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FOCUS

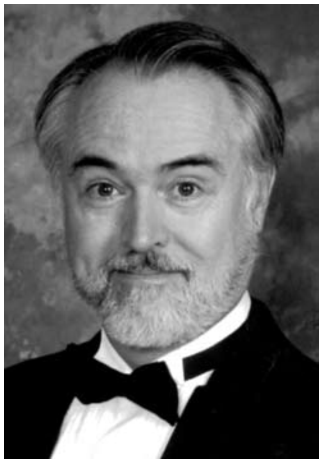
Volume 27 Issue 3

Inside

On the cover: This year's Mathematics Awareness Month Poster, highlighting this year's theme, which is Mathematics and the Brain. See page 11.

MAA National Elections Coming Up in April

David Bressoud



Frank Farris

David Stone

A new MAA President has just taken office; how can it be time for elections again? This is because the MAA allows the future Presidents a full year of President-Elect status, during which they participate in the governance of the Association and prepare for their term as President. So it is time to elect the person who will serve as President-Elect in 2008 and then as President in 2009 and 2010. We also elect the new MAA Vice-Presidents who will serve in 2008 and 2009.

The members of the Nominating Committee were Wade Ellis, Barbara Faires, Ronald Graham, Ann Watkins (chair), and Betsy Yanik. Thanks to their work, we are able to present the following candidates for the MAA national elections:

President-Elect:

David M. Bressoud
Macalester College

Frank A. Farris
Santa Clara University

David R. Stone
Georgia Southern University

First Vice-President:

Nancy Baxter Hastings
Dickinson College

William Hawkins Jr.
University of the District of Columbia

Elizabeth Mayfield
Hood College

Second Vice-President:

Reginald U. Luke
Middlesex County College

Hortensia Soto-Johnson
University of Northern Colorado

Daniel J. Teague
NC School of Mathematics
and Science

Election booklets and instructions for voting will be sent out in early April. Members will be able to vote either electronically or using paper ballots. Last date for receipt of eligible ballots is May 31, 2007. We strongly encourage all members to vote online.

Call for Suggestions for Gung and Hu Award

The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award for service offered by the MAA. This service may have been in mathematics or in mathematical education. It may comprise one or more activities. The period of service may have been short or long.

The Selection Committee maintains a list of individuals worthy of consideration for the award, and annually solicits suggestions for additions to this list. Suggestions should be sent to the Association at the address below to be forwarded to the selection committee. Names suggested by March 30 will receive current consideration; others will be considered in future cycles.

Individuals suggested for consideration should be widely known and respected throughout the MAA and the mathematical profession for the national scope and beneficial impact of their professional work and service. For this reason, suggestions should be short (at most two double spaced pages, in 12 point font) highlighting the most important aspects of the person's career and impact. It is helpful to include one or two URL's for relevant websites; it is not helpful to include multiple letters of recommendation.

Gung and Hu Awards Committee
Mathematical Association of America
1529 Eighteenth Street NW
Washington, DC 20036

An Interview with Larry Schumaker

By Joe Gallian and Michael Pearson

The Carriage House Conference Center at MAA headquarters in Washington DC opened in the fall of 2006 after an extensive renovation funded by a generous gift from Paul and Virginia Halmos. The vision was that the Carriage House could become a center for an ongoing series of mathematically intensive workshops, symposia and seminars. The programs will reflect the range of ideas and topics for which the MAA has been long known, with an emphasis on mathematical exposition.

The MAA received a grant from the National Security Agency to support a Distinguished Lecture Series intended to appeal to a general audience. The first lecture in the series, entitled *Spline Functions and Their Impact*, was given by Larry Schumaker on January 25, 2007. More information about the lecture, as well as the Distinguished Lecture Series, is available at the MAA web site.

Larry Schumaker is a Stevenson Professor at Vanderbilt University. He received a B.S. degree from the South Dakota School of Mines in 1961, and a Ph.D. from Stanford in 1966. He has also held positions at the University of Texas, Texas A & M, and many visiting appointments. At Texas A & M he received the Student Council Teaching Excellence Award. Schumaker's research areas include approximation theory and computer-aided geometric design. In particular, he has made extensive contributions to the development of the theory of splines, a subject that has largely developed since 1960. He is an author of 37 books or proceedings and 160 research papers, and has been the advisor of 11 Ph.D. students. Schumaker has won the Alexander von Humboldt Prize, and has been elected a foreign member of the Norwegian Academy of Science and Letters.

