MAA FOCUS

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WHAT'S INSIDE

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The Mathematical Association of America intends to change the name of its newsmagazine to MAA FOCUS.

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MAA FOCUS

Volume 28 Issue 8

Inside

On the cover: Frank Farris's "5D Window," created to his specifications by Hans Schepker. See the article on page 4. (Photo by Frank Farris.)

MAA FOCUS Deadlines

	February	March	April
Editorial Copy	December 10	January 14	February 9
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Employment Ads	December 10	January 14	February 9

Cross-Cultural Study About Math Attitudes a Media Hit

On Friday, October 10, 2008 the *Notices of the American Mathematical Society* published an online version of "Cross-Cultural Analysis of Students with Exceptional Talent in Mathematical Problem Solving," an article by Titu Andreescu, Joseph A. Gallian, Jonathan M. Kane, and Janet E. Mertz. The article is a comprehensive analysis of decades of data on students identified as having profound math ability. It suggests that cultural attitudes put American leadership in the mathematical sciences and related fields at risk.

Working with an advance copy, Sara Rimer of *The New York Times* and Carolyn Y. Johnson of *The Boston Globe* wrote lengthy stories about the study that included quotes from the authors, some of the students mentioned in the study, and Boston-area academics. Rimer's piece, news releases written by Allyn Jackson of the AMS and Terry Devitt of University of Wisconson-Madison together with the authors, and a Reuters story were media hits. By Saturday morning, Rimer's article was the second most emailed *Times* story, only behind an election politics op-ed. Within one day, a *Google*® search located 30 US news stories and 23 international ones related to the study, including ones written in Chinese, German, Hungarian, and Italian as well as English language stories published in India, Canada, Australia, Thailand, and Malaysia among others. Among the outlets were the *New York Post*, *Denver Post*, *AOL News*, *ABC News* (online), MSNBC.com, *Yahoo News*, and Smashit.com.

The *Orlando Sentinel* ran an editorial agreeing with the recommendations given in the study for correcting the problem. Numerous newspapers and websites in India posted the news releases or their own stories. MyNews.in used the title "US kids are math duds, so reveals a study;" *Science News* used "Numbers don't add up for U.S. girls;" ScienceDaily.com used "US Culture Derails Girl Math Whizzes;" the *Calgary Herald* used "Negative stereotyping robbing US of math talent."

The impetus for the study came in January 2005, when then Harvard President Lawrence Summers suggested that factors other than socialization and discrimination could be the primary reason there are many more men than women in high-end science and engineering positions in the US. Summers' remarks prompted Janet Mertz, a biochemist at the University of Wisconsin-Madison whose child excelled in the IMO and Putnam, to initiate the study. Mertz had long suspected that cultural attitudes are the prime reason for the under representation of women and minorities among the top finishers in these ultra-high-level math competitions, but could not find hard data to support her belief. Andreescu, former leader of the US Math Olympiad team, Gallian and Kane joined in the study. The *Notices* article can be found at http://www.ams.org/notices/200810/tx081001248p.pdf.

MAA to Host Open House During the Joint Mathematics Meetings

The selection of Washington, D.C. as home to the upcoming Joint Mathematics Meetings presents MAA members a special opportunity to see a different side of their beloved Association. MAA headquarters will host an open house that will give its members a chance to visit the Dolciani Mathematical Center, the MAA Carriage House, and Halmos River of Bricks on Wednesday, January 7th from 3 p.m. until 6 p.m.

The open house will give members a chance to see the things that make MAA headquarters so unique, such as the spiral staircase winding throughout the Dolciani building, the newly renovated Carriage House, and the way the River of Bricks duplicates the confluence of the Potomac and Anacostia rivers that outline the nation's capital. Those who participate in the open house will also be able to take a peek into the Association's past by checking out the portraits of former Presidents that line the staircase, or by thumbing through old volumes of the journals in the Begle Room.


The MAA Carriage House

Shuttle buses marked with an "MAA Open House" sign will depart from the Marriott Wardman Park Hotel's 24th Street entrance every half hour from 3:00 p.m. to 5:30 p.m. and return every half hour from 4:00 p.m. to 6:30 p.m. If meeting attendee's would rather metro to the headquarters buildings, they can catch a red-line train at the Woodley Park Station headed toward Glenmont and get off at Dupont Circle. Leave through the Dupont Circle North exit and walk east on Q Street to 18th Street. Take a right on 18th Street and left onto Church Street to get to the Carriage House at 1781 Church Street.

Refreshments will be available during the open house. Additional information will be available during the meetings at the MAA booth in the exhibit hall at the Marriott Wardman Park Hotel.

A Window on the Fifth Dimension

By Frank A. Farris

Is there enough mathematics in your home? What visual aids do you keep on hand for that inevitable moment when guests want to know why you spend your life on mathematics? Feeling a lack in this area, I commissioned glass artist Hans Schepker to produce a window — from the fifth dimension? — based on an image that came up in my research. It turned out splendidly, and you can see it on the cover of this issue of MAA FOCUS.

The story of the design involves complex functions, color as a visualization tool, and five-fold symmetry. I cannot tell it to every guest who walks in the door, but mathematicians will find it a strange combination of the familiar and the unlikely. Can this window make the fifth dimension transparent?

The Human Story

At the 2007 JMM in New Orleans, I met Hans Schepker, who was exhibiting his glass art. You may have seen his booth or visited his web site, http://glassgeometry.com. A rainbow pentagon design caught my eye and I asked whether he might be able to make me a custom window for my home. Hans was enthusiastic, but I did nothing about the project until I saw him again a year later at the JMM in San Diego.

It turned out that Hans was to spend several weeks that winter in my neighborhood, giving a geometry course at a Waldorf School, using glass art as the hook to interest students. We met at my house and talked about possibilities, settling on a computed image — approximately the one in Figure 1 —as the pattern for a window.

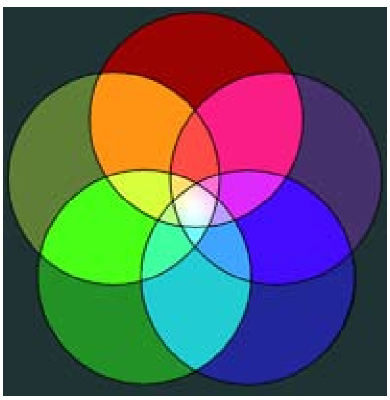

Figure 1: The original schematic design

The next, rather difficult, step was selecting the actual glass for the project. I spent a couple of hours with Hans at the Palo Alto showroom of Franciscan Glass — enough to learn the difference between pointing to a color on a screen and having a piece of glass of just that color.

A week or so later, I asked my college friend Kitty Kameon, who has taught color theory at the San Francisco Academy of Art University, to meet me at the Franciscan Glass *warehouse*, across the bay at the eastern foot of the Dumbarton Bridge. We spent more hours sorting through possibilities, which after a time ceased to appear finite. The goal was to implement certain rules from the mathematical original: Crossing one of the circular boundaries leads to a neighboring color; moving from the center outward should lead from white through pastels to vivid saturated colors and then to darkness at the outside. (The outer darkness was replaced by clear glass in order to help the window appear to float in its frame.)

In addition to finding *colors* for each of the four rings of five congruent shapes, we thought about the balance of textures. In some places, the window is not entirely symmetrical. We decided that mathematical precision could give way to aesthetic considerations, given that the glass choices were finite. The bluish sector was the easiest part; there were more than enough choices. We spent more time on the sector where greens become oranges and then reds. I especially like the gold crown at the top of the window. Overall I very much appreciate how Kitty's advice led to harmonious colors that honor the spirit of the mathematical design.

Once Hans had all this information, he faced the engineering challenge of making the window strong enough. A typical glass window has at least some straight lines — an obvious source of strength. This window has only curves. Despite the challenges of the unique design, Hans sent the finished window to me after only six weeks' work. It arrived in a custom-made plywood crate and has delighted me from my first view.

The Mathematical Story

The mathematics behind the design started with my desire to find a mathematically correct depiction of a color wheel, something I have been trying to do for years. My need for a good color wheel arose in the mid-90s, when I proposed a particular way to use color to depict complex-valued functions on the complex plane [**2**]. The first step is to color the complex plane in such a way that, theoretically at least, each point has a different color. I like to use the primary colors red, green, and blue to color the cube roots of unity, then fill in hues around the unit circle, and fade to white at 0 and black at infinity. To create a *domain coloring* for a function $f(z)$, at each z-value in the domain, paint the color corresponding to $f(z)$. More realistically, for the z-value of each *pixel* in the picture, set the color value of that pixel to $f(z)$.

For instance, using the rather crude color wheel on the bottom in Figure 2 to indicate the color scheme, I produced the domain coloring of a sixth-degree analytic polynomial beneath the color wheel. The five white areas indicate the zeroes of the polynomial, which might suggest that we are one zero short. Around the largest white area, note that the colors cycle twice around the edge: This is a *double* zero — just one example of the many phenomena revealed by domain colorings.

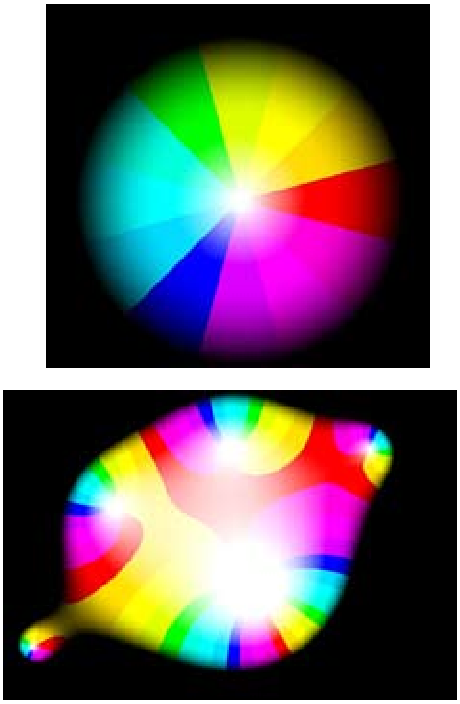

Figure 2: A choice of color wheel and the corresponding domain coloring of a sixth degree polynomial.

I have worked with several color wheels, each one a kludge. Then, in the summer of 2007, I prepared a talk for the MAA Silver and Gold Banquet at the San Jose MathFest. The new color wheel was mathematical!

Most of my computed images have been coded in a file format called PPM. It is rather an antique file format, but it is extremely simple and portable. For each pixel, the file lists three values from 0 to 255 to indicate how much red, green, and blue light should shine from that pixel. These are called RGB values and, independent of file format, most screens you view are being addressed via RGB values. The set of possible RGB values forms a rather obvious cube, with black at one corner, as (0,0,0) and white, as (255, 255, 255), at the opposite corner. This cube was the inspiration for my next steps.

New Idea: Use stereographic projection to map the complex plane to the unit sphere. Map the unit sphere inside the RGB cube, tilted so that the pole corresponding to 0 is nearest to (255, 255, 255) and the equator point that came from 1 is near the red corner. Then read off the colors.

It takes a little computation to get white in the right place, and to pursue my intention that 1 should be colored red. Here is the formula that takes complex numbers to a correctly-tilted sphere inside a cube: $z = u + iv \rightarrow (X, Y, Z) =$

$$\frac{1-u^2-v^2}{1+u^2+v^2}\frac{(1,1,1)}{\sqrt{3}} + \frac{2u}{1+u^2+v^2}\frac{(2,-1,-1)}{\sqrt{6}} + \frac{2v}{1+u^2+v^2}\frac{(0,1,-1)}{\sqrt{2}} \in [-1,1]^3$$

Readers may recognize that the coefficients of the vectors arise from stereographic projection. It is conceivable that someone might recognize the three vectors: They are unit eigenvectors of the cyclic permutation of three variables. The first one points along the main diagonal of the cube, toward the vertex that will represent white. The second and third span the plane of the celebrated regular hexagonal cross-section of the cube.

All that remains is to map this 2 x 2 x 2 cube to the color cube:

$$(X, Y, Z) \rightarrow \left(\frac{1+X}{2}, \frac{1+Z}{2}, \frac{1+Y}{2}\right).$$

This is so theoretically lovely that it should yield beautiful pictures. Unfortunately, this mathematical color wheel looks terribly bland.

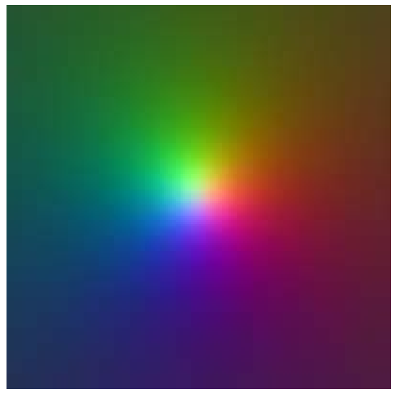

Figure 3: A color wheel derived by stereographic projection of the complex plane into the color cube.

