson Blomquist Boston College
Advisor(s): Casey Mann, University of Texas at Tyler

This project reports the results of a summer REU research project concerning the Heesch numbers of regular polygons whose edges are marked with generalized edge matching rules. The Heesch number of a tile is the maximum number of layers formed from copies of the tile that can surround a centrally place copy. Generalized edge matching rules are an extension of standard geometric and combinatorial matching rules, such as complementary bumps and nicks or matching colors. The authors developed a computer algorithm to determine if a given regular polygon marked with generalized edge matching rules tiles the plane periodically with an isohedral number less than or equal to 5, and for those tiles that do not tile the plane periodically, the algorithm attempts to find the Heesch number of the tile. This program has been used to find new examples with high Heesch numbers and has verified a Heesch number 11 example recently discovered by DeWeese and Corinaldi.

240. Univalent Harmonic Mappings and Symmetric Families

Joshua Kaminsky St Mary's College of Maryland
Advisor(s): Michael Dorff, Brigham Young University

Proving that a complex function is one to one has traditionally been a more involved process than proving a real function is one to one. For analytic functions, many different approaches have been developed in order to show a function is one to one. However, for harmonic functions, there are relatively few tools available. Existing methods rely on convexity in a direction, ϕ, or on the harmonic argument principle. This summer, we looked at symmetric harmonic maps, and in particular, maps with symmetry. In particular we developed a transformation that would increase the symmetry of a map. In order to prove univalence of these functions, we adapted existing tools. We are currently working on proving univalence from this transformation in general.

241. Geometric structures on Lie algebras associated with simple graphs

Omar Leon Undergraduate Student/ Florida International University
Lazaro Diaz Florida International University
Advisor(s): Mirroslav Yotov, Professor/ Florida International University

A well known construction associates Lie algebras to simple graphs. We study geometric aspects of such Lie algebras, for example properties of their Lie groups. We proved that even dimensional Lie algebras associated with graphs of small degree admit integrable complex structures. We also proved that the members of two infinite families of such Lie Algebras associated with trees also admit integrable complex structures. This provides the corresponding Lie groups with invariant complex structures.

242. Linear Independence of Sets of Three Algebraic Curvature Tensors

Britney Lovell University of Redlands
Advisor(s): Rolland Trapp, California State University San Bernardino

Looking at the dependence relationship between algebraic curvature tensors gives us an idea of how we can define an algebraic curvature tensor in terms of a linear combination of others. Given that φ is symmetric and ψ_i skew-symmetric, knowing when $\{R_\varphi, R_{\psi_1}, \ldots, R_{\psi_n}\}$ is linearly dependent is just one way of looking at this problem. In this study, we attempt to narrow down the problem by find some cases of when $\{R_\varphi, R_{\psi_1}, \ldots, R_{\psi_n}\}$ is linearly independent. We will show $\{R_\varphi, R_{\psi_1}\}$ and $\{R_\varphi, R_{\psi_1}, R_{\psi_2}\}$ are both linearly independent for any symmetric φ, and for any anti-symmetric ψ_1 and ψ_2.

243. Lower bounds on the agreement proportion of $(2, m)$-agreeable societies

Benjamin Maguire Lenoir-Rhyne University
Advisor(s): Thierry Zell, Lenoir-Rhyne University

A graph G on n vertices is $(2, m)$-agreeable if no subset of m vertices of G induces the empty graph (here, $n \geq m \geq 2$). If G is $(2, m)$-agreeable and if the boxicity of G is at most d, we prove that G must contain a clique of size at least $(2d)^{2-m}\, n$. This generalizes a result of Abrahams, Lippincott, and Zell (case $m = 3$), and can be

interpreted as a lower bound on the agreement proportion in the framework of convex approval voting introduced by Berg, Norine, Su, Thomas, and Wollan.

244. Linear Algebraic Properties of Simple Graphs

Alexander Moncion Baez Florida International University
Austin Nowak Florida International University
Advisor(s): Mirroslav Yotov, Florida International University

EMAILED

Given a simple graph G, a well known construction associates a two-step nilpotent Lie algebra L with it. This association actually gives rise to a conservative functor $F : \mathbf{Graphs} \to \mathbf{k-Lie-Alg}$ from the category of simple graphs to the category of Lie algebras over the field k. Our work focuses on the study of the image of the functor F. We propose a partial dictionary between properties of graphs on one side and properties of vector spaces, underlying the corresponding Lie algebras, on the other. We check our proposal for graphs of small degree as well as for graphs which are star-like trees. We formulate also a conjecture explaining when a two step nilpotent Lie algebra is associated with a tree graph.

245. Isoperimetry in the Plane with Density

Luis Sordo Vieira Wayne State University
Advisor(s): Frank Morgan, Williams College

It is well known that on $\mathbb{R}^2$ the least perimeter curve that encloses a given area is a circle. But what if we give the plane a density that weights both area and perimeter? The log convex density conjecture (LCDC) says that if the density is radial and its log is convex, circles about the origin minimize weighted perimeter for given area. We will do a quick survey of the geometry of surfaces with some different densities. We will conclude by considering the borderline case of the LCDC by investigating the plane with density e^r. Our partial results use symmetrization, generalized curvature, and the four vertex theorem to prove that a minimizer is convex and contains the origin in its interior.

246. Investigating Weighted Wet Foams as Decorating Weighted Dry Foams

Ben Thompson Cornell College
Advisor(s): David Finn, Rose-Hulman Institute of Technology

A dry foam consists of thin liquid walls separating gas cells like in soap suds. A wet foam is where the liquid in the cell walls pool at the intersections of cell walls. It is well understood how to "decorate" the intersection of a weightless dry foam to produce a weightless wet foam in its equilibrium state. What happens if the cell walls are weighted? This investigation concerns how to decorate weighted foams, where the weight of the liquid is not negligible, and the weight of the liquid pooling at the intersection will play a role in the equilibrium structure of the foam.

247. The Stratification of the Inertia Space of the $SO(2m)$-action on $\mathbb{R}^{2m}$

John Wells Rhodes College
Advisor(s): Christopher Seaton, Rhodes College

In this poster, we will discuss the structure of the inertia space of the $SO(n)$-action on $\mathbb{R}^n$, for even n. To define the *inertia space* we consider the space $\widetilde{\mathbb{R}^n}$, which is defined to be

$$\widetilde{\mathbb{R}^n} = \{(x, h) \in \mathbb{R}^n \times SO(n) | hx = x\},$$

where the group $SO(n)$ acts on this space by

$$g(x, h) = (gx, ghg^{-1}).$$

Then the inertia space is defined to be the quotient space of $\widetilde{\mathbb{R}^n}$ by $SO(n)$, denoted $SO(n) \setminus \widetilde{\mathbb{R}^n}$.

In order to look at the structure of the inertia space we look into the different isotropy types of the $SO(n)$-action on $\widetilde{\mathbb{R}^n}$, followed by determining the slices of the action for each isotropy type. A *stratification* is a decomposition of a space into smooth manifolds that have certain properties. Then using an equation developed by Farsi–Plfaum–Seaton we find an explicit statement of each stratum. Pictures of the $n = 3$ case will be used to give a better intuition of the hidden structure.

248. Graph Isomorphisms in Discrete Morse Theory

Seth Aaronson Ursinus College
Laura Stibich St. Francis University
Marie Meyer College of St. Benedict & St. John's University
Advisor(s): Nicholas Scoville, Ursinus College

A discrete Morse function f on a graph G induces a sequence of sub-graphs of G. A notion of equivalence between discrete Morse functions based on a sequence of homology groups of the corresponding subgraphs of G has been previously introduced. In this paper, we use the homology sequence to study a new notion of equivalence between discrete Morse functions. This equivalence is based on the isomorphism type of the subgraphs of G. We count the number of equivalence classes on star graphs Sn and deduce an upper bound for the number of equivalence classes for a large collection of graphs.