While at the MAA for his lecture, Professor Schumaker talked with Joe Gallian about his career.

JG: Did you begin college as a math major?

LS: I went to the South Dakota School of Mines and Technology to study engineering. I was an amateur radio enthusiast, so for the first two years I was an electrical engineering major. But I eventually decided that the lab work was too much and I found math more and more attractive, so in my junior year I switched to math. I remained interested in EE at Stanford and took a lot of graduate classes in EE. But I don't regret at all going into mathematics. It is an exciting area, and I am not at all unhappy that I made the switch.

JG: What made you choose Stanford?

LS: Well that is an interesting story. I had an NSF graduate fellowship to go to NYU. I was almost ready to go, but at the last moment I had applied to the Hughes Aircraft Company for a Masters Fellowship, and I was awarded that and it included a summer job at Hughes. So I thought I should reevaluate where I wanted to go to graduate school, and Stanford moved up in my list. I had already applied there and been accepted.

JG: When you went to graduate school were you leaning towards any specific branch of mathematics?

LS: I was definitely interested in the applied end of mathematics because of my engineering background, and because I worked at Hughes Aircraft the summer before I went to graduate school.

JG: Do you think the summer jobs at Hughes helped you in your graduate studies?

LS: Not so much as preparation, but getting out there and seeing what people were doing in the real world was certainly valuable. It was awfully nice that they gave me some freedom, so I had time to look into papers and books. In that sense I did use some of that time for preparation for graduate school. It is certainly a valuable thing for a student to do an internship between undergraduate and graduate school. I would advise it. Plus, I got paid very well and got to enjoy the LA area.

JG: Did you do any original research with a team or anything like that at Hughes?

LS: I was in a team with mathematicians who were working on certain problems dealing with radar, and in the course of the summer we wrote a couple of reports. That was the first summer I was there, and then the second summer I came back to Hughes and got back in the same group. But in that summer I was no longer allowed to look at those reports because I did not have the proper secret clearance. It had been documented and stamped with secret clearance, which I did not have, and I was no longer able to see my own work!

JG: Do you make a distinction between approximation theory and numerical analysis or is approximation theory just one branch of numerical analysis?

LS: I like to think of much of numerical analysis as approximation theory in the end, because you are trying to approximate an unknown complicated function somehow, and in the course of doing so you happen to use a computer. However, I don't think numerical analysts would call themselves approximation theorists. I'm on the borderline between the two because I like the theory. I like to prove theorems, but I also like to see the applications, and I like to write programs and do numerical computations.

JG: What about splines in particular? How did you get interested in that specific branch of approximation theory?

LS: It is a branch of mathematics that did not really exist until about 1960, which is about the time I went to Stanford. My advisor was Sam Karlin. He was interested in approximation theory, although that is not really his main area. I was actually taking a course in probability theory with Sam, and he was writing a book in a related area and asked me to read the galleys, which I did. He suggested that I might like to work on a new area which was coming on line called splines, and that is basically how I got into it. Up until about 1960, there weren't more than a handful of papers on the subject, and now there are thousands. I just started my career at the time the subject was beginning.

JG: The changes that have taken place over the years must be unbelievable.

LS: It is fascinating to see how an area of mathematics can develop, and this particular area has so many applications in science and engineering. It has been rewarding to be part of the field.

JG: It is tied pretty closely to the development of the computer.

LS: Absolutely. Numerical analysis in general as we know it couldn't have been done without modern computers, although extensive computations were done hundreds of years ago when people were trying to calculate orbits. Even in later years when people were calculating ballistics paths and such things, there were masses of people with pencil and paper doing numerical computation long before the advent of computers. And even in 1960, which was the start of the spline revolution, computers were still very primitive. There was only one computer on campus. You only got to send a job to it one time a day in a batch process. You would wait until the next day to get an answer, and you used punch cards.