This is where my pragmatic nature took over, happily leading me to unexpected consequences: I decided to take the cube root of each component of $(X, Y, Z) \in [-1,1]^3$. Why? This would drive points outward toward the edges of the cube; brights would become brighter, darks darker. Before you read on, ponder the resulting color wheel. What is Figure 4 a picture of?

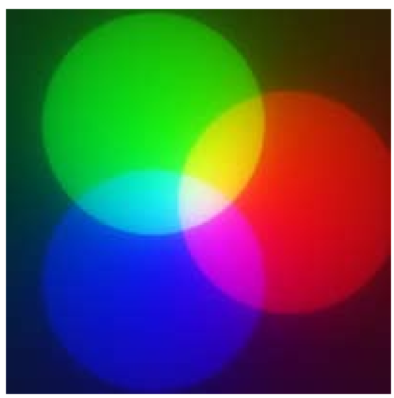

Figure 4: A color wheel derived from the previous by taking cube roots coordinate by coordinate, for no good reason.

If you are discretely minded, you might recognize the graph of a regular octahedron, viewed by stereographic projection. A continuously minded mathematician might focus on the circular boundaries, which are unstable sets of the cube root map, the traces of the coordinate planes. Let's explain each point of view.

Picture what the cube root map does to the interval [–1,1] . It moves positive values toward 1 and negative values toward –1; these points are stable attractors. The origin is unstable; points on either side, though they may be very close, are moved toward different endpoints. Now observe that I applied this map to each coordinate of a triple inside the cube. This moves points away from the coordinate planes and toward the vertices of the cube.

The reason we see eight regions of almost constant color is that all points in one of those regions have been pulled toward the same vertex of the cube, of which there are eight. The reason we see dividing circles is that color changes rapidly when we cross a place where one of the (X,Y,Z) coordinates is zero; these three loci are the intersections of the tilted sphere with coordinate planes. Appearances notwithstanding, this is indeed a *continuous* assignment of colors, and were it not for the discretization in RGB space, there would be a unique color assigned to each point in the complex plane. If we flatten the image to eight colors (my software says it contains more than 184 thousand), since each region corresponds to a vertex of the cube, we get a stereographic view of the polyhedron dual to the cube, namely the octahedron.

Using this color wheel in domain colorings gives a pleasantly art deco effect, as in Figure 5.

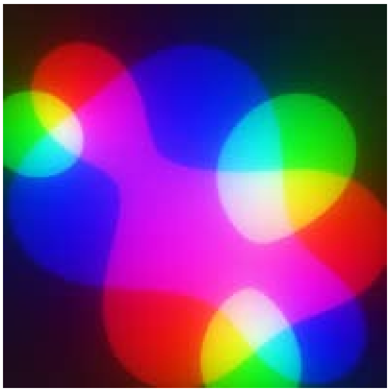

Figure 5: The cube-root color wheel used for a domain coloring of a cubic polynomial.

And now, the fifth dimension: The three-fold rotational symmetry of the cube-root color wheel is beautiful, and consistent with my original intention to place red, green, and blue around that unit circle at the cube roots of unity. Still, I wondered what other polyhedra might turn up as the result of more-or-less natural maps of the plane into the color cube. I wondered if I could produce the MAA logo, a regular icosahedron, in this manner. My first goal was to produce a color wheel with five-fold symmetry.

The most fruitful idea was to put the 2 x 2 x 2 cube into $\mathbb{R}^5$ so that it sits in a three-dimensional eigenspace of the cyclic permutation of five variables. I had observed in the past that this is a good source of five-fold symmetry. I used the components of stereographic projection as coefficients of normalizations of the vectors

$$\left(1,1,1,1,1\right),\left(1,\cos(2\pi/5),\cos(4\pi/5),\cos(6\pi/5),\cos(8\pi/5)\right),$$
$$\text{and}\left(0,\sin(2\pi/5),\sin(4\pi/5),\sin(6\pi/5).\sin(8\pi/5)\right).$$

Once the cube was placed in $\mathbb{R}^5$, I took cube roots of each component, moving each point toward the nearest vertex of the 5-cube. This pulls points off the 3-plane spanned by the three given vectors, so I simply projected them back. It was then easy to read off colors, in the same manner as before. The resulting color wheel is shown in Figure 6; actually, this one uses fifth roots instead of cube roots, which gives a more distinct coloration. Now that you know that a regular octahedron lurked in the previous color wheel, you might enjoy figuring out which polyhedron this represents.

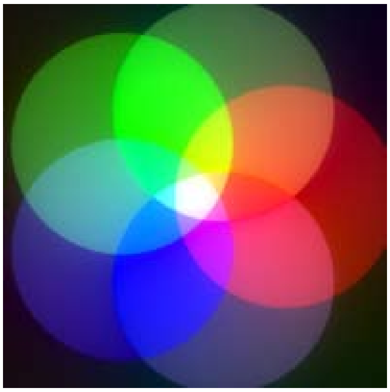

Figure 6: A color wheel with 5-fold symmetry, the source of the window design

There are 22 faces — two pentagons, ten triangles, and ten quadrilaterals. This is the dual of the 5-fold zonohedron. Given that there are 32 vertices of the 5-cube, why do points on the sphere inherit colors from only 22 of them? Computation shows that 10 of those vertices have small projection onto this 3-space; they are not sufficiently near to draw points.

I undid the stereographic projection and constructed this polyhedron in Maple, with the pentagonal faces missing so that you can look through. The faces are colored in only approximately the correct way. Figure 7 shows three views, from the top, the bottom, and along one of the great circles that corresponds to rings in Figure 6.

Figure 6 is the source of the window design, rotated for aesthetic effect. It is a window on the fifth dimension in this sense: Looking into the center, we are looking up into the portion of $\mathbb{R}^5$ where all coordinates are positive, along the vector (1,1,1,1,1).

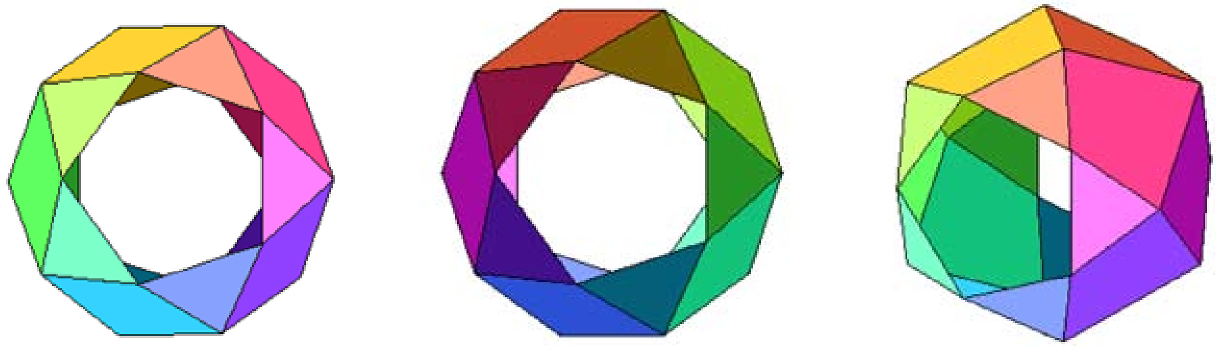

Figure 7: Three views of the dual of the 5-fold zonohedron

From there, crossing any of the five large rings takes us past a coordinate hyperplane, and one of the variables turns negative. When we have crossed all five rings, by any of the curious paths our eye can trace, we are looking back the other way, along (–1,–1,–1,–1,–1), toward darkness.

References

Frank A. Farris, Review of Tristam Needham's *Visual Complex Analysis* in *Amer. Math. Monthly*, 105:6 (1998), 570–576. (Supplementary materials, including color versions of the published images, appear at http://www.maa.org/pubs/amm_complements/complex.html.)

Frank A. Farris, Vibrating Wallpaper, *Communications in Visual Mathematics*, vol. 1 (later vol. 0 of *JOMA*).

Frank Farris teaches at Santa Clara University. He is currently interim editor of Mathematics Magazine. *His home page is* http://math.scu.edu/~ffarris/.

Marcia P. Sward (1939–2008)

By Linda Rosen

The mathematics community lost a champion and a friend with the death of Marcia Peterson Sward on September 21, 2008. She died with dignity and grace from kidney cancer that was diagnosed just weeks before her death.

Marcia served as the first Associate Executive Director of the Mathematical Association of American (MAA) between 1980 and 1985. She returned to the MAA as Executive Director in 1989. Between her two periods at the MAA, she served as the Executive Director of the Mathematical Sciences Education Board (MSEB) at the National Research Council of the National Academy of Sciences.

Marcia graduated summa cum laude and first in her class at Vassar College with a degree in mathematics. She and her husband of one year, Gil Sward, each pursued graduate study in mathematics at University of Illinois at Urbana-Champaign. Although she was one of the few females in the program, Marcia approached her work with zest and dedication. Under Edward Scott, Marcia was awarded her Ph.D. in 1967 for her dissertation on *The Mixed Boundary Value Problem Along the Line of Parabolicity for a Certain Class of Hyperbolic Partial Differential Equations*.

After graduation, Marcia spent a year at Catholic University in Washington, DC as a housemother while job hunting. She began her academic career at Trinity College, also in Washington, DC. Although she loved teaching, Marcia was always looking for new causes to wrestle and new heights to scale. She accepted a one-year visiting appointment at the National Highway Traffic Safety Administration, where she began to hone her executive and management skills.

At that time, the MAA was creating a new position for an Associate Executive Director that would, among other tasks, direct publication of the organization's three journals and create a newsletter to serve members' needs. Intrigued by

the possibility of combining her love of mathematics, her dedication to helping young people learn, and her leadership skills, Marcia applied for the job. To get a feel for the place, Marcia decided to hand-deliver a cover letter for her application early on the morning of her interview. To her surprise, there was a man sitting alone on the doorstep, waiting for the building to open. She had to explain her somewhat unorthodox behavior, only to learn that she was talking to Don Kreider, the chair of the search committee for her position! It was no surprise that her drive, talent and contagious enthusiasm led to a job offer as the first MAA Associate Executive Director.

Employing paper, red pencil, and a typewriter, she managed to bring out the first issue of FOCUS in March 1981. The debut of FOCUS "was worth waiting for," Ralph Boas (Northwestern University) later wrote. "You are setting a high standard for yourself." Ivan Niven (University of Oregon) hailed "the balance of articles and announcements, of ideas on the one hand and facts on the other." And Don Albers, then Editor of the *Two-Year College Mathematics Journal*, exclaimed, "FOCUS is terrific… Finally, something that I can say I've read from cover to cover." Lynn Steen, who co-chaired the FOCUS editorial board at that time, recalls many substantive discussions with Marcia about the possible inclusion of letters to the editor in the newsletter. Should they be encouraged? Should they be limited to MAA issues or to responses to FOCUS issues? There were no MAA precedents; her groundbreaking work on letters to the editor and other FOCUS features persist to this day. Marcia served as editor of FOCUS until September 1985, when she left the MAA.

As her next professional venture, Marcia agreed to become the first Executive Director of the MSEB, which was established by the National Research Council in response to the David report, "Renewing U.S. Mathematics: Critical Resource for the Future." This was a risky venture. There was seed money from the National Research Council (NRC) to keep the organization afloat for only a year, and its mandate was proactive, unlike other groups within the NRC. Working with Kenneth Hoffman (then the director of the Joint Policy Board for Mathematics Office of Governmental and Public Affairs) and Shirley Hill (Past-President of the National Council of Teachers of Mathematics), Marcia convened a stellar advisory board and raised sufficient funds to see the MSEB grow enormously in influence. The seminal report, "Everybody Counts: A Report to the Nation on the Future of Mathematics Education" remains to this day a key resource for K–12 mathematics education. Always persuasive and cheerful, she recruited Lynn Steen, just finishing his last meeting as Past-President of the MAA, to serve as author of that report.

Under Marcia's tutelage, the MSEB served as a focal point for the mathematical community on issues related to K–12 teaching and learning of mathematics. I was then Director of Policy Studies at MSEB, and remember that Marcia displayed acumen, an incredible work ethic, talent for collaboration, passion for the cause, and an unwillingness to seize personal glory. Ken Hoffman noted that, "Beyond her intellect, Marcia's special gifts were her warmth and vitality. They were infectious, impacting every project and every person she worked with."

Marcia returned to lead the MAA in 1989 on the occasion of Alfred Willcox's retirement. Although many people tried to persuade her to stay with MSEB, citing her unique skill set and record of success, she admitted that her heart belonged to the MAA.