249. Relations in the Dyer-Laskof Algebra for Morava E-theory

Louis Atsaves MIT
Advisor(s): Mark Behrens, MIT

In the field of algebraic topology, certain operations act naturally on the (generalized) cohomology of topological spaces. In the case of ordinary cohomology, these operations are well understood, and are called Steenrod or Dyer-Lashof operations. They satisfy explicit but complex relations called the Adem relations. Recent work of C. Rezk has shed light on the corresponding operations for an important class of cohomology theories (Morava E-theories). The relations are encoded in certain maps of rings, but in many cases a complete and explicit description is missing. In my research, I find explicit formulas for these Adem relations and use them to prove that u, a ring map, is injective. I also use these explicit formulas to find the kernel of u dual.

250. Persistent Homology of Transmembrane Proteins

Alisa Aylward College of the Holy Cross
Advisor(s): David Damiano, College of the Holy Cross

In this project we investigate the persistent homology of Veitoris-Rips complexes of transmembrane proteins. Persistent homology measures the lifetime of generators in the simplicial homology of a filtered simplicial complex. The Veitoris-Rips construction creates a filtered simplicial complex for a data cloud. Interpreting the locations of the carbon atoms of a protein as a data cloud, allows for the application of the persistent homology algorithm. Protein information can be extracted directly from the Protein Data Bank. Javaplex software, developed by Andrew Tausz and Henry Adams, is used in combination with our Matlab code to analyze the homology of transmembrane and other proteins. Our goal is to reveal details about the topological structure of these proteins.

251. On Calculations of p-Typical Formal Group Laws

Eddie Beck University of Georgia
Advisor(s): Niles Johnson, University of Georgia

Formal group law theory provides computational tools with which to explore algebraic topology and homotopy theory. This paper studies the formal sum and the cyclic power operation for p-typical formal group laws, specifically to reduce prohibitive computation times through algorithm and time complexity analysis. We provide a combinatorial algorithm that directly computes terms of arbitrary degree using Mahler partitions. We also provide an *online* algorithm for computing the cyclic power operation, meaning that the precision of the calculations can be increased without restarting the computations. We measured the time complexity by counting the number of monomial multiplications required. These algorithms are at worst sub-exponential on the degree of the precision. Our algorithm substantially reduced previous computation times and shows that the McClure formula on MU_{17}, MU_{19} and MU_{23} are non-zero.

252. Upper Bounds on the Volume of Hyperbolic, Closed 3-Braids

Alyson Bittner SUNY Geneseo/REU at California State, San Bernardino
Advisor(s): Rolland Trapp, California State, San Bernardino

We find a family of links whose complement is made up of all regular, ideal tetrahedra. We prove that fact by a specific decomposition of the complement of the braid. We estimate the volume of certain closed 3-braids and proceed to show the volume density is $2v_3$ (where v_3 is the volume of a regular, ideal tetrahedron).

253. On Constructible Sets

Tyler Bongers Colorado State University - Pueblo
Walter Morales California State University Bakersfield
Advisor(s): Jorge Garcia, California State University Channel Islands

Given an initial family of sets, we take unions of two sets in the family, intersections of two sets in the family and complements of the sets in this family in order to form a new collection of sets; continuing recursively, in the limit, we obtain an algebra of sets through this construction process. Some problems encountered in this field include the termination of the construction process, as well as a characterization of constructed sets. Additional issues include the minimum size of a generating family for a given family, as well as the number of sets that can be generated by a given family. We define a class of simple families which do not contain any elements constructible in n steps from the other elements of the family; we call this property n-minimal constructibility. We prove several new results about n-minimal constructible families; one major result is that every finite algebra has a generating family which is n-minimal constructible for each $n \in \mathbb{N}$. Further results include techniques for finding a generating family of minimal size for any finite algebra. This work has a number of possible applications, particularly in the fields of economics and computer science.

254. Γ-Euler-Satake characteristics of non-orientable 3-orbifolds

Ryan Carroll Rhodes College
Advisor(s): Christopher Seaton, Rhodes College

The Euler characteristic of a manifold is an invariant which provides an abundance of information about the topological space's shape. An orbifold is a topological space much like this except with singular points induced on its surface by a group action such as a reflection or rotation. The singular points of an orbifold necessitated a new invariant be defined to take these singularities into consideration; this "upgraded" invariant is known as the Euler-Satake characteristic which considers the singularities as fractions of a space. More recently, Carla Farsi and Christopher Seaton generalized the work of these past authors, exploring more recent invariants known as Γ-Euler-Satake characteristics. Instead of considering the orbifold itself, this approach applies the Euler-Satake characteristic to the pieces of an orbifold broken down strategically by some finite group Γ. This talk will discuss this extension of past results to non-orientable 3-orbifolds and what information Γ-Euler-Satake characteristics might contain.

255. Orbifolds and Buildings

Samir Chowdhury Tufts University
Advisor(s): Genevieve Walsh, Tufts University

Buildings are often visualized as simplicial complexes consisting of subcomplexes called apartments, which are put together following certain rules. For spherical buildings of type n, each apartment can be represented as a tessellation of S^{n-1}. This creates a canonical correspondence between apartments of spherical buildings of type n and spherical reflection orbifolds, which we formalize in this paper. We conclude by constructing an exotic building from the orientable spherical orbifold $S^2(2,3,3)$, which is the quotient of S^2 by the rotational symmetry group of the tetrahedron.

256. Homology of $S_n([p])$

Derek DeSantis California State University Channel Islands
Rebecca Meissen Worcester Polytechnic Institute
Wesley Hough Hanover College
Advisor(s): Brant Jones, James Madison University

We explore the relationship between topology and partially ordered sets, specifically the symmetric group S_n under the Bruhat partial order. Bjrner and Wachs showed that any interval in S_n is homotopic to a single sphere. Babson and Hersh introduced a method for analyzing the homology of a poset via Discrete Morse Theory. We use these methods to examine the topology of $S_n([p])$, the symmetric group on n letters which avoids a specific permutation pattern p.

257. Classifying f-vectors of Manifolds with Boundary

Jonathan Hahn Carleton College
Advisor(s): Stephen Kennedy, Carleton College

Given a 3-dimensional manifold with boundary, such as a ball or solid torus, we can represent it with sets called simplicial complexes made of points, line segments, triangles, and tetrahedrons; in such a representation we can count the number of faces in each dimension and encode this information in an f-vector. Until recently, out of all 3-dimensional manifolds with boundary, we had only characterized the f-vectors of the ball. This summer at the Cornell REU led by Ed Swartz, using the idea of central retriangulation from David Walkup, we characterized the f-vectors of several others, including the solid torus, solid Klein bottle, thickened sphere, and other handle bodies, as well as a 4-dimensional manifold.

258. Characterizing Moves in Hexagonal Regions of Hextile Knot Mosaics

Andreana Holowatyj Victoria University of Wellington, New Zealand
Advisor(s): Geoff Whittle, Victoria University of Wellington, New Zealand

A hexagonal knot mosaic is a knot diagram that lies in a hexagonal grid in a particular way such that the intersection of the knot diagram and the interior of one of the hexagons is one of 27 basic types. Within the knot mosaic there exist simply connected regions of finitely many hextiles known as hexagonal regions. We enumerate and characterize planar isotopy moves in n-hextile regions; then explore methods used to condense the theorized infinite number of hextile moves on a hexagonal grid. We explain parameters that limit and determine the range of planar isotopy moves possible in cases of specific hexagonal regions.

259. Virtual Birack Shadow Modules and their Link Invariants

Nicole Sanderson University of California, Davis
Advisor(s): Sam Nelson, Claremont McKenna

We introduce an algebra Z[X; S] associated to a pair X; S of a virtual birack X and X-shadow S generalizing the birack shadow algebra. We use modules over Z[X; S] to define enhancements of the virtual birack shadow counting invariant. As an application, we show that the new invariant can detect vertical mirror images and hence is not determined by the fundamental biquandle.