JG: A very high percentage of your 160 some papers are jointly authored. Is there any disadvantage to that?

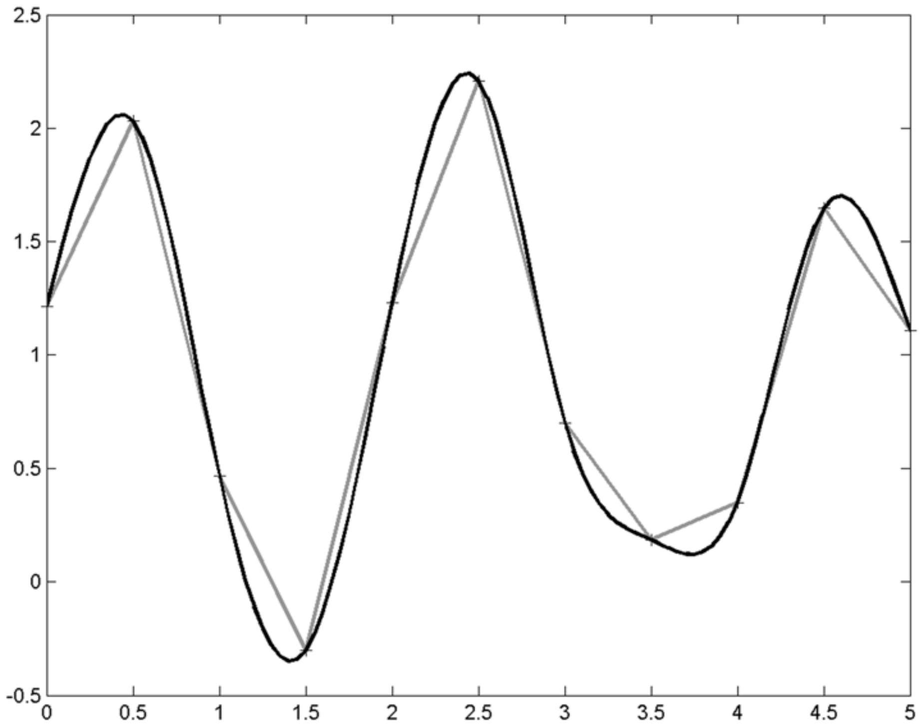
LS: I think that joint work is exciting! I find it extremely rewarding to work with other people, and I find it very efficient because the claim that two minds are better than one is true. People have different backgrounds. The give and take you have with a colleague standing at the blackboard is something very valuable. I enjoy doing it.

JG: I noticed that you have very strong ties with the Norwegian math community. How did that get started?

LS: My first PhD student was Norwegian. This was at the University of Texas at Austin. Tom Lyche was only a few years younger than me. He was interested in splines and I was the only person there doing splines, so we hooked up and he turned out to be an extremely strong student. He went back to Norway and became a full professor, and later a member of the Norwegian Academy. Over the years we have organized numerous conferences together and written many papers. He has become very influential in the Norwegian math community, and has had a huge number of graduate students, so by now I even have Norwegian great grandchildren.

Splines

Introductory numerical analysis classes usually study cubic splines in one variable; that is, functions S(t) defined over a collection of intervals a = $t_0 < t_1 < t_2 \ldots < t_n$ = b such that S restricted to any subinterval $[t_i, t_{i+1}]$ is a third-degree polynomial, and such that S'' is continuous on the whole interval [a, b]. These are used to smoothly interpolate specified values. According to Wikipedia, "the term derives from a design procedure called 'lofting,' a technique used in the British aircraft industry during World War II to construct templates for airplanes by passing thin wooden strips (called 'splines') through points laid out on the floor of a large design loft… The thin wooden strips provided an interpolation of the key points into smooth curves." More generally, splines may be defined locally by higher-degree polynomials (or other classes of functions) in one or more variables in such a way that they have some degree of smoothness across boundaries. The theory of splines developed largely since 1960, and has found a tremendous range of applications in science and engineering.



A spline curve generated by using data points for t = 0, 0.5, 1, ..., 5. The points were connected by straight lines (lighter gray) and then interpolated with cubic splines

JG: How did you end up at Vanderbilt?