Marcia with Mary Kay Abbey and Tina Straley on the ferry to Samos, May 2003. Photograph by Liz Teles.

The following presidents benefited directly from Marcia's support — Lida Barrett, Debbie Haimo, Don Kreider, Ken Ross, Gerald Alexanderson, and Tom Banchoff. Ken Ross remembers, for example, attending a workshop put on by the American Society of Association Executives with Marcia. Its express goal was to help them analyze their own and each other's working style. He notes that the workshop was "…a good idea and got us off to a good start." It meant enough to Marcia that she fondly mentioned the workshop in her last conversation with Ken this summer. Lida Barrett had similar impressions of Marcia. "Because of her past role at MAA, she offered wonderful support to me in my role as President. Her wide contacts in the mathematics profession, many from her time at MSEB, helped expand MAA's interactions and influence with other organizations and activities," Barrett noted.

Under Marcia's leadership, the MAA increased its membership, programs and revenue. She was instrumental in initiating new programs, many of which were grant-supported. Chris Stevens recalls that when she and Jim Leitzel were developing the idea for Project NExT, they wondered where they might secure outside funding. Marcia interested the ExxonMobil Foundation in the concept; in fact, Marcia was so persuasive that the Foundation indicated their willingness to provide support and requested a formal proposal primarily for recordkeeping purposes. Stevens said that "Without her successful sales pitch, Project NExT might never have become a reality." Marcia's support did not end with her fundraising; she took a keen interest in the progress of Project NExT, even after she retired from the MAA.

Another project that benefited from Marcia's leadership and dedication was SUMMA (Supporting Undergraduate Minority Mathematics Achievement). Steen wrote that "She was a perfect example, in Robert Kennedy's memorable image, of a person who dreams of things that never were and asks why not? MAA programs such as SUMMA and Project NExT are living memorials to her energy, effort and commitment."

Her collaboration with Ed Dubinsky to develop special interest groups within MAA came to fruition in 1999 with SIGMAA-RUME. Dubinsky said that "First and foremost, Marcia was for me, during the many years of her work with the MAA and after, the "face" of the organization. Aside from her many concrete accomplishments; she made the MAA a warm, humane, professional society. It was a pleasure to interact with her and she created a very positive atmosphere in all aspects of the organization."

"It is time for me to move on to new challenges," Marcia said on the occasion of her retirement in 1999. "I have had wonderful opportunities at the MAA." Gerald Alexanderson observed, "It is difficult to imagine the MAA without Marcia at the helm."

Nonetheless, Marcia continued her association with the MAA, regularly participating in the winter meetings, special events at the Carriage House, and MAA study tours. MAA staff member Lisa Kolbe treasures her memories of time spent with Marcia on the MAA's first tour to Greece. "I remember her dancing high in the mountain village of Vourliotes in the lovely white Greek outfit she bought on the island of Samos."

After her retirement, Marcia went sailing for a year in the Bahamas, a sport she had grown to love as a child. Upon her return, she found that she was not quite ready to retire. Her pioneering spirit and her love of nature led her first to the National Environmental Education and Training Foundation, whose mission is to provide objective environmental information to help Americans improve their quality of life.

Until her death, Marcia was Deputy Executive Director of Environmental Education at the Audubon Naturalist Society. She helped design programs to reach over 8,000 children through preschool, school, camp, scout and family programs. Among many other programs, Marcia provided the inspiration and leadership to establish a preschool where children unravel the wonders of the natural world through a balance of self- and teacher-directed activities.

Marcia is survived by two sons, Douglas and David Sward and their wives, Honey and Erika as well as her former husband, Gil Sward. She is also survived by two older brothers, Walt and Reeve Peterson, their wives, Judy and Georgia, as well as many nieces and nephews. Her first grandchild, a boy, is due in October. Despite her many professional accomplishments, what mattered most to Marcia was her family.

Marcia's family has requested that donations in her memory be made to:
Marcia Sward Tribute Fund
Audubon Naturalist Society
8940 Jones Mill Road
Chevy Chase, MD 20815.

Preparing Students for a Life in Math or Computer Science

By Donna Pierce and Peter A. Tucker

"I like math, I just don't know what I would do with a math degree." We've all heard this statement from undergraduates. Students may be aware of a few vocational directions, but are largely unaware of other opportunities. They are even more unaware of how those vocational directions will influence their lives. We have found that students need more than to be referred to web sites; they need to find out that there are bigger issues involved. Each student will require different answers to vocational questions. Thus, we realized that students need more complete guidance to help them do the exploration. We decided to develop a one-credit course on career preparation for Math/CS students.

Certainly, not all careers are the same — a course that simply tells students what career they should pursue won't work. Students need experience in researching careers, and they need time to consider these careers in all their detail. The course had to guide students toward learning and understanding their own specific strengths and values, so that they could see what kinds of vocational opportunities are consistent with those strengths and values. We wanted each student to consider the life they are looking toward and how their career fits into that life. In other words, we wanted students to be concerned with their entire career, and not just with landing the first job. We wanted them to think beyond their potential salary, and to consider environment, personal values, ethics, contribution to society, and personal satisfaction as well.

Toward those goals, we set up the following components for our course:

- Individual strength and value research (e.g. Meyers-Brigg)
- Career and opportunity research
- Internship/REUs and graduate school research
- Career profiles, "day in the life" articles
- Field trips and informational interviews
- Guest speakers
- Ethical investigation
- Final paper booklet

The target audience for this course is sophomores, for two reasons. They have just decided on this major and we want them to be thinking ahead. At this point, they might be thinking about their plan for the next four years, but we want them to see why each piece of that plan is important and how certain choices can help them as they consider future directions. Sophomores aren't always ready for internships/REUs, but they need to start thinking about and preparing for them. The course should be informal, and therefore we made it a one-credit course. The seminar style, with weekly cookies, also helps. We encourage discussion, both in small groups and with the whole class. Finally, students each write a five-page paper on a particular career path, incorporating all that they have learned throughout the course. We collect these papers and bind them into a booklet. Each student receives a copy of that booklet, as do the faculty in math and computer science.

Two components are critical to the success of this course: internships or REUs, and field trips and informational interviews.

One of the best ways for our students to learn what they can do with a math degree is to have them participate in an REU or internship. Thus, we have the students research REUs. Many students do not know what an REU is or how to apply. We ask them to find three different internships or REU opportunities using a worksheet to guide their investigation. The worksheet asks students to find such things as: title and description of internship, skills and background required, stipend, and the application process. Additionally, students who have done REUs in the past come and discuss their experiences with the class, describing their experience and sharing what they have learned about career opportunities and how to prepare for them. Interacting with people in their field on a day-to-day basis gives students opportunities to ask questions that help them think more holistically about career choices, considering such things as personal satisfaction, personal values, and contribution to society. Through this assignment students learn more about opportunities available to Math/CS students and the background skills required for qualifying for these opportunities.

Informational interviews and the field trip are one of the most eye-opening experiences for students, especially when it comes to learning about the day-to-day characteristics of a career. For the informational interview, students have to find their own interviewees. They choose someone in a career of interest to them, find out about the day-to-day facets of the interviewee's job, specific issues that might arise, and gather valuable career advice. We find that our alumni are a great resource, as they are personally invested in the school and show interest in helping the next generation of graduates do well.

The field trips are always the highlight of our course. We try to include interaction with research and with industry on these trips. Students interact with research through a visit to a graduate school or to a research conference, and with industry by visiting a workplace. In both cases, we try to set up panels for open, informal discussion. We find that it is very worthwhile to have a lot of diversity on these panels, including gender and experience.

We have offered this course for the past four years and have noticed the following benefits: Students have started thinking more about careers, and preparing for those careers, before their senior year. They know some of the questions they need to ask and how to find answers independently. The research assignments, informational interviews and field trips give students a wider and more complete view of what's out there and what working in their field looks like. More students are applying and participating in REUs and internships. Students report more interest in, and less fear about, graduate school. The course is also an asset in our recruiting efforts. Now when students say, "What would I do with a math degree?" we point them to this course. Finally, this course serves as a model for other departments whose goal is to integrate career preparation into a student's major.

One challenge we have run into is a disconnect between student expectations for this course and our own. Students would like us to tell them what career path they should take and walk them through the process until they get the job. We want students to think more holistically about career choices, but students tend to focus more on details. Course evaluations tend to say: "More help with resume writing and interviewing techniques." In our view, this is training available at the career center and our course should go beyond that. Another evaluation sentiment echoes that disconnect: "Don't spend as much time on ethics." We understand the students' needs for practical help. We have made adjustments to the course schedule to address some of those needs. However, on some course objectives (ethics, holistic informed career choice, fostering independence) we remain stubborn.

As we consider future course offerings, we want to make our course expectations clearer throughout the semester. We have two specific strategies for the next time: First, we will invite alumni of our course to return as guest speakers, to express how (we hope) the course has helped them. Second, we want to reconsider the homework assignments. Rather than have students simply report on what they find, we want students to be more reflective on how this information corresponds to their personal skills and values. We think these strategies will improve a course we are already proud of.

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Teaching Time Savers: The Microphone Is Mightier than the Pen

By Emily Dryden

In my upper-level courses for math majors, I am fond of assigning projects. These multi-stage assignments include a brief proposal, a written report, and an oral presentation. While these projects create an excellent opportunity for students to explore new mathematical topics and take ownership of their learning, they also add significantly to my grading pile. On the written components of the project the bulk of my feedback takes the form of end comments, in which I try to point out both strengths of the document and areas for improvement. I am extremely careful, almost obsessive, when I write these comments. After all, if I want my students' writing to improve, I should model good writing. Each set of end comments takes about 20 minutes to compose.

Technology to the rescue! Using standard recording software (see below for details), I find that I am able to *say* my end comments in two to three minutes. It takes me a few minutes to compose my thoughts, and I sometimes listen to my comments to make sure that I struck the right tone. But I now spend at most ten minutes producing substantially the same feedback as before, just in a different medium. And the medium makes all the difference — my speaking isn't perfect, but I'm less obsessive about it. Also, I find it easier to convey criticism in the oral format without the harshness that a written comment can sometimes carry. While I am no longer explicitly modeling the mechanics of good writing, I am still emphasizing the importance of formulating thoughts clearly and in an audience-appropriate manner, which is the guidance my students seem to need most.

Of course, this dose of grading relief is applicable to more than just comments on papers. While probably not appropriate for offering feedback on computationally-heavy problem sets, the method can be used for work that is expository or includes substantial abstract arguments. You can read a student's submission and write numbers in the margin at spots where you have a question or comment, then simply refer to those numbered spots in your recorded feedback. For the more technologically inclined, *Microsoft Word* and *Adobe Acrobat Professional* allow you to embed audio files throughout a document; if students submit their assignments electronically in one of these formats, you can directly replace written margin comments with audio clips.

How do students respond to this change? Well, my students are already used to being bombarded by audio, and they haven't reported any problems with listening to my comments. I can tell that they do listen, because later submissions in the process incorporate my suggestions. Since students are frequently asked to submit work electronically, it seems natural to receive electronic feedback.

Okay, so how can you do this? First you need to check whether your computer has recording software and a microphone. I use *Audacity*, which is available for free download for various operating systems including Mac OS X, Windows, and Linux; go to http://audacity.sourceforge.net. Your computer may have a built-in microphone; otherwise, a basic plug-in microphone will cost you about $15. Plug in the microphone, open the software, and you're ready to record. I usually export my comments as MP3 files, since my students are accustomed to listening to these. Then you can either e-mail the files to students, or place them in students' personal directories if there is a "drop box" system in place at your institution.

Time Spent: A one-time investment of 10–30 minutes to find/download recording software and get comfortable using it, plus at most 10 minutes per student per assignment to provide feedback.

Time Saved: At least 10 minutes per student per assignment.

Emily Dryden is an assistant professor at Bucknell University in Lewisburg, PA. Technology has not yet invaded all aspects of her life, as she prefers to cycle to work and exist without a cell phone.

Teaching Time Savers are articles designed to share easy-to-implement activities for streamlining the day-to-day tasks of faculty members everywhere. If you would like to share your favorite time savers with the readers of FOCUS, then send a separate email description of each activity to Michael Orrison at orrison@hmc.edu. Make sure to include a comment on "time spent" and "time saved" for each activity, and to include pictures and/or figures if at all possible.

MAA Online Introduces NumberADay and MinuteMath

The MAA has added two new features to its web site; NumberADay (http://maanumberaday.blogspot.com) and MinuteMath (http://maaminutemath.blogspot.com). Every workday, the NumberADay blog spotlights a different number and some of its interesting properties, while MinuteMath will exercise your mind with a new problem selected from the American Mathematics Competition's bank of contest problems. Your comments, suggestions, and additions are welcome.