260. Stick and Edge Numbers of Composite Knots

Geoffrey Schuette Sul Ross University
Sara Melvin The University of Texas at Tyler
Hans Chaumont Pomona College
Advisor(s): Jennifer McLoud-Mann, The University of Texas at Tyler

Upper bounds for the stick number and edge number of a composition of knots in the simple cubic lattice were established by Adams, et al. This poster will include similar results for the simple hexagonal, face centered cubic, and two variations of the body centered cubic lattices. In addition, upper bounds for the composition of n trefoils are given for each lattice.

261. On the Contractibility of Finite Co-Hopf Spaces

Ryan Vaughn University of Mary Washington
Advisor(s): Randall Helmstutler, University of Mary Washington

We use results from a 1966 paper of Stong on finite topological spaces to prove that finite co-Hopf spaces are necessarily contractible. This result is dual to a theorem of Stong on the contractibility of finite Hopf spaces, though neither proof dualizes to give the other. As a consequence, for any fixed finite space X, the set of homotopy classes $[X, Y]$ does not admit a non-trivial natural group structure in Y.

262. Connectivity at infinity for braid groups on graphs

Liang Zhang Lafayette College
Advisor(s): John Meier, Lafayette College

Let $C_m(\Gamma)$ be the discretized m-point configuration space on a simplicial graph Γ. The fundamental group of $C_m(\Gamma)$ is sometimes referred to as the m-strand braid group on Γ, and it can also be viewed as the group of movements of m robots on the graph Γ, without collision. Asymptotic properties of the universal cover, $\widetilde{C_m(\Gamma)}$, are invariants of the fundamental groups. Thus one can ask, for example, which of these groups are 1-ended? That is, what conditions on m and Γ ensure that the complements of compact subsets of $\widetilde{C_m(\Gamma)}$ are connected? We establish that these spaces are 1-ended unless one of two combinatorial conditions holds, and we settle the question when Γ is a complete graph. In the case of complete graphs we are also able to establish higher connectivity at infinity properties. In particular, we prove that the universal cover of $C_m(K_{m+n})$ is $(v_{m,n}-2)$-connected at infinity, where $v_{m,n} = \min\left\{m, n, \left\lfloor \frac{m+n+1}{4} \right\rfloor\right\}$.

263. On The Ramsey Number and Writhe of Knots

Patrick Cummings Marist College
Advisor(s): Rolland Trapp, California State University, San Bernardino

In this poster we use a particular linear embedding of a complete graph to improve the bound on the Ramsey number for a collection of knots. In hopes to further improve our understanding, we then find a bound on the writhe of any knot in that particular embedding.

264. On the Order Type of the Floer Chain Complex Filtration of the Knot Concordance Group

Michael Newman Columbia University
Stephen Hancock Columbia University
Advisor(s): Jennifer Hom, Columbia University

For a special family of knots $K \subset S^3$ called L-space knots, the Alexander polynomial of K determines the knot Floer complex of K. We define a reduced basis for this $\mathbb{Z}\oplus\mathbb{Z}-$filtered chain complex, which associates to each knot a unique complex whose vertices are basis elements in their corresponding chain groups and whose edges are the boundary maps between those basis elements. This complex is a knot invariant, and the set of such complexes can be given a group structure with multiplication induced by the tensor product such that there exists a homomorphism from the concordance group of knots into this group $\mathcal{F}_{algebraic}$. We prove results on the coarse order type of the image of L-space knots and connected sums of L-space knots in $\mathcal{F}_{algebraic}$, in the hopes of using this homomorphism to ultimately study the structure of the concordance group.

265. Minimizing Maximum Indegree in Strongly Connected Graphs

Antonio Ochoa Cal Poly Pomona
Jenny Iglesias Harvey Mudd College
Advisor(s): Glencora Borradaile, Oregon State University

Given an undirected graph, one can assign directions to each of the edges of the graph, thus orienting the graph. We consider the problem of orienting an undirected graph to minimize either the maximum indegree or the lexicographic order of the indegree sequence. In particular, we also stipulate the resulting graph to be strongly connected. This problem has applications in telecommunications network design where minimizing the load on each node can improve rerouting time in the face of network-edge failure. Our results build on previous work done on minimizing the maximum indegree of a graph without constraints. As part of our results we prove that we can minimize the maximum indegree of a graph while maintaining strong-connectivity using a simple greedy algorithm. We do this by showing that this greedy algorithm results in an orientation that matches a lower bound on the maximum indegree of a strongly connected graph. We also conjecture that a slight modification of this algorithm allows us to minimize the lexicographic order of the indegree sequence.

266. Equality Between Zero Forcing Number and Maximum Nullity of a Complete Edge Subdivision Graph

My Huynh Arizona State University
Advisor(s): Leslie Hogben, Iowa State University

For a simple, undirected graph G the zero forcing number $Z(G)$ is the minimum number of black vertices initially needed to force all vertices in G black according to the color change rule. The color change rule states that for G with all vertices colored either black or white, if a vertex v is black and an adjacent white vertex w is the only white neighbor of v, then v can force w to be colored black. The minimum rank $\mathrm{mr}(F, G)$ is the smallest possible rank over all symmetric matrices over a field F whose ijth entry (for $i \neq j$) is nonzero whenever $\{i, j\}$ is an edge in G and is zero otherwise. The maximum nullity $\mathrm{M}(F, G)$ of G is $|G| - \mathrm{mr}(F, G)$ and it is known that $\mathrm{M}(F, G) \leq \mathrm{Z}(G)$ for all G. Zero forcing number is a useful tool in the study of minimum rank problems. Examples are known of graphs for which $\mathrm{M}(F, G) < \mathrm{Z}(G)$. To subdivide the edge e between vertices u and v, delete e and add a new vertex w adjacent to exactly u and v. The complete subdivision graph $\overline{G}$ is obtained from G by subdividing each edge once. A positive answer to the open question of whether $\mathrm{M}(F, \overline{G}) = \mathrm{Z}(\overline{G})$ for all fields F and all graphs G is presented.

267. Omega Graphs

Sara Chauvette Franklin College
Rebecca Thompson Colorado College
Advisor(s): Cindy Wyels, CSU Channel Islands

We will explore the topic of radio labeling, a specific type of graph labeling. We ask which radio numbers are achievable by graphs on n vertices. To address this question, we define a two-parameter family called Omega graphs. By using the distance maximizing method of finding lower bounds and developing labeling schemes to produce upper bounds, we determine the radio numbers for over half the Omega graphs, therefore filling in many values on the achievable radio number line.

268. Extremal Graphs Without 4-Cycles

Frank Firke Carleton College
Evan Nash University of Nebraska-Lincoln
Peter Kosek The College at Brockport, State University of New
Advisor(s): Jason Williford, University of Wyoming

Determining the largest number of edges in a C_4-free graph on n vertices is a problem that remains unsolved for general n. However, we extended previous work by Füredi to prove an upper bound for the number of edges in a C_4-free graph on $q^2 + q$ vertices for q even. This upper bound is achieved if and only if there is an orthogonal polarity graph of a projective plane of even order q.