LS: That is a little complicated. I was at the University of Texas from 1968 until 1979 and I became very interested in politics. I was actually interested in politics all my life, but I felt that you had to devote 100% of your time to one thing or the other, so I really didn't get involved. But in 1979 I felt pretty strongly that I wanted to try to do something in the world of politics. I saw where there could be an opportunity, namely, to go back to my home state of South Dakota and run for the U.S. Senate against a pretty famous person that I felt was going to lose his seat in 1980. I went back to South Dakota and resigned my position at the University of Texas.

JG: That was a gamble wasn't it?

LS: Yes, definitely, and my wife wasn't very happy about it. This was the Democratic primary. I told the party people that I saw at the Democratic meetings, that the incumbent was going to lose his seat in the 1980 election, and they needed to nominate somebody else. They were of course not very happy about the idea that I was going to run against George McGovern — the guy who built the Democratic party in South Dakota.

JG: So you ran against McGovern in the primary?

LS: That's right. He had never had a primary opponent, and of course he had already been a presidential candidate. I hadn't lived in South Dakota for more than 20 years, but my family still lived there. So I still had connections, but obviously I had no name recognition, and I had to face a fair amount of hostility from the party itself. I spent most of the year organizing around the state putting together something that paralleled the party structure, since they wouldn't do anything for me. We were on a pretty tight budget. I spent about $50,000, and have been told that George McGovern spent about $1,000,000 in that primary election. They said that I would get about 5% of the vote, but I ended up with almost 40%.

JG: Wow! That's quite good, especially since you had so little money.

LS: Very little money and very little exposure. Name recognition was a real problem since I hadn't served as a mayor or a state legislator or anything else in the state. So I think my basic premise was correct. That race could have been won, but I didn't have enough time or money. In the end George McGovern did lose his seat to a Republican congressman.

JG: Looking back, was it a wasted year or do you think it was well spent?

LS: It definitely was not a wasted year. I am not sure it helped me be a better mathematician, other than perhaps making me appreciate mathematics more than I had before, but it was educational and really very interesting to see how American politics works and what is involved in putting a campaign together. In the sense that it required a lot of organization, it helped me to grow. I think it is a very broadening experience, which is not a bad thing for mathematicians who tend to live very narrow lives. It was an opportunity to go out and do something completely different.

After the election I moved to Texas A&M University, but my family didn't want to leave Austin. So beginning in 1980 I began teaching at Texas A&M, which was a two and half hour drive. I continued to live in Austin, and went over on Tuesday and came back on Thursday night. But after about eight years of that, we agreed we couldn't continue that way. So when this job opportunity at Vanderbilt came up in the *Notices*, it looked like it might solve our problem, and it turned out to be a very good thing because I really enjoy Vanderbilt.

JG: There are two applied math journals that I look at. It seems to me that they are in theory applied. It doesn't seem to me that they have real applications. I wonder if this is peculiar to these two journals or if a lot of applied mathematics only has applications way down the line.

LS: I would say that many applied math journals do have lots and lots of mathematics, and lots of theorems, usually with only potential applications. It is left up to engineers and scientists to find that mathematics and go out and apply it. Certainly it would be a misperception to think of all applied mathematicians as working on real world problems in labs alongside engineers and scientists. Certainly there are people doing that, but in the mainline applied math journals you will see mathematics and theorems.

JG: What about the stuff you do? Does it have closer connections with real applications?

LS: No, I would say that I am in the category of people who write applied mathematics papers with the idea that somebody will hopefully come along and use it in the future. I do occasionally get input from people in the real world that have specific problems, and it informs what I am working on. Some people at NASA contacted me recently with some specific problems related to modeling the corona of the sun. I didn't actually work on that problem or write papers with those guys, but the problems they wanted to solve led me to some interesting mathematics that in the end they could use for what they wanted to do.

JG: Do you know of anything you ever worked on that led to patents? People who work in industry often end up with some kind of patents.