A Book on PhDs in Mathematics Education

The presentations from the recent National Conference on Doctoral Programs in Mathematics Education have been published by the Conference Board of the Mathematical Sciences. The book is entitled *U. S. Doctorates in Mathematics Education: Developing Stewards of the Discipline*. It was edited by Robert Reys and John Dossey and is in the *Issues in Mathematics Education* series co-published by the American Mathematical Society and the Mathematical Association of America. A review is forthcoming on *MAA Reviews*.

Notice of MAA Policy for Interviews at the National Meetings

The MAA recognizes the important role that our national meetings play in the hiring process, both for those seeking jobs and for departments. Especially for those on the job market, the application and interview process can be a stressful time, and it may be awkward for applicants to express their desire to modify the setting in a situation that they feel is less than ideal. Therefore, the MAA Board of Governors has adopted the following policy:

"The MAA strongly discourages the use of personal hotel sleeping rooms as the site for professional interviews of prospective employees. This practice is intimidating for some job-seekers, particularly those who find the situation uncomfortable and possibly unsafe."

Found Math

…And as N increases, the value of 2^N rises exponentially.

From "Quantum Computing with Ions,"
Scientific American,
August 2008, p. 67.
Thanks to Jonathan Sondow

An Intuitive Approach to the Borsuk-Ulam Theorem

By Alex Bishop, Adam Cimpeanu, Kyle Flood, Bianca Homberg, Steven Homberg, Eric Marriott, Jeffrey Roth, Linus Schultz, William Sherman, Alex Smith, Geoffrey Smith and James Tanton

Each semester I (Tanton) offer extra-curricular mathematics courses for math-interested students, ages 11–18, keen to experience mathematics as a creative and organic enterprise. These courses, dubbed research classes, introduce students to the joys — and frustrations — of the research experience, of not knowing, of "feeling around in the dark" and of finding the fortitude of mind to dwell on complex issues for sustained periods (weeks, not just minutes). This semester the co-authors of this paper established the classic Borsuk-Ulam Theorem in a way that is slick, intuitive, and completely accessible. Their success speaks to the joy and value that can be found for all in allowing mathematical creativity to bubble forth, no matter the age and the background experience of the mathematical artists!

The Theorem

Informally, the Borsuk-Ulam theorem states:

At any instant there exist two antipodal points on the Earth's surface with identical air temperature and air pressure.

Here we are assuming that air temperature and air pressure vary continuously over the surface of the Earth. A more formal statement of the theorem would be that any continuous map $f : S^2 \to \mathbb{R}^2$ identifies a pair of antipodal points, but the above interpretation with $f =$ (temperature, pressure) is usually offered in texts as an appealing and quirky consequence.

Here is the students' proof:

For a point P on the surface of the Earth let P^* denote its antipodal point (and so $P^{**} = P$), and for a continuous temperature/pressure map $f = (f_1, f_2) : S^2 \to \mathbb{R}^2$ consider the map $g : S^2 \to \mathbb{R}^2$ given by the differences of temperatures and pressures for pairs of antipodal points:

$$g\left(P\right) = \left(f_1\left(P\right) - f_1\left(P^*\right), f_2\left(P\right) - f_2\left(P^*\right)\right)$$

Notice that for any point P we have that $g(P^*) = -g(P)$, so if we plot the set of all values of $g(P)$ in the plane $\mathbb{R}^2$ we obtain a set that is rotationally symmetric 180° about the origin.

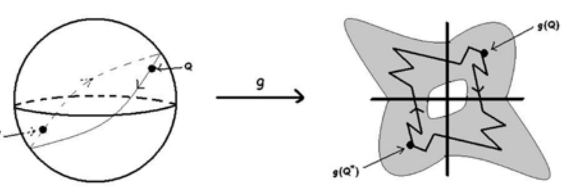

Figure 1

Pick a particular point Q on the sphere and walk along a great circle to its antipode Q^*. This gives a path in the image set. Walking back from Q^* to Q along the second half of the great circle (along all the antipodes of the first half) gives the rotational image of this path in the image set. We now have a loop in the image set starting and ending at $g(Q)$.

Holding the point Q fixed, shrink (that is, contract) the great circle over the surface of the sphere down to the point Q. The image loop starting and ending at $g(Q)$ shrinks to the point $g(Q)$. Some intermediate loop during this contraction must pass through the origin so, despite what the diagram suggests, there must be a point R for which $g(R) = (0,0)$. QED

Down a Dimension

The one-dimensional version of the theorem states that for any continuous map on a circle there exists antipodal points adopting the same value. As my students discovered, this can be generalized. Their approach is novel.

For any continuous map $f : S^1 \to \mathbb{R}$ and any value k there exists a value θ such that $f(\theta) = f(\theta + k)$.

(Here, a real value θ corresponds to the point $(\cos\theta, \sin\theta)$ on the unit circle.)

To see why this must be true, think of the function f as the height function of a fence built along a circle and let θ^* correspond to the location of the maximum height of the fence. Attach a rod to the top of the fence at position θ^* and at position $\theta^* + k$.

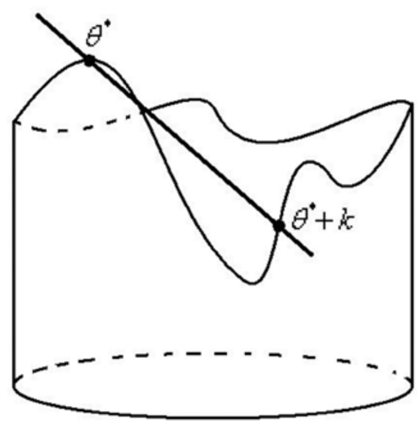

Figure 2

Now slide the rod around the fence, keeping the pivot points of the rod at positions θ and $\theta + k$ until the second end of the rod reaches position θ^*. The slope of the rod has changed sign and so the rod must, at some position, have been horizontal.

(Question**:** Is there a two-dimensional analog of this result?)

Ham-Sandwiches

A famous consequence of the (two-dimensional) Borsuk-Ulam theorem states, somewhat informally:

At any instant there exists a plane that divides the volume of Pluto, of the Eiffel Tower, and of this magazine each exactly in half simultaneously.

This result is known as the Ham-Sandwich theorem for it can also be phrased in terms of slicing two pieces of bread and a slab of ham each exactly in half.

To see why this it is true, imagine that we have three sets S_1, S_2, and S_3, in space each with a well-defined volume. Note that each point P on a fixed sphere with center O determines a vector $\overrightarrow{OP}$ and that there certainly exists a plane with $\overrightarrow{OP}$ as its normal that divides the volume of S_3 in half. Let $f_1(P)$ be the volume of the portion of S_1 that lies on the side of this plane that contains $\overrightarrow{OP}$ and $f_2(P)$ the volume of S_2 on this same side. We have defined a function $f: S^2 \to \mathbb{R}^2$ and, by the Borsuk-Ulam theorem, there exists a point R such that $f(R) = f(R^*)$. But note that the planes corresponding to the points R and R^* are the same but with opposite orientation. This common plane is the one that does the slicing trick.

Although this argument for proving the ham-sandwich theorem is the standard one, it was also developed by the students.

A Context for Technicalities

To experienced mathematicians it is clear that many fundamental assumptions of continuity were brought into play here: the intermediate-value theorem, the extreme-value theorem, an interplay between connectedness, path connectedness and simple connectedness, the use of continuous images of connected sets, of compact sets $f(S^2)$ and the like. And many of these issues were up for discussion during the course. For example, in exploring the Borsuk-Ulam theorem students at first contemplated the image set and were wondering if it can have "holes?" In math-speak, they were really asking: Can the continuous image of a simply-connected compact set fail to be simply connected? If this were a college course I would have been wildly grateful for this opportunity to leap in and begin a speech on beginning homotopy theory: a burning question has just provided a context and a need for a theory.

very easy as an educator to fall into the trap of answering questions that have not been asked. Playing with ideas first and setting technicalities aside can provide fodder for real questions and give context and meaning to technicalities that, in the end, cannot be ignored. Having a context is key to developing true understanding and I sincerely hope as an educator that I more often than not have the means to help students create a context. My young high-schoolers, in a real sense, are now ready for a course on algebraic topology. Given that they are now also wondering what continuity really means and whether it is possible to say that a fence will always have, somewhere, a maximum height they are clearly ready for a course on real analysis too! I hope never to underestimate the value of intellectual play.

(By the way, the students achieved the results outlined in this paper in four one-hour sessions. The first four sessions of the semester were devoted to studying Sperner's lemma. My hunch was that this combinatorial result could be used to prove the Borsuk-Ulam theorem, but my hunch, as it turned out, was irrelevant!)

James Tanton teaches at the St. Mark's School in Southborough, MA, and is the founding director of the St. Mark's Institute of Mathematics; see the web site at http://www.stmarksschool.org/math.

The Courants and I

By Edwin Rosenberg

It was years before I realized I had missed the greatest opportunity of my life: to become a protégé of Richard Courant. In 1940 a hand had been extended to me, perhaps tentatively; but ....

In 1935, my parents bought a house in New Rochelle, a short train trip from New York, where my father worked. My older brother Fred and I commuted by car to the Ethical Culture-Fieldston School in the West Bronx, a 40 minute drive — then. A kilometer from us lived the Richard Courants. He had founded, and from 1920 to 1933 directed, the Göttingen Mathematics Institute in Germany, the acme of European mathematics.

Because of his Jewish background, Professor Courant was expelled from Göttingen by the Nazis. He, his wife Nina, and their elder son went to England in 1934, and briefly considered settling there. The other three children were planted with friends of the family, the younger son living in Copenhagen with the family of Harald Bohr, younger brother of Niels Bohr. Soon, however, Courant was invited to New York University and the family came to the United States. Richard became a professor and again established a mathematical institute, modeled after the one at Göttingen and named for him when he retired in 1958.

The Courants, Papa and Mama [accents on the second syllables], settled in New Rochelle in a sprawling house on Calton Road. Mama monitored a vegetable and flower garden far behind the house, which became a center of refugee and musical activity. Mama also monitored their children: Ernst, Gertrud, Hans, and Leonore (called Lori).

Professor Courant already knew where his children should go to school. He had heard about a "wonderful school, Fieldston ... [with] a really great science teacher, Augustus Klock." This information came from an American who, in 1926–1927, studied under Max Born at Göttingen, obtaining a doctorate when

Richard Courant and one of his sons.

twenty-three: J. Robert Oppenheimer. "In 1934, ... [my father] remembered this ..., and promptly enrolled Ernst in that school," says Hans Courant (email, March 14, 2008). Gertrud and then Hans followed at Fieldston; Lori went to public school in New Rochelle.

However the connection was made between the Courant and Rosenberg parents, two person commuting became four person. My brother and I, studying German at school, enjoyed practicing with, or on, the Courants. Our standard inquiry to Ernst was: "Ein schöner Tag, nicht wahr?" ("A lovely day, not so?") In return, the Courants used us to improve their English.

Ernst also used these trips to practice a wooden musical instrument new to my brother and me: a Blockflöte, or recorder. Holding the instrument vertically to the lips, its player could not avoid introducing into it saliva that brass players delicately dump from valves. Here there was no valve, and much was the ribbing Ernst took as liquid dripped onto whatever protection he put beneath it.

This was my introduction to a musical family far above my background. In their living room was rooted a massive grand piano that Papa played while around him gathered other members of chamber ensembles, some family, some visitors often famous beyond New Rochelle. Nina was a violist whose specialty was the viola da gamba, an obscure Renaissance instrument. (In 1997, at my 50th reunion at MIT, I sat next to Professor Victor Weisskopf, who had come close to a Nobel Prize in physics. He had been Ernst's PhD advisor in physics at the University of Rochester, and recalled being at and in some of the Courants' musical gatherings.)

Performances were held at the Courants' and friends' homes. My mother, invited to hear Mrs. Courant play the viola da gamba, arrived late and rushed in. Just as Mama's presentation began, a policeman entered and asked whose car, with a specified license plate, was blocking the driveway. My mother's, of course: by the time she got her coat, exited, moved the car to a legal but distant location, and returned, Mama's performance had ended. No further opportunities arose.

Once, when I wandered into the house, a battered but serviceable French horn was sitting bell down on the piano. "Whose is that?" I asked Hans. "Oh," he proudly said. "It's mine. Papa just got it, and it's my turn to learn a new instrument." What an attitude!