269. Radio Numbers for 4th Power of Paths

David Gomez CSU San Bernardino
Veronica Chavez CSU San Bernardino
Jessica Luna CSU San Bernardino
Marcos Reyes CSU San Bernardino
Advisor(s): Min-Lin Lo, CSU San Bernardino

Let G be a connected graph. For any two vertices u and v, let $d(u, v)$ denote the distances between u and v in G. The maximum distances between any pair of vertices is called the diameter of G and denoted by $\mathrm{diam}(G)$. A radio labeling of a connected graph G is a function f from the vertices, $V(G)$, of G to the natural numbers such that for any two distinct vertices u and v of G: $|f(u) - f(v)| > \mathrm{diam}(G) - d(u, v) + 1$. The span is the maximum difference of the labelings on vertices. The radio number of G is the minimum span of a radio labeling of G. The 4th power of G is a graph constructed from G by adding edges between vertices of distance four or less apart in G. The radio number for paths, square paths, and cube paths were solved. In this poster we will discuss the progress we made towards finding the radio number for 4th power of paths during a 2011 MAA summer research program funded by NSA (grant H98230-11-1-0215) and NSF (grant DMS-0845277).

270. Obtainability of Strong Orientations: Creating an Efficient Network of One-Way Streets

Cassidy Griffin St. Lawrence University
Richard Powell St. Lawrence University
Brendan Gorman St. Lawrence University
Advisor(s): Patti Frazer Lock & Robin Lock, St. Lawrence University

A strong orientation of a graph is a way to orient the edges so that the resulting directed graph is strongly connected, meaning it is possible to get from any vertex to any other vertex while following the arrows. In his book *Graph Theory and Its Applications to Problems of Society*, Fred Roberts describes a depth-first search algorithm (which we call the Roberts Algorithm) for putting a strong orientation on a graph. We call a strong orientation *obtainable* if it is possible to arrive at that orientation using some application of the Roberts Algorithm. We present results about obtainable and unobtainable orientations, including classes of graphs which have unobtainable orientations and classes of graphs for which all orientations are obtainable. In addition, we discuss relative efficiency of orientations which are obtainable or unobtainable, using four different ways to measure efficiency or optimality.

271. Stars and a Generalization of Farey Numbers

Matthew Jacobson University of Colorado - Boulder
Advisor(s): Nathaniel Dean, Texas State University - San Marcos

The screen size of the graph $K_{1,n-1}$ (i.e. a star of order n) is the smallest number such that the $k \times k$ interger lattice suports a drawing of $K_{1,n-1}$, where each vertex is a lattice point and the edges are drawn as non-overlapping line segments. We find that the screen size of a star is related to the number of visible points on an integer lattice (i.e. the screen). In particular, given a screen of odd size $s = 2k + 1$, $k \in \mathbb{N}$, if $K_{1,n-1}$ is centered in the middle of the screen, then $n \leq 8F(\frac{s-1}{2}) + 9$, where $F(i)$, called the Farey number of i, is the length of the Farey sequence for i. However, the center of the screen is not necessarily the point that will fit the largest possible star. Thus, we derive a 2-dimensional generalization of Farey numbers that gives the exact number of visible points from any point of an integer lattice by looking at the number of co-prime pairs in a rectangle and applying the sieve principle. We then generalize Euler's totient function in order to investigate the asymptotic properties and provide bounds for our new function.

272. Extremal problems for graph homomorphisms

Nicholas Kass Montclair State University
Advisor(s): Jonathan Cutler, Montclair State University

Many problems in extremal graph theory correspond to questions involving homomorphisms into a fixed image graph. Both the hard-core and Widom-Rowlinson models in statistical physics can be stated in these terms, as can graph coloring. Recently, there has been interest in maximizing the number of homomorphisms from graphs with a fixed number of vertices and edges into small image graphs. For the image graph H_I, the graph on two adjacent vertices one of which is looped, each homomorphism from G corresponds to a unique independent set in G. It follows from the Kruskal-Katona theorem that the number of homomorphisms to H_I is maximized by the lex graph, whose edges form an initial segment of the lex order. Loop-threshold image graphs, which include H_I, have the property that there is a threshold graph which maximizes the number of homomorphisms. The only extremal homomorphism problem with a loop-threshold image graph on at most three vertices not yet solved is $H_I \cup E_1$, where extremal graphs are the union of a lex graph and an empty graph. While we cannot give an exact answer for every number of vertices and edges, we give an asymptotic bounds for $\ell(e)$, the number of vertices in the lex component of the extremal graph with e edges and at least $e + 1$ vertices.

273. Book Thicknesses of Subdivision of Graphs

Jessica Oehrlein Oklahoma School of Science and Math
Advisor(s): Aaron Potechin, Massachusetts Institute of Technology

A book embedding of a graph is an embedding of the graph in multiple half-planes called pages. The vertices of a graph G are labeled and placed in order on a straight line called the spine. A page is the spine and some subset of the edges of G such that no two edges cross. The minimum number of pages necessary to embed G in a book is the book thickness or stack number of G, denoted $sn(G)$. A subdivision of a graph G is the addition of vertices to existing edges of G. We show that the book thickness of any subdivision of G is at most 2 $sn(G)$.

274. Indications: A Matching-Like Structure on Directed Graphs

Allison Oldham The College of William and Mary
Advisor(s): Rex Kincaid, The College of William and Mary

A matching, as defined on an undirected graph, is a collection of non-adjacent edges. Adjacency and, as a result matching, does not easily extend to directed graphs. However, following the article *Controllability of Complex Networks* by Liu, Slotine and Barabasi, we define an *indication*, a matching-like collection of directed edges. Further, we outline a method for finding a maximum cardinality indication in a directed graph. We then explore several properties of *indications* as well as some of the interactions between these properties. Finally, we discuss an analogue of the Marriage Problem on a directed graph whose solution is a *complete indication*. Algorithms and proofs for the results rely heavily on converting a directed graph into an undirected bipartite graph.

275. The Best Mixing Time on Trees

Jeanmarie Youngblood Macalester College
Advisor(s): Andrew Beveridge, Macalester College

Consider a random walk on a graph $G = (V, E)$ starting at vertex $i \in V$. Under some mild assumptions, the distribution of the current location of our walk converges to the *stationary distribution* π of the graph. Quantities that capture the convergence rate are called *mixing measures.* We study the *best mixing time*. This mixing measure is defined via the theory of stopping rules developed by Pitman, Aldous, Lovász and Winkler. Using this framework, we employ an *optimal stopping rule* to guide our random walk from $i \in V$ to obtain an exact sample from π. Let $H(i, \pi)$ be the expected length of such an optimal stopping rule. Define the *best mixing time* $T_{\text{bestmix}} = \min_{i \in V} H(i, \pi)$. We restrict our attention to trees. Fixing $n = |V|$, it is interesting to find the trees that minimize and maximize a given mixing measure. Typically, the star and the path are the extremal structures. We resolve this extremal problem for T_{bestmix}. We show that the star minimizes T_{bestmix}. The maximization problem gives an unexpected result. For even n, the path is the maximizing structure. However, for odd n, the maximizing structure is an $(n-1)$-path with a single leaf adjacent to a central vertex.

276. The Extremal Permutations for the Routing Number on Cycles

Luis Valentin College of William and Mary
Advisor(s): Gexin Yu, College of William and Mary

Let G be a connected graph on n vertices and π be a permutation on those vertices. The routing number of π on G, denoted $rt(G, \pi)$, is defined as the minimum number of steps necessary for the following procedure to terminate: For each vertex v of G, place a "pebble" destined for vertex $\pi(v)$. While not all pebbles are at their destination, select a set of disjoint edges in G and exchange the pebbles at the endpoints of each edge. The routing number of G, denoted $rt(G)$, is equal to the maximum routing number of all permutations on G. The routing number of graphs were first defined by Alon, Chung, and Graham. Li, Lu, and Yang proved that $rt(C_n) = n - 1$. They conjectured that for $n \geq 5$, if $rt(C_n, \pi) = n - 1$, then π is either the rotation $(12 \ldots n)$ or its inverse. We give a proof to this conjecture.