LS: Not patents explicitly, but occasionally I receive letters and emails from people asking to use some particular piece of my mathematics in commercial applications. I am not 100% sure if they patented any of it, but I know some of my work on splines went into medical equipment that is used quite heavily in hospitals. It was published in the open literature, so basically, it was impossible to say no. I have colleagues who are working on splines, one in particular who was at A&M, who has patented a number of spline algorithms. It is kind of an interesting concept that you could patent a piece of mathematics. I am sure the guys at Stanford patented many of the algorithms that are related to cryptography. It is not something that mathematicians think about doing very frequently. You would have to be in the more applied end, although the work on cryptography is really pure number theory.

JG: I'm wondering if you think over the next 50 or 75 years applied mathematicians will become involved with using mathematics to study environmental problems.

LS: I definitely think so, because I can see it happening already in biology. Many universities are starting to bring together mathematicians and biologists to work on problems that mathematicians can contribute to. I can't see any reason why that kind of activity shouldn't also happen with respect to the environment, ecology, and all the other areas of science that perhaps haven't really been mathematized. I think it's going to happen. Definitely biology will continue to be mathematized because there are a lot of hard problems, and the job opportunities are out there. In the future there will be plenty of funding for those areas because it is something that the general public understands.

JG: The connection between mathematics and biology now is probably similar to when you got started in splines and computers in the early 60s. You can get in on the ground floor.

LS: That's true. That's one reason I have suggested to some of my graduate students to look at biology, because getting into a field early is a big advantage. In some sense the easiest paths and more obvious problems can be found in the early stages of development of the mathematics. The remaining problems are the hard problems that no one can solve, or they may be a little more on the fringe. If you get in early, you have the opportunity to break new paths that take people off into directions that hadn't been thought of before. That is pretty exciting.

JG: Well, can you think of anything else we should talk about that is interesting?

LS: For young students contemplating entering the world of mathematics, I think it is interesting that I could start out in a very small town (population 1500) in the plain states, graduate from high school in a class of 25, and still go on to have a career in mathematics.

JG: Do you think it is easier today than it was 40 years ago?

LS: I am not quite sure, because now to get into graduate school you have a bigger pool to contend with, given the huge number of very smart students coming from overseas. In that sense it might be harder than it used to be for kids from a small town in the plain states. A career in mathematics is really in some sense a privilege because you are able to work on the things you really want to work on. A lot of times you won't really have any useful output that you can point to, since mathematics only produces something theoretical. It is kind of a privilege do to that kind of work and get paid for it. And there are plenty of perks that go with an academic career: traveling around the world, meeting people at conferences, having the opportunity to manage your time. If I were counseling young students who were considering a career in mathematics, I would say that it may not be the career that brings the largest salary compared to business on something else, but if it is something you really enjoy doing and you want to have a most rewarding life, I think mathematics is wonderful.

JG: I agree with you 100%

Joe Gallian is President of the MAA. Michael Pearson is the MAA Director of Programs and Services.

The MAA Distinguished Lecture Series

The MAA, with the generous support of the National Security Agency, is proud to present a series of public lectures. The series features some of the foremost experts within the field of mathematics, known for their ability to make current mathematical ideas accessible to non-specialists, and provides a learning opportunity for both professionals and students, as well as anyone interested in learning more about current trends in mathematics and the relationship between mathematics and broader scientific, engineering and technological endeavors.

The lectures will take place in the Carriage House, the MAA's conference center. Visit http://www.maa.org/dist-lecture/ for more information, including abstracts.

Scheduled Lectures

Larry Schumaker, Vanderbilt University - January 25, 2007
Spline Functions and their Impact

Doron Zeilberger, Rutgers University - February 20, 2007
The Many Paths of Alternating Paths

Trachette L. Jackson, University of Michigan - March 13, 2007
Building Models of Tumor Heterogeneity: Insights into Prostate Cancer and the Cancer Stem Cell Hypothesis

Bernd Sturmfels, University of California, Berkeley - May 17, 2007
The Joy of Solving Equations

David Bressoud, Macalester College - September 19, 2007
Proofs and Confirmations: The Story of the Alternating Sign Matrix Conjecture

Discussing the "Bridge Course"

By George Exner

What fraction of U.S. colleges and universities require or offer a "transition to upper level mathematics" or "bridge to proofs" course? The reader is encouraged to make an estimate, for comparison with some data later in this article. The question itself was one of many raised — and one of the few settled — at a panel discussion on "The Bridge Course" at the recent Joint Meetings in New Orleans.