Occasionally I was invited upstairs to Papa's study. A high point was his showing me how to physically determine the surface of minimum area bounded by an irregular curve. He started simply, making a circle with easily malleable wire and dipping it into a soap solution. When he removed the wire frame from the soap, a bubble remained which, as it dripped, soon reduced itself to just one surface: the thin disc inside the wire. Surface tension, he pointed out, forced that surface to be the one of minimum area for the boundary. Then he twisted the frame into increasingly complicated three dimensional closed curves, and showed me that the principle still prevailed. Though the shapes seemed weird, the bubbles always contracted to the minimal area for their boundaries. I was fascinated. His aim was to mathematically obtain minimal areas for any boundaries. A nice problem that, like many mathematical problems, is easy to comprehend; but not easy to solve! Decades later, I learned that some of his work was based on the Problem of Plateau. (Cf. Constance Reid, *Courant in Göttingen and New York*, Springer-Verlag, 1976, pp. 189–191, 263.)

On the floor, near and partially obstructing the living room entrance, was an eight-foot long, sixteen inches square wooden crate. When I inquired, its top was removed, revealing a refractor telescope, which on the next clear evening was taken outside. With Ernst now at Swarthmore, Gertrude and Hans instructed me. The main tube was mounted on a tripod and a small sighting scope attached atop the tube. That was my introduction to astronomy: planets were discs, stars remained points, and galaxies fuzzily swam into view. If care were taken to align the mount with north, one could pull gently on a cord that rotated the telescope so it stayed focused on objects such as the moon and planets, which otherwise moved rapidly across the field of view. I was hooked: I wanted to become an astronomer.

In winter, when observing was best, though cold, we sometimes built igloos large and stable enough for non-watchers to crawl in together for warmth. How nice, I learned, to snuggle against a female close to my age.

The years passed. Gradually we went in different directions. In 1943 Hans, like I, came to MIT, but by then I was in the Army. My parents moved into New York in 1947; after that I was in but intermittent touch with the Courants.

When his father died in early 1972, Hans considerately invited my wife and me to the memorial service, fittingly held at the Courant Institute. We listened to speakers from many countries, starting with Richard's first doctoral student in Germany and continuing, by order of age, to a grandson, who told of his grandfather taking him walks into the neighboring woods "and always teaching, whether we were walking or sitting on a log." To me, the atmosphere in the auditorium was palpably electric, reflecting the tremendous intellectual powers of those assembled. I was awed.

Ernst, Hans, Mama, and Gertrud, probably in 1927.

Richard Courant in his first car, a Rohr, in 1927.

A few years later we were asked to the family retreat on Loon Lake, in far northern New York State. It was a delightful occasion to reminisce in a good-sized cottage on the shore, perhaps a third of a mile across an inlet to the far shore. There we heard stories about Papa's repeated trips to Germany following World War II, where he had gone to assist mathematics departments' redevelopment, and to recruit promising young mathematicians for the NYU Institute. The search had been narrowed one year by the recent loss of the family ensemble's cello player. Jürgen Moser, who, imprisoned by the Russians after World War II, had maintained sanity and honed mental skills by constructing logarithm tables in his head, not only filled both requirements but, in time, married Gertrud and became head of the Institute.

Nina passed away some six months short of her 100th birthday. At her memorial service, also at the Institute, speakers included her children and grandchildren. Gertrud mentioned a visit to the vacation home when her mother was more than ninety. While making coffee early one morning, Gertrud looked out the kitchen window and noticed a small dark speck in the water, two-thirds of the way across the lake. "What is a seal doing here?" she wondered, and then realized it was a person swimming. She ran down to the dock, got into a rowboat, and caught up to the swimmer near the far shore. Mama, of course.

Alarmed that her mother was alone and unsupervised, the daughter asked her how she was going to get back to the cottage. "Swim, of course," answered Mama. The daughter urged her to get in the boat and return, at which suggestion Mama became incensed, and told her daughter that she swam over and back every morning, intended to complete her swim this morning, and wanted to be left alone.

The eldest grandson, who had been frequently cared for by his grandparents, said that when he was a little kid, Mama had put a stick into the ground near the back of the house and another one out by the garden. Then she raced him from one stick to the other and back, running sideways or backwards, always exhorting him to go faster, faster. This continued for years.

"You know," concluded the grandson, "I became a fairly good athlete and played lots of sports: team sports, individual sports. Naturally, I won some and I lost some. But I'll tell you one thing: never again in my life did I experience the thrill I got the first time I beat my grandmother!"

Another grandson recited his embarrassment as a teenager when one day he asked his grandmother, then well in her eighties, what it was like to be an old lady. "As soon as I said it, I realized it was rude and started to apologize. But it didn't faze her.

"Her face lit up and her eyes shone. 'Oh,' she replied, 'I'm very, very busy. There are lots of things I used to do that I can't do any more, and those I still can do take me so much longer that I'm very, very busy.'"

Such stories brought down the house. Sitting in the back of the sloping auditorium, I watched heads nodding at the tales. "Just like her," all agreed.

The four Courant siblings at a family wedding in 1998. From left to right: Gertrud, Hans, Ernst, and Lori.

The service ended with a moving performance by Lori, a professional violist, of a favorite piece of her mother's, which Lori had played for her mother during her last days. Part of Bach's Violin Sonata #3 in C major, on Lori's viola it was in F major. How many recollections the service, and that music, brought back. Tears of laughter turned to tears of reminiscences. Youth had fled, but not memories.

Clearly, Papa's genes have not receded. His two sons became physicists; Gertrud obtained a PhD in biology; Lori, who married two mathematicians, edited mathematical papers. A daughter of Gertrud's is a mathematician at the University in Dijon, France; one of Hans' has a PhD in mathematics. Now for the next generation.

Oh, yes, that missed opportunity. When I was a high school junior, Professor Courant asked me to comment on a chapter of some math book he was co-authoring. I took a sheaf of papers home, but I was too busy with other things such as: the school newspaper, girls, friends, sports, even my high school courses.

Perhaps you have heard of the book *What Is Mathematics?* Still, despite that major missed opportunity, I did teach mathematics. Yet how often have I wondered: what if...?

Edwin Rosenberg taught mathematics over thirty years, at Western Connecticut State University in Danbury, CT, and for two years (1976–78) at Community School in Tehran, Iran. He has had articles published about mathematics, history, and sports. He expresses his grateful appreciation to the Courant siblings for their suggestions and assistance: in particular to Lori for the loan of photographs and permission to use them.

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November

8/11/08

Write About *Mathematicians* in Non-Major Courses

By Karl-Dieter Crisman

In recent years we've (rightly) emphasized writing skills in math major curricula. Isn't this just as natural in service courses or so-called 'core' courses? Of course!

Attempting this (especially in truly low-level classes) encounters the usual complaints and fears about content, or flowery responses on now-discredited topics like the universality of the golden ratio in art. It's difficult to bridge the gap between our goal of giving students a broader perspective of what math is (and is useful for) and students' persistence in viewing it as a collection of formulas to be memorized and used under controlled conditions. So instead of having the students write about math, I've done a 'bait-and-switch' to encourage learning. There is still a paper to write, but it's about a mathematician, not about mathematics per se. (See the handout on page 21.)

The assignment is to write a paper about a mathematician and then to describe (accurately and with detail) what the student finds compelling and/or interesting about that mathematician. To get students started, I provide a list of common possibilities organized into rubrics like "Female Mathematicians" or "Mathematicians who Died Young." Usually students find at least one of the topics intriguing, and search through several possibilities to find one they will enjoy; I'm always sure to provide explicit (and reliable) library or web starting points.

The point isn't that the assignment is hard — indeed, it isn't. Instead, the idea is to build a bridge between students' conceptions and the reality of mathematics. The assignment is not much affected by 'math anxiety,' since the topic is the person, not the math. Students typically end up doing quality work. Finally, since it focuses on their thoughts, I have fewer worries (though still some) about simple internet plagiarism.

My experience is that essentially all students (often to their surprise) report that they learned something quite interesting about the individual they chose. More to our purposes, since most of the people in question are closely linked to their work, students often learn about some truly new mathematical idea. Most gratifying to me was that many students also seemed to gain real perspective about their own studies and future from learning about the challenges these real people faced in life, which promoted a better attitude toward mathematics in general by the end. I'll give a few of my favorite examples.

Archimedes is almost always a top choice, and notions of notation from "The Sand-Reckoner" and his amazing applied side consistently wow students. (You should limit how many students in one section get to pick him, because he is so popular!) I've had students from non-European backgrounds discover significant mathematics from the history of their own cultures; this had the extra benefit that, since there is often not as much easily accessible documentation about these mathematicians, the students were forced to try to understand the math. As one might expect, students finding topics close to their interests improved the outcome. In one class of a pre-science track, a number of women found real role models in Julia Robinson and other female mathematicians who faced great challenges.

As with all assignments, there are practical suggestions. Students take more ownership of their thinking when the paper is connected to sharing it with peers, whether a group presentation on related figures or just a quick blurb from their seats. Timing is crucial too, so that there is enough time to do it, but not enough to forget about it — I like assigning it over a break if I know they will have little other homework, since it isn't too onerous.

Have them submit early samples if you think they might have difficulty using real arguments to express their preference; if you do use a draft system, then online submission is essential.

A final key asset of this idea is that one can easily tailor the assignment to one's needs. There are plenty of mathematicians out there to restrict the assignment, for example, to those who thought about faith issues, or to a specific time period. One can also more explicitly require some description or understanding of the math as opposed to just the person.

I've used this assignment at colleges ranging from open admission to highly selective, and consistently found it to have very high impact for its length. The only caveat is that students in a particular section must have at least a *minimal* interest in the class. If most students' goal is truly a gentleman's D (I am not exaggerating), this assignment won't help. But the majority of students had plenty of innate desire to do a decent job, which led to learning irrespective of their backgrounds. With a little bit of forethought, this assignment can open eyes and even change attitudes.

Karl-Dieter Crisman (karl.crisman@gordon.edu) is Assistant Professor of Mathematics at Gordon College in Wenham, MA. A version of this appeared in the Proceedings of the 16th Conference of the Association of Christians in the Mathematical Sciences, and this material has been presented at both MAA and ACMS sessions in the past.

Write About a Mathematician

You will do a three page essay on a historical mathematician. Note: this does not necessarily have to be about math! It just has to be about what you find interesting/compelling about this person. Here are ideas for you!

You can go to the library and check out books about interesting mathematicians – this would be in QA 21–31. Many general history of math books will be good. E. T. Bell's *Men of Mathematics*, despite its dated (and unfortunately accurate) title, is a fun first resource; some older books by Dirk Struik or Howard Eves are good, too. There are several good websites, especially the MacTutor site (http://www-groups.dcs.st-and.ac.uk:80/~history/) and MathWorld (http://mathworld.wolfram.com).

Who on earth would you write about? Here is a very short and very incomplete list of some mathematicians and/or mathematically minded people whom you could write about: Euclid, Pythagoras, Archimedes, the Bernoullis, Euler, Fibonacci, Pascal, Fermat, Mersenne, Gauss, Abel, Galois, Dirichlet, Weierstrass, Fourier, Condorcet, Borda, Cauchy, Cantor, Hilbert, Ramanujan, Hardy, Laplace, Galileo, Newton, MacLane, Dickson, Jacobson, Einstein, Noether, Grothendieck, Gromov, Wiles, Tao...

By the way, there are a few mathematicians who have appeared in popular books recently, like Srinivasa Ramanujan, Paul Erdös, and John Nash. If you do a report on Nash, the movie *A Beautiful Mind* has been fictionalized, so don't use it as a resource. The book the movie was based on is fine. Also, *Good Will Hunting*, *Pi*, the TV show *NUMB3RS*, and others are **fiction**, so don't use them!

To help you get some ideas, here is a table with common names encountered under categories people often find interesting. Remember, find something interesting!

Category	**Some Famous Names**
(Old) Greek Mathematicicans	Pappus, Archimedes, Apollonius, Euclid, Zeno, Democritus, Pythagoras
Female Mathematicians	Emmy Noether, Julia Robinson, Karen Uhlenbeck, Karen Smith, Sophie Germain, Fan Chung, Ingrid Daubechies
Non-Western mathematicians/mathematical texts	Ramanujan, Al-Khwarizimi, Omar Khayyam, Liu Hui, Brahmagupta, G. Shimura, H. Hironaka, the Rhind Papyrus, Plimpton 322
Mathematicians who died young or violently or may have had mental illness	Nash (not deceased yet), Ramanujan, Riemann, Erdos, Boltzmann, Cantor, Galois
Mathematicians who thought (well or poorly) about God	Pascal, Descartes, Galileo, Laplace, Kronecker, Kepler, Russell, Hardy, Euler, Riemann, Ramanujan, Erdos, Cauchy
"Mathematicians" primarily known for something else	Pascal, Descartes, Mersenne, Galileo, Newton, Einstein, Pauli, Zeno, E.T. Bell, Stephen Wolfram
Some recent Fields medalists of interest	Gromov, Drinfeld, Thurston, Witten, McMullen, Voevodsky, Tao, Perelman

Archives of American Mathematics Spotlight: The Lawrence Biedenharn Papers

By Carol Mead

The Archives of American Mathematics at the Center for American History has recently processed and made available the papers of prominent physicist Lawrence Christian Biedenharn, recognized worldwide as one of the leaders of modern theoretical physics. The papers consist of Biedenharn's notes, publications, conference talks, teaching materials from Duke University, correspondence, and personal documents. Biedenharn's wife, Sarah, donated the papers to the University of Texas at Austin in 2007.