277. Convex Combinations of Minimal Graphs

Missy Lucas Rice University
Advisor(s): Michael Dorff, Brigham Young University

A minimal surface in $\mathbb{R}^3$ is a surface whose mean (or average) curvature vanishes at every point on the surface; bending in one direction is "canceled out" by opposite bending in the perpendicular direction. Complex-valued, planar harmonic mappings of the form $f = h + \overline{g}$ can be used along with the Weierstrass representation to provide a formula to parameterize minimal graphs in $\mathbb{R}^3$. We present our results on conditions under which known minimal graphs can be combined to form new minimal graphs and continuous transformations between them.

278. Domination and Independence on the Triangular Honeycomb Chessboard

Hong Lien Tran Kennesaw State University
Advisor(s): Joe DeMaio, Kennesaw State University

Puzzles on the chessboard have long been studied by mathematicians. Across the Board: The Mathematics of Chessboard Problems by John Watkins is an indispensable collection of mathematically themed chessboard problems.

We do not restrict ourselves to the standard 8×8 chessboard. Generalizations are quickly made to the square board of sides other than n=8, m×n rectangular boards and other variant surfaces. Chessboard problems are most frequently set in the context of Graph Theory. Two classic problems in Graph Theory that appear again and again are those of dominating sets of minimum cardinality and independent sets of maximum cardinality. For chessboards the question of a minimum dominating set transforms into how to threaten or occupy every square on the board with the fewest pieces. Maximum independent sets become the problem of how to place the most non-attacking pieces. Our project explores these two combinatorial problems on the variant triangular honeycomb chessboard for the rook, bishop, knight and king.

279. Periods of Orbits for Maps on Graphs Homotopic to a Constant Map

Adriana Johnson Bard College
Zachary Gaslowitz Harvey-Mudd College
Whitney Radil College of St Benedict
Advisor(s): Christopher Bernhardt, Fairfield University

Our work proves two theorems concerning the set of periods of periodic orbits for maps of graphs that are homotopic to the constant map and such that the vertices form a periodic orbit. The first result is that if v is not a divisor of 2^k then there must be a periodic point with period 2^k. The second is that if $v = 2^k s$ for odd $s > 1$, then for all $r > s$ there exists a periodic point of minimum period $2^k r$. These results are then compared to the Sharkovsky ordering of the positive integers.

280. Bounding Ranks of Elliptic Curves

Cecylia Bocovich Macalester College
Advisor(s): David Bressoud, Macalester College

An elliptic curve E is defined as an equation

$$y^2 = x^3 + ax + b$$

Where a and b are coefficients that satisfy the property $4a^3 + 27b^2 \neq 0$. The rational solutions of the curve, points of the form (x, y) satisfying the equation, form a group. This group, denoted $E(\mathbb{Q})$ is known as the Mordell-Weil group. Mordell proved that this group is isomorphic to $\mathbb{Z}^r \oplus E(\mathbb{Q})_{tors}$ where the group of rational torsion points consists of all points of finite order. The rank r is difficult to compute and the main goal of this research is to determining a lower bound of ranks on equations of the form $E = y^2 = x^3 - m^2x + 1$. This is an expansion and application of a paper written by Ezra Brown and Bruce T. Myers entitled *Elliptic Curves from Mordell to Diophantus and Back*.

281. Egyptian Fractions and Pythagorean Spirals

Haoqi Chen St. Norbert College
Advisor(s): Teena Carroll, St. Norbert College

An Egyptian Fraction is a representation of a rational number as the sum of distinct unit fractions. These representations are not unique. We count the number of representations of one using even denominators. Though the total number of Egyptian fraction representations of one using 9 or more terms is not known, we are able to give the number of even representations of size 9, as well as give an expression for the number of even representations up to size 13. The idea finding triples of integers which are the lengths of sides of right triangles has interested mathematicians from Ancient Greece all the way up to modern high school students. We generate infinitely many disjoint sequences which each encode an infinite number of Pythagorean triples. The recurrence relation which generates these sequences is the same as the one used to find the Egyptian fraction representation of one of a given length using the largest denominator. Furthermore, when we restrict our interest to even denominators we see a physical correspondence to lengths in a geometric object called a Pythagorean Spiral which is also determined by the same sequence. This connection to Egyptian Fractions is striking because it combines ideas developed separately in Ancient Greece and Ancient Egypt.

282. Pell's Equations for Multidimensional Continued Fractions

Krishna Dasaratha Harvard University
Cornelia Mihaila Wellesley College
Advisor(s): Thomas Garrity, Williams College

A classical problem in number theory is finding integer solutions of Pell's equation $x^2 + dy^2 = 1$, where d is a square-free integer. Pell's equation has infinitely many solutions, which can be derived from the continued fraction expansion of $\sqrt{d}$. We have recently developed a large family of generalized continued fractions. These include, after some manipulation, well-known multidimensional continued fraction algorithms such as the Brun Algorithm, the Fully Subtractive Algorithm, the Triangle Map, the Güting Map, and the Mönkemeyer Map. For each multidimensional continued fraction in this family, we construct a three-variable analogue to the original Pell equation. The multidimensional continued fraction algorithm generates infinitely many solutions to this generalized Pell equation, and these solutions are units in the corresponding cubic number field. We then extend our results to multidimensional continued fractions in higher dimensions.

283. Primes and Polynomials

Justin DeBenedetto Wake Forest University
Advisor(s): Jeremy Rouse, Wake Forest University

Euler's quadratic $x^2 + x + 41$ has many unique properties. Notably, it contains the longest known succession of prime numbers without repetition. The Green-Tao Theorem states that arithmetic progressions of primes exist of any chosen finite length. By expanding upon the Green-Tao Theorem, we have proven that there exist quadratics which yield longer successions of primes. Exploring quadratics of the form $x^2 + x + A$, we have proven that if $x^2 + x + A$ is prime for $0 \leq x \leq \frac{4\sqrt{A}}{\sqrt{3}}$, then it must also be prime over the larger range $0 \leq x \leq A - 2$. Furthermore, we have found a 60,000 digit long prime number of the form $x^2 + x + 41$, suggesting that Euler's quadratic takes on infinitely many prime values.

284. A Family of Multidimensional Continued Fractions Yielding A Solution to the Hermite Problem

Laure Flapan Yale University
Matthew Stoffregen University of Pittsburgh
Advisor(s): Thomas Garrity, Williams College

In 1848, Charles Hermite asked Carl Jacobi for a method to represent real numbers as sequences of nonnegative integers such that the algebraic properties of a number are revealed by the periodicity of its sequence. Attempts to provide such a method are called Multidimensional Continued Fractions. We construct a large family of Multidimensional Continued Fractions and show that this family generalizes many well-known algorithms such as the Mönkemeyer, Brun, Parry-Daniels, and Güting algorithms. We then show that every cubic number field contains an infinite number of cubic irrationals α, β in the number field such that (α, β) is periodic with respect to one of the Multidimensional Continued Fraction algorithms in our family. We then demonstrate how this result provides a method for approaching the Hermite Problem for cubics.

285. Analysis of Nonpowers

Mark Gerads Winona State University
Advisor(s): Joyati Debnath, Winona State University

In this paper, we will examine the natural numbers that are nonpowers, i.e. any natural number that is not a perfect power. Here by natural numbers, we mean the elements of the set $\mathbb{N} = \{1, 2, 3, ...\}$, and by perfect powers, we mean any number a^b where a and b are natural numbers and b is greater than 1. We define $\mathbb{S} = \{a^b \mid a, b \in \mathbb{N}, b > 1\}$, and nonpowers, $\mathbb{M} = \mathbb{N} \setminus \mathbb{S}$. We will deduce properties of nonpowers, and then find new formulae for recognizable functions such as the Riemann zeta function.

286. Sudoku Hypercubes: Construction and Bounds

Emily Hill Metropolitan State College of Denver
Advisor(s): Diane Davis, Metropolitan State College of Denver

Much is known about latin squares and sets of mutually orthogonal latin squares. Sudoku squares are a specialized, more restrictive type of latin square. In this talk, we will provide a definition of Sudoku cubes, and a method for

constructing them. Additionally, we will prove that the maximum number of Sudoku cubes of order n^3 that can be constructed by this method is given by $(n^3 - n)(n^3 - n^2)$, a novel result. Finally, we will generalize our results to higher dimensions.

287. The Generalizations of the Golden Ratio: Their Powers, Continued Fractions, and Convergents

Saarik Kalia MIT PRIMES
Advisor(s): Tanya Khovanova, MIT PRIMES

The relationship between the golden ratio and continued fractions is commonly known throughout the mathematical world: the convergents of the continued fraction are the ratios of consecutive Fibonacci numbers. The continued fractions for the powers of the golden ratio also exhibit an interesting relationship with the Lucas numbers. In this paper, we study the silver means and introduce the bronze means, which are generalizations of the golden ratio. We correspondingly introduce the silver and bronze Fibonacci and Lucas numbers, and we prove the relationship between the convergents of the continued fractions of the powers of the silver and bronze means and the silver and bronze Fibonacci and Lucas numbers. We further generalize this to the Lucas constants, a two-parameter generalization of the golden ratio.

288. Counting matrices with restricted position by rank over a finite field

Aaron Klein Brookline High School/PRIMES
Advisor(s): Alejandro H. Morales, MIT

We look at the general problem of finding the number of matrices over a finite field with a certain rank and with support that avoids a subset of the entries. These matrices are a q-analogue of permutations with restricted positions (i.e. rook placements). For general sets of entries these numbers of matrices are not polynomials in q (Stembridge 98); however, when the set of entries is a Young diagram, the numbers are polynomials with nonnegative coefficients (Haglund 98). In particular, the polynomials are the sum over rook placements of $q^{-\text{inversions}}$ of each placement. In this study, we extend this result to skew Young diagrams. We apply this result to the case when the set of entries is the Rothe diagram of a certain permutation by giving a condition on the permutation for the set to be a skew Young diagram up to rearrangement of rows and columns. We also give conditions for Rothe diagrams to be inverse skew Young diagrams up to rearrangement of rows and columns.

289. Pell's Equation Through Dynamical Systems

Austin Mack University of North Carolina Asheville
Timothy Sawicki University of North Carolina Asheville
Advisor(s): Samuel Kaplan, University of North Carolina Asheville

Much work in number theory has been done since the 18th century on finding integer solutions to equations of the form $x^2 - Dy^2 = 1$, where D is a non-square integer. These equations, called Pell's equations, have been shown to have infinite number of solutions that require finding the first non-trivial solution for each value of D. There is no simple relationship between the parameter, D, and the size of its smallest non-trivial solution. In the late 18th century, Euler generated an iterative method to compute non-trivial solutions. We will reduce Euler's method to a function on three integers that can be iterated. Using more modern tools of dynamical systems, we will explore the dynamics of this map as well as the stopping time of the algorithm. The stopping time will give an indication of the size of the smallest non-trivial solution. Through these discoveries, we find the solutions to Pell's equations have an iterative relationship simplifying the process into a product of matrices defined by terms in the iteration. The iteration as well gives way to a sequence of terms that relate to the continued fraction of $\sqrt{D}$, allowing us to solve Pell's equations from a different approach.

290. Continued Fractions and Nathanson Heights

Hunter Merrill Mississippi State University
Advisor(s): Jian Shen, Texas State University

Let p be a prime, and let $\mathbb{Z}_p$ denote the field of integers modulo p. The *height* of a point $\mathbf{r} = \langle r_1, ..., r_d \rangle \in \mathbb{Z}_p^d$ is defined as $h_p(\mathbf{r}) = \min\{\sum_{i=1}^{d}(kr_i \bmod p) : k = 1, ..., p-1\}$; however, an explicit formula for height was not previously known for all points, even for the case $d = 2$. The formula for height for $d = 2$ was improved and extended to all integers m (not necessarily prime) by using continued fractions, where the height is given by

$h_m(\langle 1, r\rangle) = \min\{q_{2k}(r+1) - p_{2k}m : 0 \le k < n/2\}$, where p_i/q_i is the i^{th} convergent of the continued fraction expansion of r/m. We use continued fractions to give an explicit formula for finding height.

291. Multiplicative Partitions

Alice Nadeau Grinnell College
Bingxi Wu Grinnell College
Advisor(s): Marc Chamberland, Grinnell College

Multiplicative partitions, the multiplicative analogue to the widely understood additive partitions, are increasing in importance in the world of number theory. We studied various integer sequences and their Dirichlet series generating functions which helped us discover many new results about multiplicative partitions. These results include various convolution identities, a new recursive formula for multiplicative partitions, and new representations for the Riemann zeta function. The similarities between the multiplicative partition generating function and the Riemann zeta function may help us discover more about the Riemann zeta function.

292. Representation of an Integer as Harmonic Means

Kerri Nunnamaker Washington & Jefferson College
Advisor(s): Roman Wong, Washington & Jefferson College

We investigate problems related to the 1948 *Erdös-Straus Conjecture* which says that for every positive integer n, $\frac{4}{n}$ can be written as a sum of reciprocals of at most three distinct positive integers. Using the work of Solomon Golomb, we showed that every positive integer can be represented as the harmonic mean of two positive integers, and that the number of these representations can be determined. We also looked into the Egyptian fractions mentioned in the *Ahmes Papyrus* and examine algorithms that can express rational numbers such as $\frac{3}{n}$ as a sum of three or more reciprocals.

293. Discrete Approximations of Differential Equations via Trigonometric Interpolation

Kristin Heysse Concordia College
Michael Dyrud Concordia College
Austin Bren Concordia College
Advisor(s): Oksana Bihun, Concordia College

We use trigonometric interpolation to approximate solutions of a differential equation $Qu = f$, whose differential operator Q with domain $D(Q)$ is a formal polynomial of operators $\{1, x, d/dx\}$. A solution u is projected onto the space T_n of trigonometric polynomials of degree n. The projection Tu, defined as a trigonometric interpolant of u, is identified with a vector $\hat{u}$ of its values at partition points via an isomorphism π. The operator Q is represented by a square matrix $\hat{Q}$ defined implicitly by $\hat{Q}\hat{v} = \pi TQTv$ for all $v \in D(Q)$. The original equation is approximated by a system of linear equations $\hat{Q}\hat{u} = \hat{f}$, where $\hat{f} = \pi Tf$.

We prove that if $Q = a_0 + a_1\frac{d}{dx} + \ldots + a_s\frac{d^s}{dx^s}$, then rank $\hat{Q} = \dim \hat{Q} + |\text{sign } a_0| - 2m - 1$, where $m \ge 0$ is the number of solutions, in the set $\{1, 2, \ldots, n\}$, of a certain system of polynomial equations. Our numerical tests show high accuracy and fast convergence of the method applied to several boundary and eigenvalue problems for differential equations.

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294. Using Games to Approximate Solutions to Partial Differential Equations

David Sacco Student/California State University Monterey Bay
Advisor(s): Rachel Esselstein, Professor/California State University Monterey Bay

For this project, we explore the interesting connection between Game Theory, Monte Carlo, and numerical solutions to certain Partial Differential Equations. As it turns out, it is known that there is a clear link between probability theory and the heat equation. What is known is that one can devise a game that computes random walks on a bounded lattice to approximate solutions to the heat equation at interior mesh points. The point of this research is one, to gather information about the relationship between probability theory, game theory, numerical analysis, and the heat equation into one source, and two, to investigate the other cases of partial differential equations, being parabolic and

hyperbolic partial differential equations, as well as elliptic equations with variable coefficients. The investigation involves looking to see if one can devise games that would also, by similar means, approximate solutions to those equations.