Biedenharn was born in Vicksburg, Mississippi, in 1922. His undergraduate studies at the Massachusetts Institute of Technology (MIT) were interrupted by World War II, which he spent in the U.S. Army Signal Corps. He received his Bachelor of Science degree in absentia from MIT in 1944 and applied for graduate studies at MIT while stationed in Tokyo in 1946. He returned to MIT in 1946 and completed his PhD in theoretical nuclear physics in 1950.

After graduation, Biedenharn worked for two years as a research assistant at the Oak Ridge National Laboratory in Tennessee. His teaching career began in 1952 when Yale University hired him as an assistant professor. He joined the faculty at Rice University in 1954, becoming an assistant professor there in 1956. Five years later, he moved to Duke University where he became the youngest full professor on the Duke faculty at age 38. He worked at Duke, supervising 24 PhD students, until becoming Emeritus in 1992. At that time, Biedenharn moved to the University of Texas at Austin where he continued to teach as an adjunct professor until his death from cancer in 1996.

Throughout his life, Biedenharn received many honors and awards, including the Fulbright and Guggenheim awards in 1958, and the Alexander von Humboldt Foundation Senior Scientist Award in 1976 and 1987. Biedenharn's 70th birthday was celebrated with a symposium held in his honor in Bregenz, Austria. The journal *Foundations of Physics* published a memorial edition dedicated to Biedenharn in 1997. The University of Texas at Austin established the Lawrence C. Biedenharn Endowed Chair in Physics with a gift by Sarah Biedenharn to honor her husband's contributions to theoretical physics.

Biedenharn's colleagues and his many students knew him as an intelligent, cultured man with varied interests. In addition to his extensive scientific work, which continued unabated throughout the years, and even increased towards the end of his life, Biedenharn pursued many other interests. Among other things, he had a great love for music, particularly chamber music.

The highlight of the Biedenharn papers is his serial notes. Over the course of almost 50 years, Biedenharn kept research notes related to his work on symmetries in nuclear physics, time reversal, relativistic quantum mechanics, and Coulomb Excitation, about all of which he wrote and published extensively. Also included in the serial notes are printed material related to his teaching activities and his participation in various conferences around the world.

In addition to the serial notes, the collection contains copies of hundreds of his research articles in the fields of nuclear physics and later mathematical physics along with related notes, his conference talks covering a span of 40 years, and his teaching materials used at Duke University. The personal series of the papers includes his thesis from MIT, military service documents, personal correspondence, the proceedings of a symposium in honor of Biedenharn, and the memorial issue of the *Foundations of Physics* journal dedicated to Biedenharn.

A digital version of *Lawrence Biedenharn's Biography*, including photographs and a publication list, can be found online on Duke University's website at http://www.phy.duke.edu/about/Lawrence-Biedenharn.

The finding aid for the Lawrence Biedenharn Papers is available online at http://www.lib.utexas.edu/taro/utcah/01016/cah-01016.html.

The Archives of American Mathematics is located at the Research and Collections division of the Center for American History on the University of Texas at Austin campus. Persons interested in conducting research or donating materials or who have general questions about the Archives of American Mathematics should contact Carol Mead, Archivist: carolmead@mail.utexas.edu, (512) 495-4539. The Archives web page is at http://www.cah.utexas.edu/collections/math.php.

20/VI/62

$$H_1 = -\frac{\sqrt{3}}{6} S_1^0 \qquad E_{\pm\alpha} = \frac{\sqrt{5}}{4} S_2^{\pm 2}$$

$$H_2 = -\frac{\sqrt{5}}{4} S_2^0 \qquad E_{\pm\beta} = \frac{i}{6\sqrt{5}}\left(\sqrt{3}\, S_1^{\pm 1} + \sqrt{5}\, S_2^{\pm 2}\right) \qquad E_{\pm\bar\beta} = \frac{i}{6\sqrt{5}}\left(\sqrt{3}\, S_1^{\pm 1} - \sqrt{5}\, S_2^{\pm 1}\right)$$

We may for the subgp $u_1 \times u_2$ construct the invariants:

(a) $H_1, H_2, H_1^2 + E_\alpha E_{-\alpha} + E_{-\alpha} E_\alpha \equiv \Lambda$ from $u_1 \times u_2$

(b) $E_{+\beta} E_{-\beta} + E_{-\bar\beta} E_{\bar\beta} \equiv$ spinor invariant from outside $u_1 \times u_2$

$\{E_\alpha, E_{-\alpha}, H_1\} \cdot$ ('vector' spinor combination)

This expresses the fact that of the operators $E_{\pm\beta}, E_{\pm\bar\beta}$ only 2 independent (of $u_1 \times u_2$) numbers exist.

Now a different splitting of the u_3 gp would be to use:

$\{S_1^0, S_1^{\pm 1}\}$ and then two invariants are $I = (S_1^0)^2 + S_1^{+1} S_1^{-1} + S_1^{-1} S_1^{1}$

$II = \sum S_2^q S_2^{-q}$

Consider the tri-linear invariant.

$$I_{nv} \equiv -E_{-\alpha}\left(E_\beta E_{\bar\beta}\right) + H_1\left(\frac{E_\beta E_{-\beta} - E_{-\bar\beta} E_{\bar\beta}}{\sqrt{2}}\right)$$

$$- E_\alpha \left(E_{-\beta} E_{-\bar\beta}\right)$$

$$[E_\beta, I_{nv}] = \frac{\pm}{\sqrt{6}} E_{-\bar\beta} E_\beta E_{\bar\beta} + \frac{\pm 1}{\sqrt{2}}\left(E_{-\alpha} E_\beta E_\alpha\right)$$

$$+ -\beta_1 E_\beta \left(\frac{E_\beta E_{-\beta} - E_{-\bar\beta} E_{\bar\beta}}{\sqrt{2}}\right)$$

$$+ \frac{H_1 E_\beta\, \beta\cdot H}{\sqrt{2}} - \frac{H_1 E_{-\bar\beta} E_\alpha}{(\pm)\sqrt{2}} - E_\alpha\, \beta\cdot H\, E_{-\bar\beta}$$

Page from Biedenhahn's serial notes "I," 1962.

Reflections on Montclair State University–Beijing Connection

By Mika Munakata and Aihua Li

At this year's national meeting for the NSF GK–12 graduate fellows program, NSF Director Arden Bement cited international collaborations as a focal point of NSF efforts. Scientists and mathematicians must be able to function within the global setting and must seek opportunities to engage in cross-cultural partnerships. In particular, there is an urgent need to expose undergraduate and graduate students to international research opportunities. NSF's commitment to international collaborations is demonstrated by funding for international projects. Recently, our GK–12 *Fellows in the Middle* program at Montclair State University (NSF Award #0638708) received a supplement to establish an international component to our existing project.

Welcome banner to US-Sino Workshop in Xi'an

Supported by the supplement and by considerable contributions from the university, in January 2008 17 GK–12 project participants spent two weeks in Beijing, China. The group included four graduate students (Fellows), three middle school teachers, two university faculty members, four project staff, and the superintendent of a local school district. The main goal of our visit was to form international partnerships for all of our participant groups. Less tangible goals included deepening the participants' understanding of cultural differences and having them practice effective cross-cultural communications and collaborations.

During our stay in Beijing, the Fellows, teachers, and university faculty often moved in different circles. The Fellows each spent about four days at a university in Beijing. Our partner universities were University of Science and Technology Beijing, Beijing Normal University, Beijing Jiaotong University, and China University of Geoscience. Each Fellow was paired with a member of the research faculty at one of these universities and was introduced through him/her to graduate student partners. The science Fellows learned new techniques of biological research and saw state-of-the art laboratories. The mathematics Fellows worked with Chinese graduate students on reading and analyzing papers and presenting their research to small groups. One mathematics Fellow commented that "it was very interesting for me to see the types of problems the math graduate students were working on. For example, they were working on optimization problems involving traffic — which they can really use." The Chinese students were particularly interested in presenting their research and to practice their English communication skills. Aside from the research experiences they gained, the US Fellows established some important contacts. As one Fellow put it, "meeting with fellow researchers at Beijing Normal University was a tremendous experience where I was able to develop international collaborations that I hope will last my entire research career."

The teachers visited several middle and high schools. There, they met teachers and students, observed classes, and gave sample lessons to the students. We were all intrigued by some of the differences we saw. The Chinese teachers we met teach just two classes each day, but teach about 60 students in each class, which means that the total number of students taught by one teacher per day is not too different between the two countries. But having only two teaching periods means that the teachers are able to meet more often with their colleagues. They collaborate on lesson planning, assessment, and reflection on lessons. They visit each

Middle School students at Xisi Middle School in Beijing.

other's classes frequently and offer feedback.

While the Chinese teachers seemed accustomed to working together outside the classroom, they were interested in learning about our project's models for co-teaching interdisciplinary lessons. In fact, in keeping with the structure of the GK–12 program, one pair of US teachers team-taught an interdisciplinary lesson on natural selection that relied on statistics. One of the presenting teachers commented that "traveling to China and being immersed in a culture so different to me was unforgettable. However, as a teacher, visiting the schools and being invited to present a lesson to a class of Chinese middle school students was an experience I do not think I will ever equal."

The US university faculty members met with Chinese researchers and gave several presentations. The topics included descriptions of our grant activities, mathematical problem solving, research projects in Antarctica, and astronomy. We also participated in several colloquia with Chinese educators. Of particular interest to us was the panel discussion with members of the Ministry of Education, the Institute of Education at Beijing Normal University, and several mathematics textbook authors. These discussions lent a new perspective to our observations of the Chinese mathematics classrooms.

In addition to our professional activities, Montclair State University's GK–12 team visited many historical sites such as the Great Wall, Forbidden City, and the Terra Cotta Warriors in Xi'an. We also visited mathematics and science-related sites including the Peking Man geological site and the Beijing Planetarium. With the Beijing Summer Olympics fast approaching, it was fun to be able to see some of the landscape and city life that later arrived through our television sets.

While we were busy with research activities, visits to schools, and sightseeing, the most important element of the experiment was the cultural exchange. We wanted to expose the graduate students to the opportunities and challenges associated with international collaborations. Before our visit, Aihua (who is originally from Beijing) led four afternoon Chinese language and culture workshops. While these workshops helped all participants learn basic Chinese greetings, everyone was humbled by the difficulties associated with being immersed in a different language. It is important for those unaccustomed to cross-cultural and cross-linguistic interactions to be aware of these challenges and to think about how to adjust to them. Similarly, we wanted the participants to think actively about how differences in culture can often impact how people learn, teach, conduct research, and communicate.

US mathematics graduate student Daniela Kitanska presenting her research to a Chinese mathematics graduate student.

The group at the Wild Goose Pagoda in Xi'an.

This May, we hosted three colleagues from Beijing: a physicist, a professor of mathematics education, and a biology professor. During their 12 days in New Jersey, they visited middle schools; presented at various venues including a professional development workshop, an Optimist's luncheon, and at our middle school Math and Science Day; and met with us about possible collaborative projects. They were also able to visit tourist sites around New York City and were invited to many barbecues.

Next January, we will return to Beijing with different participants to further strengthen our partnership. It is the hope that our partnership with the four universities in Beijing will serve as a springboard for sustainable international programs for our science and mathematics graduate programs. In Years three and four of our funding period, we will partner with colleagues in another country in a different part of the world.

Our experiences in Beijing were eye-opening for all of our participants. (For several of the participants, this was their first experience abroad.) The teachers gained new insights into the teaching of mathematics and science. The graduate students became exposed to the idea of international collaborations. The university faculty established connections that will undoubtedly lead to collaborations in the future. Regardless of the individual experience, however, all of us are very grateful to our Chinese hosts. From the graduate students who took our Fellows shopping, to the middle school teachers who shared their experiences with us, to our Chinese colleagues who helped us organize and coordinate the visit – everywhere we went, we were met with a warm welcome. Their hospitality and common interests make us hopeful that this will be a long-lasting partnership.

Mika Munakata is an associate professor at Montclair State University, where she specializes in mathematics education. She received her PhD from Columbia's Teachers College in 2002.