295. Nabla Discrete Fractional Calculus

Lucas Castle Lamar University
Kevin Ahrendt University of Nebraska-Lincoln
Advisor(s): Allan Peterson, University of Nebraska-Lincoln

We will first define positive and negative nabla fractional differences for functions defined on $N_a = \{a, a+1, a+2, \cdots\}$. The corresponding discrete nabla Laplace transform will be defined and we will prove several properties of these nabla Laplace transforms including the nabla Laplace transform of fractional Taylor monomials. The discrete nabla Mittag-Leffler function will be introduced and we will derive a variation of constants formula for solving nabla discrete fractional initial value problems. Finally we will show how to use our Laplace transform results to prove composition properties of nabla fractional differences.

296. Limit and Morse Sets for Deterministic Hybrid Systems

Thomas Rudelius Cornell University
Sijing Shao Iowa State University
Xavier Garcia University of Minnesota - Twin Cities
Advisor(s): Wolfgang Kliemann, Iowa State University

The term "hybrid system" refers to a continuous time dynamical system that undergoes Markovian perturbations at discrete time intervals. In this paper, we find that under the right formulation, a hybrid system can be treated as a dynamical system on a compact space. This allows us to study its limit sets. We examine the Morse decompositions of hybrid systems, find a sufficient condition for the existence of a non-trivial Morse decomposition, and study the Morse sets of such a decomposition. Finally, we consider the case in which the Markovian perturbations are small, showing that trajectories in a hybrid system with small perturbations behave similarly to those of the unperturbed dynamical system.

297. Invariant Measures for Hybrid Stochastic Systems

Emily Speranza Carroll College
Jennifer Kunze Saint Mary's College of Maryland
Anthony Sanchez Arizona State University
Advisor(s): Wolfgang Kliemann, Iowa State University

In this paper, we seek to understand the behavior of dynamical systems which are perturbed by a parameter which changes discretely in time. If we impose certain conditions, we can study these systems as Markov processes by themselves or we can study the certain embedded systems within our hybrid systems as Markov processes. In particular, we can conclude the existence of an invariant measure for the former case and for the latter we can find invariant measures for each embedded system and relate the various measures through the flow.

298. Axiomatizing J_3 Paraconsistent Logic

Kaetlin Taylor Rhodes College
Advisor(s): Michael Sheard, Rhodes College

Paraconsistent logic expands upon classical logic by removing the *ex falsi principle*, i.e. the rule that anything may follow from a contradiction. In doing so, we no longer requiring propositions to be absolutely true or absolutely false. We develop a new axiomatization of J_3 paraconsistent logic in a natural deduction system, i.e. a system containing rules of inference but no logical axioms. We also develop a system that allows us to determine when a logical statement can be represented using alternate quantifiers, allowing us to eliminate dependent schema in our axiomatization.

299. Generating Non-negative Matrices with Specified Row and Column Sums

Daniel Grier University of South Carolina - Columbia
Advisor(s): Peter Johnson, Auburn University

A recent question answered by Michelle Foster and Peter Johnson is as follows: Given an alphabet $S = \{s_1, \ldots, s_m\}$, an integer $k > 1$, and a k-dimensional array $[f] = [f(i_1, \ldots, i_k); 1 \leq i_1, \ldots, i_k \leq m]$ of non-negative numbers, under what conditions does there exist a statistically stable source producing text over S such that whenever $1 \leq i_1, \ldots, i_k \leq m$, $f(i_1, \ldots, i_k)$ is the relative frequency of $s_{i_1} \ldots s_{i_k}$ among blocks of k consecutive letters in the source text. The *consistency condition* is such a requirement: for any $i_1, \ldots, i_{k-1} \in \{1, \ldots, m\}$, $\sum_{i=1}^{m} f(i, i_1, \ldots, i_{k-1}) = \sum_{j=1}^{m} f(i_1, \ldots, i_{k-1}, j)$.

We first show that we can generate matrices ($k = 2$) satisfying the consistency condition. Using this construction we also show that if A and B are non-negative matrices with corresponding row and column sums, then there exists a finite sequence $T_1, \ldots, T_k$ of matrices such that $B = A + \sum_{i=1}^{k} T_i$ and $A + \sum_{i=1}^{j} T_j$ has non-negative entries for each $j \in \{1, \ldots, k\}$. We then state a more general theorem about k-dimensional arrays of non-negative real numbers, which is simply an extension of the $k = 2$ case.

300. Diagram vectors of frames and the tight frame scaling problem

Martin Copenhaver Georgia Institute of Technology
Cortney Logan Stonehill College
Kyanne Mayfield University of Portland
Jonathan Sheperd University of Notre Dame
Advisor(s): Sivaram K. Narayan, Central Michigan University

A frame is a redundant spanning set. A tight frame is a generalization of an orthonormal basis. A notion of digram vectors associated to frames in $\mathbb{R}^2$ has yielded many results about tight frames in $\mathbb{R}^2$. We provide a generalized notion of diagram vectors which allows for significant developments in the theory of tight frames in finite dimensions. In particular, we completely answer the *tight frame scaling problem* - that is, given a set of unit vectors $\{f_i\}_{i=1}^{k}$ in a finite dimensional Hilbert space $\mathcal{H}_n$, when do there exist positive scalars $c_1, \ldots, c_k$ so that $\{c_i f_i\}_{i=1}^{k}$ is a tight frame? When such scalings do exist, we provide a means of determining coefficients using a specific formulation for $\mathbb{R}^2$ as well as a general approach for $\mathcal{H}_n$ using techniques from computational geometry.

301. Irreducibility of Iterates of $f(x) = x^2 - q$

Katharine Chamberlin College of the Holy Cross
Advisor(s): Rafe Jones, College of the Holy Cross

For most rational numbers q, all iterates of the polynomial

$$f(x) = x^2 - q$$

are irreducible. Yet, there is an infinite set of q's such that $f(x)$ is reducible, and an infinite set of q's such that the second iterate $f^2(x)$ is reducible when the first iterate of $f(x)$ is irreducible. We have identified the conditions which define this latter set of rational numbers. We give conditions necessary for the third iteration, $f^3(x)$, to be reducible when prior iterates, namely $f^2(x)$ and $f(x)$, are irreducible. We consider whether this set, and its analogues for higher iterates, may be finite.

302. The Cohen-Macaulayness of Edge Ideals of Clutters Generated from Paths of a Tree

Thomas Polstra Georgia State University
Ryan Gunderson University of Nebraska-Lincoln
Advisor(s): Susan Morey, Texas State University

Let $R = K[x_1, x_2, \ldots, x_n]$ be the polynomial ring with n variables over a field K. Define the edge ideal of a clutter $\mathcal{C}$, denoted $I(\mathcal{C})$ to be the ideal of R made by assigning each vertex in $\mathcal{C}$ to a variable in R and taking the generators of $I(\mathcal{C})$ to be monomials formed by taking the product of the variables in an edge.

Let T be a tree and let $\mathcal{C}_{\ell,T}$ be the clutter whose vertex set is the vertex set of T and whose edge set is defined to be all sets of vertices in T that define a path of length ℓ in T. For T a tree, Rafael Villarreal has shown a necessary and sufficient condition for $I(\mathcal{C}_{1,T})$ to be Cohen-Macaulay. Villarreal's necessary and sufficient condition was for

T to be something we define as a tree suspension of length 1. Using methods different from that of Rafael Villarreal we define a more general tree suspension of length ℓ and show that for T a tree, $I(\mathcal{C}_{\ell,T})$ is Cohen-Macaulay if and only if T is a suspension of length ℓ. In order to reach this result, properties of ideals such as height, dimension, and depth are studied. Also, we prove that for F a forest, $\mathcal{C}_{\ell,F}$ necessarily has the König property.

303. The $\bar{X}$-S Synthetic Chart

Leah Birch Loyola University New Orleans
Advisor(s): Maria Calzada, Loyola University New Orleans

The newly developed $\bar{X}$-S Synthetic Chart is a statistical quality control procedure that jointly monitors the mean and standard deviation of a production process. This chart can be optimized for a user specified in-control average run length or ARL_0 and out of control condition (δ, γ), where $\mu = \mu_0 + \delta\frac{\sigma_0}{\sqrt{n}}$ and $\sigma = \gamma\sigma_0$ are the process mean and standard deviation and μ_0 and σ_0 their in-control target values. The process is considered out-of-control when $\delta > 0$ or $\gamma > 1$. When the process is in control, $ARL = ARL_0$, but once a shift in μ or σ occurs, the $\bar{X}$-S Synthetic chart should quickly notify the user that the process is out of control, ideally making $ARL << ARL_0$. The $\bar{X}$-S Synthetic Procedure's performance can be analyzed by comparison to other well known quality control charts, such as the $\bar{X}$-S, the EWMA-S, and the CUSUM-S procedures.

304. Analysis of Visitation Data with Demographic Projections for the EcoTarium Museum in Worcester MA

James Boyne College of the Holy Cross
Advisor(s): Catherine A Roberts, College of the Holy Cross

The EcoTarium Museum in Worcester MA is concerned with changing demographics in the state and how it might impact future visitation and membership levels. It is also interested in creating new exhibits that will interest people in the near future and wonders if these demographics will play a role in determining what sorts of exhibits will be designed. Visitation data and membership data is analyzed to understand the geographic distribution of visitors to the museum. This will help the museum target advertising to specific towns. Regression analysis of census data determines which factors influence visitation rates the most. This informs the museum staff as they plan future exhibits and programming.

305. Which Rotor-Routers are Universal?

Xiaoyu He Acton-Boxborough Regional High School
Advisor(s): Tanya Khovanova, Massachussetts Institute of Technology

Rotor-routers are a blossoming new field in dynamic combinatorics, giving natural deterministic analogues for stochastic processes such as diffusion-limited aggregation. In 2010 J. Propp defined a concept of universal rotor-router classes. I will introduce these topics and present powerful new tools in the subject. I will discuss the compressor configuration, which has rare period-preserving properties and decides the universality of wide classes of two-state rotor-router types. I will also state the Reduction Theorem that reduces rotors of n states to rotors of two states for the purposes of this problem. I will state the counter-intuitive conjecture that all unboppy rotor types are universal and present strong empirical evidence for it. Through work with the compressor I also give a number of results about string manipulations of independent interest.

306. Statistic Methods accounting for the missing observations in confounding variables and comparison of their efficiency based on simulation study

Weici Hu Smith College
Advisor(s): Nicholas Horton, Smith College

Multiple regression is used to assess the relationship between predictor variables and outcomes. Confounders are a common problem in many observations because they cloud causal associations. An additional problem is that potential confounding variables may be missing for some subjects. We explored different methods to account for partially observed confounding variables in a series of simulation studies in a setting where the explanatory variable, confounder and outcome were all multivariate normal. We imposed missingness using a series of different ignorable and nonignorable assumptions, then compared the results from a variety of approaches (truth [no missingness],

complete case only, omission of the missing confounder, missing indicator approach (Jones, JASA 1996) and multiple imputation. Less principled methods (variable exclusion and missing indicators method) yielded substantial bias while complete case methods were quite inefficient. Use of principled methods such as multiple imputation are preferred.

307. A Geographical and Statistical Analysis of Childhood Leukemia Deaths Relating to the Locations of Nuclear Power Plants

Sarah McGinnis Maryville College
Whitney Thompson Simpson College
Rebecca Pettit Notre Dame of Maryland University
Jean Sexton Southern Methodist University
Advisor(s): Monica Jackson, American University

It has been suggested that environmental factors such as radiation emitted by nuclear power plants may be to blame for the alarming increase in childhood leukemia rates. This study examines whether or not there is a geographical pattern in cancer rates based upon the locations of nuclear power plants throughout the United States. A spatial analysis is conducted to look for global and local clusters with an increased mortality rate due to childhood leukemia along with a statistical analysis that examines which factors contribute significantly to this rate. Our study finds no evidence to prove that nuclear power plants are responsible for the recent increase in childhood leukemia rates.

308. Markov vs. Ito Processes with Applications in Finance

Yue Pan Arizona State University
Advisor(s): Bruno Welfert, Arizona State University

In the simulation of differential equations random variations in parameters can be modeled with Markov processes (chains) whose state is determined by a given transition probability (matrix). Such process typically varies on a faster time scale than required from numerical integration considerations. Our goal is to reduce fast scale variations of the process to a (cumulative) variation on the numerical time scale by means of an Ito approximation of the Markov process, which is scalable and can be easily implemented alongside the deterministic problem. Applications to problems arising in finance such as bond and option pricing illustrate our strategy.

309. Linear Tests of Uniformity for Data Defined on Polygons and Finite Tori

Laura Passarelli Scripps College
Connor Ahlbach Harvey Mudd College
John Choi Harvey Mudd College
Advisor(s): Michael Orrison, Harvey Mudd College

Suppose respondents in a survey are asked to choose an element from a finite set X. If we assume their responses are governed by an underlying probability distribution P, then it is natural to wonder whether P is actually the uniform distribution defined on X. In this poster, we present the results of our study of linear tests of uniformity when X is the set of points of a discretized circle or torus. In particular, we construct several straightforward tests of uniformity, derive formulas for their associated degrees of freedom, and run the tests on two example data sets to demonstrate their usefulness.

311. The Effects of Hurricane IKE on a Natural Wetland Ecosystem

Glenda Thammavongsa University of Houston-Downtown
Conrado Cacho University of Houston-Downtown
Advisor(s): Brad Hoge, University of Houston-Downtown

This study utilizes two taxonomic groups (diatoms and arcellaceans) and statistical analyses to study changes in a coastal wetland before and after a hurricane. Anahuac National Wildlife Refuge is a natural wetland adjacent to Galveston Bay, which was struck by Hurricane Ike in 2008. Saltwater intrusion was expected to change the species diversity in the marsh. Alpha and Beta diversity studied to determine changes in species richness and relative abundance. Shannon's index was used to exhibit the alpha diversity of both groups. Spearman's similarity and Bray-Curtis dissimilarity indices were used to study beta diversity. The death assemblage was used to show changes before and after Ike. Stable assemblages were present before the hurricane. Drastic changes were present for two years after

the impact. Recovery was apparent by the third year. Shannon's and Spearman's indices displayed a pattern of low diversity prior to Ike, a significant increase two years after, and a gradual decrease in the third year. It was not possible to show the amount of change was significant compared to trends in previous years for Spearman's Index, however, Bray-Curtis results showed dramatic changes in beta diversity due to the introduction of rare genera.

312. Yelling in Circles: A Generalization of the Game ''Look Up and Scream"

Sarah Warkentin Harvey Mudd College
Lucia Petito University of Rochester
Advisor(s): Anant Godbole, East Tennessee State University

We examine the mathematics in the game "Look Up and Scream!" We extend the definition of a "win" to the formation of a k-cycle for a fixed k. We show that the number of yells Y_k is approximately Poisson distributed (with parameter 1/k), and give a bound on the error of the approximation. Then we discuss the joint distribution of k-cycles for all k up to a fixed b, and show that the random variables Y_k can be approximated as independent Poisson random variables.

313. Cutting in Queues

James York-Winegar Austin Peay State University
Advisor(s): Matthew Jones, Austin Peay State University

We investigate the first-in-first-out $M/M/1$ queue with arrivals "cutting line" at random locations in the queue. Arrivals cutting in line frustrate previously arriving customers, causing them to leave the queue. We derive the stationary distribution of the queue length, average queue length in steady state, and average waiting times to departure, probabilities of obtaining service, and average waiting time to service conditioned that an arrival actually makes it through service. We also study the "cutting line" effects on the $M/M/s/K$ queue.

Index

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