Dr. Aihua Li received her PhD in mathematics in 1994 from the University of Nebraska-Lincoln. Previously she taught at Loyola University New Orleans for nine years. She has been teaching at Montclair State University since 2004.

Mathematicians Playing a Role in Math Education: What We Learned at the IME/MIME Workshop

By Anna Bargagliotti, Rama Chidambaram, and Gizem Karaali

In Hollywood, some actors are regularly cast as mean, others as sweet and endearing, and some typically play innocent big-eyed youths who inevitably succeed after awakening to the particular facts of life that their producer wants them to awaken to. It is unusual and difficult for actors to cross the bridge between different types on a regular basis. However, there are always exceptions to the rule.

In the seemingly unrelated world of academics, mathematics faculty may find themselves playing different roles. People with different skills and interests strive to balance their careers in ways that will be uniquely fulfilling to them. Many choose to play multiple roles within their research fields and within the mathematical community. However, some typecasting naturally happens in our midst as well, and switching roles becomes difficult. Who knew mathematicians and Hollywood celebrities had anything in common?

Typecasting among mathematicians is worth investigating. Looking at mathematics departments across the country we see mathematicians making many different choices. Some choose to dedicate most of their energy to the advancement of mathematics through research. This kind of role can come in many distinct flavors: undergraduate research, historical research, subject area specific research, etc. Others excel in classroom teaching and focus their energy on developing interesting coursework. Many dedicate themselves to innovate and improve the process of teaching mathematics. Recently, there has been an emergence of yet another group: a relatively small self-selected minority who successfully cross the bridge between mathematicians and mathematics educators called "mathematicians who also do mathematics education." This group's scholarly research bridges and enriches both fields.

This is a story told by three junior mathematicians who have not yet gone through the Sorting Hat, but are interested in playing many different roles in the mathematics community. Our tale reflects our experiences at the "Mathematicians in Mathematics Education" (MIME) workshop at the Institute of Mathematics Education (IME), University of Arizona, which we attended March 20–22, 2008.

Let us give some background and context: We were all intrigued by the MIME workshop announcement that promised to orient mathematicians to key issues of mathematics education. Each of us was convinced by the argument that "the demand is increasing for mathematicians who can constructively contribute to work in mathematics education, such as standards development, validation of tests, curriculum design, textbook review, and the preparation and professional development of teachers." We found our separate ways to Tucson, AZ, hoping that, with some guidance, we too could find our own unique ways to contribute to the discussion and to work on current issues in mathematics education.

We were looking to make our first crossover. In graduate school, we were trained to be research mathematicians. We are of course grateful for our training and will continue to do research mathematics, but we would like also to have the chance to be involved with mathematics education research. We respect and admire the handful of mathematicians who have taken this path ahead of us, and want to follow in their footsteps. The philosophy of the workshop was precisely that this could be done: university mathematicians could contribute in meaningful ways to the work of mathematics educators while still maintaining their role as mathematicians.

Three prominent mathematicians and an accomplished mathematics educator organized the workshop. The participants were mathematicians at various stages of their careers. Many had dedicated years to several key issues related to mathematics education. Others, like the three of us, were mostly new to the conversation, but eager to get involved and ready to begin exploring the world of mathematics education.

We started with a working dinner in which everyone had the chance to meet informally. After dinner, we had the opportunity to listen to Roger Howe of Yale University, who has "worked diligently over the years to broaden and professionalize the involvement of a research mathematician in educational reform, to lead us towards the goal where involvement of mathematicians in education is viewed as a well-informed professional activity by mathematicians and educators alike," as the citation for his 2006 AMS Award for Distinguished Public Service says. Many mathematicians know him for his work in representation theory, and many others are familiar with his contributions to mathematics education. For us novices, his talk was inspiring, not only because of his reputation as a research mathematician, but also because his message was very compelling.

Howe gave us several simple examples of how to approach problem solving in the context of mathematics education. He discussed the difference between knowing the definition of number as a mathematician and actually coming up with a definition that an elementary school student can understand and use in a meaningful way. He also suggested specific ways a mathematician could become involved in mathematics education: collaborating in the development of various teacher preparation programs, designing professional development opportunities for teachers, writing about mathematics to provide motivation and insight to pre-service and in-service teachers, participating in various education-related program panels and committees of IES or NSF, reviewing or writing educational materials such as textbooks, being a consultant on education proposals (such as NSF curriculum change proposals), etc.

The next morning, renowned mathematics educator Deborah Ball led us through the present landscape of mathematics education in the US. Following her lead, we embarked on a discussion about mathematical subject knowledge and Mathematical Knowledge for Teaching (MKT). Pretty soon we discovered that MKT is not just about knowing mathematics, but also includes everything teachers do to support student learning: lesson planning, choosing the right examples, asking good questions that lead to classroom discussions, assessing student work, etc. Each of these tasks involves pedagogical skill as well as a considerable amount of mathematical proficiency, skills of mathematical reasoning and communication, and fluency with examples and terms.

To illustrate this point we watched a video of students in an elementary school classroom. Students in the video were discussing which fraction is larger: 4/4 or 4/8. While watching the class discussion, our task as mathematicians was to observe the mathematics in a child's explanation. After an hour of deliberations, we had filled up a large whiteboard with mathematical ideas such as the knowledge of unit measure, relationship between divisor and dividend, equivalence classes, etc. In the end, this turned out to be a surprisingly challenging and undeniably exhilarating experience. We enjoyed our task and came to appreciate the nuances and subtleties involved in identifying mathematical ideas in a child's work.

That afternoon, Hyman Bass encouraged us to think about how to help school students understand the meaning of a standard procedural algorithm used in performing mathematical operations. Our task was to use pictures and manipulatives to represent the standard procedures such as borrowing and carrying used to add, subtract, multiply, and divide in a way elementary school students could understand. This again proved to be challenging for a group of approximately twenty PhDs in mathematics.

In their presentations, Bass and Ball illustrated to us the large amount of mathematical reasoning and skill that is needed to develop the necessary knowledge base for teachers. They emphasized that this is an area of education that mathematicians are qualified to comment on, provided they develop a keen sense of observation and listening to identify the problems involved in teaching. Collaborating with educators, we discovered, mathematicians could use their knowledge about mathematics to develop more effective ways to teach teachers.

At the end of the day we were assigned homework! We were asked to examine and critique the treatment of the concept of a function in various algebra textbooks. We were given sample materials from three textbooks and were asked to review them overnight so that we could continue the discussion.

The action-packed day ended with a fantastic dinner at William McCallum's house. As the beauty of the southwestern sunset came upon us, many informal conversations about mathematics education took place. For example, one of us chatted with Deborah Ball about education as a research field and more specifically about questions like: What should being a "mathematician who also does mathematics education work" really mean? How can this be defined in terms of scholarship? What is the relevant research component? How can this research be incorporated into the more traditional research portfolio of an academic mathematician? Who is the audience interested in this type of research? Discussing these types of questions with Deborah Ball, Hyman Bass, and William McCallum provided interesting perspectives about this vast territory, which remains largely uncharted. We felt lucky to be at this workshop and be led through this geography by these pioneers.

On Saturday morning at 8:30 am McCallum, a 2005 NSF distinguished teaching scholar award winner and director of IME, initiated a discussion on developing a framework for evaluating school textbooks. We learned to give constructive criticism to the authors in order to ensure correct mathematics is being taught throughout all grades. In addition, we also had a chance to view some state high school curriculum standards, which touched on issues mentioned in Deborah Ball's discussion of the landscape of mathematics education in the US. The workshop ended after a discussion regarding our future work as mathematicians in mathematics education.

We left Tucson full of excitement, hope, and promise that in the future we would be able to cross the bridge between mathematicians and math educators doing scholarly research in both fields. It was clear to us that doing research in mathematics education is engaging, challenging, and requires skills substantially different from just knowing and successfully teaching mathematics at the university level. We also learned that with a lot of motivation and keen sense of observation and listening, we could make significant contributions to the field of mathematics education.

With mathematics educators like Deborah Ball and mathematicians like Hyman Bass, Roger Howe, and William McCallum, paving the way to make this transition possible, we believe crossing over should be a lot easier for mathematicians than for Hollywood actors. However, in order for the work of a mathematician in mathematics education to be regarded as valuable research, the end product must meet the scholarly standards in two fields: mathematics and mathematics education. Clearly, this is no small task. How it can be done is an open question that we are eager to attempt to answer.

For information on IME, check out: http://ime.math.arizona.edu. For an introduction to MKT see: Ball, D. L., Hill, H.C, & Bass, H., "Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide?" *American Educator*, Fall 2005, pp.14–22, 43–46; also available online at Deborah Ball's home page at http://www-personal.umich.edu/~dball/.

Anna Bargagliotti is assistant professor of mathematics at the University of Memphis. Rama Chidambaram is assistant professor of mathematics at the University of Michigan-Dearborn. Gizem Karaali is assistant professor of mathematics at Pomona College.

Employment Opportunities

CALIFORNIA

California State University, Fresno
The Department of Mathematics at the California State University, Fresno, invites applications for a tenure-track assistant professor position beginning August 2009. A Ph.D. (or A.B.D) in mathematics is a requirement. Exceptional faculty at a higher rank may be considered. All application materials will be processed through mathjobs.org. For full consideration, the application must be complete by January 15, 2009. For more information about the position or institution, please visit: http://www.csufresno.edu/aps/vacancy/science_math.shtml

CONNECTICUT

Fairfield University
Assistant Professor
Department of Mathematics and Computer Science
The Department of Mathematics and Computer Science at Fairfield University invites applications for one tenure track position in mathematics, at the rank of assistant professor, to begin in September 2009. We seek a highly qualified candidate with a commitment to and demonstrated excellence in teaching, and strong evidence of research potential. A doctorate in mathematics is required. The teaching load is 3 courses/9 credit hours per semester and consists primarily of courses at the undergraduate level. The successful candidate will be expected to teach a wide variety of courses from elementary calculus and statistics to graduate level courses; in particular, Fairfield University's core curriculum includes two semesters of mathematics for all undergraduates.

Fairfield University, the Jesuit University of Southern New England, is a comprehensive university with about 3,200 undergraduates and a strong emphasis on liberal arts education. The department has an active faculty of 14 full-time tenured or tenure track members. We offer a BS and an MS in mathematics, as well as a BS in computer science. The MS program is an evening program and attracts students from various walks of life secondary school teachers, eventual Ph.D. candidates, and people working in industry, among others.

Fairfield offers competitive salaries and compensation benefits. The picturesque campus is located on Long Island Sound in southwestern Connecticut, about 50 miles from New York City. Fairfield is an Affirmative Action/Equal Opportunity Employer. For more information see the department web page at http://www.fairfield.edu/macs_index.html

Applicants should send a letter of application, a curriculum vitae, teaching and research statements, and three letters of recommendation commenting on the applicant's experience and promise as a teacher and scholar, to Matt Coleman, Chair of the Department of Mathematics and Computer Science, Fairfield University, 1073 N. Benson Rd., Fairfield CT 06824-5195. Full consideration will be given to complete interviewing at the Joint Mathematics Meetings in Washington DC, January 5-8, 2009. Please let us know if you will be attending.

WASHINGTON, DC

American University
Two tenure-track positions in the Mathematics/Statistics department at American University at the rank of Assistant Professor or Associate Professor, beginning Fall 2009. Qualified candidates will have a strong background in mathematics or statistics with a PhD, teaching experience is required. American University is an EEO/AA employer. Minority and women candidates are encouraged to apply. See math.american.edu/positions, or contact the Department of Mathematics and Statistics at (202) 885-3120 for details.

ILLINOIS

University of Illinois at Chicago
Department of Mathematics, Statistics, and Computer Science
The Department has active research programs in a broad spectrum of centrally important areas of pure mathematics, computational and applied mathematics, combinatorics, mathematical computer science and scientific computing, probability and statistics, and mathematics education. See http://www.math.uic.edu for more information.

Applications are invited for the following position, effective August 16, 2009, subject to budgetary approval.

Research Assistant Professorship. This is a non-tenure track position, normally renewable annually to a maximum of three years. This position carries a teaching responsibility of one course per semester, and the expectation that the incumbent play a significant role in the research life of the Department. The salary for AY 2008-2009 for this position is $54,500, the salary for AY 2009-2010 may be higher. Applicants must have a Ph.D. or equivalent degree in mathematics, computer science, statistics, mathematics education or related field, and evidence of outstanding research potential. Preference will be given to candidates in areas related to number theory or dynamical systems.

Send vita and at least three (3) letters of recommendation, clearly indicating the position being applied for, to: Appointments Committee; Dept. of Mathematics, Statistics, and Computer Science; University of Illinois at Chicago; 851 S. Morgan (m/c 249); Box R; Chicago, IL 60607. Applications through mathjobs.org are encouraged. No e-mail applications will be accepted. To ensure full consideration, materials must be received by December 31, 2008. However, we will continue considering candidates until all positions have been filled. Minorities, persons with disabilities, and women are particularly encouraged to apply. UIC is an AA/EOE.

University of Illinois at Chicago
Department of Mathematics, Statistics, and Computer Science
The Department has active research programs in a broad spectrum of centrally important areas of pure mathematics, computational and applied mathematics, combinatorics, mathematical computer science and scientific computing, probability and statistics, and mathematics education. See http://www.math.uic.edu for more information.

Applications are invited for the following positions, effective August 16, 2009, subject to budgetary approval.

Tenure track positions. Candidates in all areas of interest to the Department will be considered. The position is at the Assistant Professor level.

Tenured position. Candidates in mathematical logic, with a preference for model theory or descriptive set theory, will be considered. The position is at the Associate Professor or Professor level.

Applicants must have a Ph.D. or equivalent degree in mathematics, computer science, statistics, mathematics education or related field, an outstanding research record, and evidence of strong teaching ability. The salary is negotiable.

Send vita and at least three (3) letters of recommendation, clearly indicating the position being applied for, to: Appointments Committee; Dept. of Mathematics, Statistics, and Computer Science; University of Illinois at Chicago; 851 S. Morgan (m/c 249); Box T; Chicago, IL 60607. Applications through mathjobs.org are encouraged. No e-mail applications will be accepted. To ensure full consideration, materials must be received by November 11, 2008. However, we will continue considering candidates until all positions have been filled. Minorities, persons with disabilities, and women are particularly encouraged to apply. UIC is an AA/EOE.

KANSAS

Pittsburg State University
MATHEMATICS ASSISTANT PROFESSOR - The Department of Mathematics at Pittsburg State University invites applications for a tenure-track position starting August 2009 at the Assistant Professor level. The candidate must have an earned doctorate in mathematics or a related discipline by time of appointment. Preference will be given to candidates with interest in applied mathematics such as operations research, numerical analysis, or stochastic processes. Salary is $45,000 with rank commensurate with experience. The successful candidate will teach undergraduate and graduate courses and direct master's-level research projects. The standard teaching load is 12 credit hours per semester. Faculty are also expected to continue a program of scholarly activity commensurate with the teaching load and to contribute to the department and university service missions.

Pittsburg State University is a comprehensive regional university with an enrollment of approximately 7,000 students. The Department of Mathematics is one of 14 departments in the College of Arts and Sciences. The department is comprised of a diverse faculty of 12 full-time members who are active in many areas of research and service. Mathematics degrees offered include a BA, a BS, a BS with an emphasis in Actuarial Science, a BS in Education, and a MS. Applications must include a vita, transcripts of graduate work, a statement on teaching philosophy, evidence of effective teaching (if applicable), and four letters of recommendation with at least one addressing teaching. Electronic applications will not be accepted. Please send hardcopy materials to:

Search Committee
Department of Mathematics
Pittsburg State University
1701 S Broadway
Pittsburg, KS 66762

For full consideration, candidates should submit complete applications by December 1, 2008. Review of applications will continue until the position is filled. For additional information see the position description at http://www.pittstate.edu/eoaa/jobs.html. Questions may be directed to Dr. Tim Flood at tflood@pittstate.edu or 620-235-4400. Employment will require a criminal background check. Pittsburg State University is an Equal Opportunity/Affirmative Action Employer

KENTUCKY

Berea College
Berea College announces a full time, tenure-track position in the Mathematics and Computer Science Department, beginning September, 2009. A Ph.D. in Computer Science or a Ph.D. in the mathematical sciences is required. Preference will be given to those with a willingness to teach at least some courses in both disciplines. A strong commitment to teaching is essential. Responsibilities center on mathematics and computer science teaching ranging from introductory to advanced undergraduate. Above all we are seeking candidates who can achieve excellence in teaching and who, in an undergraduate environment, will find ways to grow professionally. All faculty in the Department are expected to interact with students on a one-on-one basis and in such activities as summer faculty/student research, independent studies, or senior capstone projects. The Department is supportive of all forms of scholarship

Applicants should send a cover letter, resume, transcripts of graduate and undergraduate work, a statement of personal teaching philosophy, and three letters of recommendation to Professor James Blackburn-Lynch, Chair, Mathematics and Computer Science Department, CPO 2146, Berea College, Berea, KY 40404. Applications will be accepted until search is concluded but full consideration is guaranteed for those received by November 1st. Minority candidates are encouraged to apply.

Berea College, in light of its mission in the tradition of impartial love and social equality, welcomes all people of the earth to learn and work here.

MAINE

University of New England
Assistant Professor of Mathematics
The Department of Mathematical Sciences at the University of New England invites applications for a tenure-track Assistant Professor beginning with the 2009-10 academic year. Applicants should have an earned doctorate in Mathematics or Applied Mathematics. The Mathematics Department is developing new programs focusing on mathematical applications in the Biological, Marine, and Health Sciences and candidates with applied concentrations related to these areas are preferred. Additional qualifications include teaching experience at the collegiate level, interest in teaching a variety of undergraduate mathematics courses, experience using technology in mathematics instruction, and an active research program.

To apply, send CV, cover letter describing how your qualifications match position requirements, a list of mathematics courses taken in graduate school, a teaching philosophy statement and description of current research (3-5 pages for each), and three letters of reference, including at least one that addresses teaching effectiveness to: Ms. Cindy Locke, Human Resources, University of New England, 11 Hills Beach Road, Biddeford, ME 04005. Application deadline: November 30, 2008. UNE is an EEO/AA Employer.

MARYLAND

Salisbury University
Assistant Professor, Mathematics,
The Department of Mathematics and Computer Science (http://www.salisbury.edu/mathcosc/) at Salisbury University is inviting applications for a tenure-track Assistant Professor position in Mathematics beginning August 12, 2009.

For a detailed position description and information about applying please visit our web site at http://www.salisbury.edu/HR/Jobs/default.asp?asearch=faculty

Salisbury University is an EEO employer

and is strongly committed to recruiting and retaining a diverse faculty, staff and student body

MASSACHUSETTS

Williams College
The Williams College Department of Mathematics and Statistics invites applications for one tenure track position in mathematics, beginning fall 2009, at the rank of assistant professor (in an exceptional case, a more advanced appointment may be considered). We are seeking a highly qualified candidate who has demonstrated excellence in teaching and research, and who will have a Ph.D. by the time of appointment.

Williams College is a private, coeducational, residential, highly selective liberal arts college with an undergraduate enrollment of approximately 2,000 students. The teaching load is two courses per 12-week semester and a winter term course every other January. In addition to excellence in teaching, an active and successful research program is expected.

Applicants are asked to supply a vita and have three letters of recommendation on teaching and research sent. Teaching and research statements are also welcome. Applications may be made on-line http://www.mathjobs.org/jobs). Alternately, application materials and letters of recommendations may be sent to Olga R. Beaver, Chair of the Hiring Committee, Department of Mathematics and Statistics, Williams College, Williamstown, MA 01267. Evaluation of applications will begin on or after November 15 and will continue until the position is filled. For more information on the Department of Mathematics and Statistics, please visit http://www.williams.edu/Mathematics.

Williams College is committed to building and supporting a diverse population of faculty, staff and students, to fostering a varied and inclusive curriculum, and to providing a welcoming intellectual environment for all. As an EEO/AA employer, Williams encourages applications from all backgrounds. To learn more about Williams College, please visit http://www.williams.edu.

NEW JERSEY

Rowan University
Assistant Professor, Tenure-Track, Ph.D. in Statistics (preferred) or related discipline, Department of Mathematics, Rowan University, Glassboro, NJ 08028. Visit http://www.rowan.edu/jobs/ for details. Requirements: a letter of application, curriculum vita, transcripts, and three letters of reference (which as a group must attest to statistical qualifications and to teaching abilities). Completed application deadline (for guaranteed consideration): January 15, 2009. Submit to Dr. Hieu Nguyen, Chairperson, nguyen@rowan.edu. Rowan University values diversity and is committed to equal opportunity in employment.

NORTH CAROLINA

Wake Forest University
Applications are invited for two tenure track positions in mathematics at the assistant professor level beginning August 2009. We seek highly qualified candidates who have a commitment to excellence in both teaching and research. A Ph.D. in mathematics or a related area is required. Candidates with research interests in Number Theory, Combinatorics, or Algebra will receive first consideration. The department has 20 members and offers both a B.A. and a B.S. in mathematics, with an optional concentration in statistics, and a B.S. in each of mathematical business and mathematical economics. The department has a graduate program offering an M.A. in mathematics. A complete application will include a letter of application, curriculum vitae, teaching statement, research statement, graduate transcripts and three letters of recommendation. Applicants are encouraged to post materials electronically at http://www.mathjobs.org. Hard copy can be sent to Stephen Robinson, Wake Forest University, Department of Mathematics, P.O. Box 7388, Winston-Salem, NC 27109. (sbr@wfu.edu, http://www.math.wfu.edu) AA/EO employer.

PENNSYLVANIA

Bryn Mawr College
The Department of Mathematics invites applications for a continuing non-tenure track position of Math Program Coordinator to begin July 1, 2009. The position is a three-year appointment. It can be renewed for multiple terms. An MA/MS in Mathematics is required, though a PhD in Mathematics is preferred. We are seeking an enthusiastic individual with excellent teaching, communication, and administrative skills. The successful candidate will work with our department to provide our students with a comfortable transition from high school to college, will encourage them to pursue mathematics beyond the elementary level, and will coordinate departmental organizations and activities. Applicants must share our dedication to opening young women's minds to mathematics while enhancing their abilities to think both deeply and broadly. See http://www.brynmawr.edu/math for a more detailed description of the department and the position.
Applicants with excellent teaching and administrative skills should arrange to have a cover letter, a curriculum vita, a statement of teaching interests and philosophy, a list of mathematics courses taken and taught, official undergraduate and graduate transcripts, and at least three reference letters sent to: Search Committee, Department of Mathematics, Bryn Mawr College, 101 N. Merion Avenue, Bryn Mawr, PA 19010-2899. Applications may also be submitted online through http://mathjobs.org. Applications should be complete by **December 15, 2008**.

Located in suburban Philadelphia, Bryn Mawr College is a highly selective liberal arts college for women who share an intense intellectual commitment, a self-directed and purposeful vision of their lives, and a desire to make meaningful contributions to the world. Bryn Mawr comprises an undergraduate college with 1,200 students, as well as coeducational graduate schools in some humanities, sciences, and social work. The College participates in a consortium together with Haverford and Swarthmore Colleges and the University of Pennsylvania. Bryn Mawr College is an Equal Opportunity, Affirmative Action Employer. Minority candidates and women are especially encouraged to apply.

Penn State University
Penn State York invites applications for a tenure track Assistant Professor of Mathematics position. Teach undergraduate math courses, primarily the first two years of college mathematics (pre-calculus, techniques of Calculus I, Calculus I, II and III, linear algebra, and differential equations; 18 credits per year). Assignments may include day, evening and Saturday classes.

Research and service expected. Ph.D. in pure or applied mathematics is required by time of appointment. Evidence of potential for excellent teaching, research and publication in high-quality journals, and professional growth is expected. To learn more about the campus and Penn State, visit http://www.psu.edu/ur/cmp-coll.html. To learn more about the position and how to apply, visit http://www.psu.jobs/Search/Opportunities.html and follow the Faculty‰ link. AA/EOE.

WASHINGTON

Pacific Lutheran University
Tacoma, WA
Department of Mathematics
We seek to fill two tenure-track assistant professorships, one in mathematics and one in statistics, beginning September 2009. PLU is a comprehensive university offering a curriculum integrating the liberal arts and professional programs. A Ph.D. or doctoral degree in mathematics, statistics or a related field is required, as is evidence of exemplary teaching. For further information visit our website at www.plu.edu/employment. AA/EOE.

WISCONSIN

University of Wisconsin Oshkosh
University of Wisconsin Oshkosh invites applications for two tenure-track Assistant Professor positions in Mathematics Education. Ph.D. in Mathematics with experience in and commitment to mathematics education or Ph.D. in Mathematics Education (or related field with equivalent of Masters degree in Mathematics), at least two years experience teaching undergraduate mathematics (including graduate TA experience), ability to apply technology in classroom, potential to continue scholarly development, and student advisement required. Submit application letter, CV, statement of teaching philosophy and research interests, three current letters of recommendation and transcripts (official or photocopy) to: Chair, Mathematics Department, University of Wisconsin Oshkosh, Oshkosh, WI 54901-8631 by January 16, 2009. Employment requires criminal background check. AA/EO employer. For additional information: www.uwosh.edu/mathematics.