The goal of the session was to begin a conversation by getting together both interested mathematicians who teach the course and mathematics education researchers who might study it. Panelists David Bressoud (mathematics, Macalester College, and chair of CUPM, the MAA Committee on the Undergraduate Program in Mathematics), Amy Cohen (mathematics, Rutgers University) and Barbara Edwards (mathematics education research, Oregon State University) gave brief presentations. Their remarks indicate the range of issues raised by the course.

Amy discussed the work of Lara Alcock (University of Essex) on *example-based* and *proof-formality-based* approaches to the course. She noted that faculty at Rutgers seemed to want four things from a bridge course: skills in "instantiation" (creating examples and non-examples), understanding that formal structure implies proof strategy, creative thinking, and critical thinking. The anomaly, however, is that they spend a lot of time on only one of these — how the formal structure implies strategy.

David spoke strongly for the CUPM point that bridging can't be left to a single course but must occur throughout the major. He believes that learning the writing of mathematical proof begins by giving students lots of practice in communicating mathematics, since proof writing is at bottom a form of communication, subject to certain special rules. Barbara talked about ways that mathematicians and education researchers might study the course and its issues cooperatively: we can, right now, ask students, watch students, make a list of goals, and assess some little part of our efforts to achieve the goals. She emphasized as well that there is useful literature studying at least some aspects of the bridging agenda.

The discussion then became general among the audience. The comments were enthusiastic and wide-ranging. Some asked "What are the two sides of the bridge, particularly for different institutions?" Some pointed out other possible roles for the bridge course besides proof, while some felt that a dedicated bridge course — one without real mathematical content — is ineffective because students learn how to do proofs by looking at good proofs *in context*. Still others wondered how to motivate students to desire to prove things at all; as Barbara Edwards put it, some students don't have a need in their hearts for proof.

In spite of the fragmented nature of the conversation — which may well reflect the fragmented and partial understandings of the bridging issues — a few things became clear. One was the general agreement with David Bressoud that, even if a bridge course is useful or necessary, it can't carry all the weight of the transition to higher mathematics on its own. Another was that while the issue was discussed among small groups of acquaintances, there has been no large-scale discussion. Finally, there are various resources out there — web sites, research literature, problem compilations — often known only to a few people.

With about twenty minutes remaining the conversation was forcibly wrenched to "What should come next?" Here there was more agreement. Amy Cohen asked for a source of problems or statements "obvious" or "plausible" but not quite true as stated, and rich enough to allow students to explore such statements in various ways to refine them to become true. It was quickly evident that such a collection would prove popular and useful. Discussion then turned to the desirability of a web site to compile such problems. The proposed web site then proceeded to grow exponentially: might it include useful links to other sites? Lists of relevant research papers? Possible texts with a line or two about the emphases of each? Places for people to look for collaborators in research projects to study the course or bridging issues in general? In self defense, those fearful of responsibility for such a site proposed a Wiki format, and it was agreed that such a resource and format would be valuable. In the interim there is an email group; those interested should send an email to George Exner at: exner@bucknell.edu.

Along with the web resource (now under development) more opportunities for conversation lie ahead. Diane Herrmann (University of Chicago) and Carol Schumacher (Kenyon College) are already engaged in planning for a panel on bridging issues for the Joint Meetings in 2008 in San Diego. There is also a strong possibilty of a special paper session. Those interested in working on a proposal for a session at the 2009 meetings are encouraged to make contact with Chris Leary (SUNY Geneseo).

Work is also beginning on collecting papers from one or both sessions in a volume that might appear in the *MAA Notes* series. There is interest in all sorts of discussion of transition matters, ranging from approaches and techniques that worked well in some offering of the course to formal mathematics education research. Collaborative studies involving both mathematicians and mathematics educators are particularly encouraged as we begin to use the strengths of both to assemble some community wisdom.

Gathering some wisdom would be a good thing because the mathematical community is already heavily invested in these issues. At the panel in New Orleans, Mike Ward (Western Oregon University) summarized a paper — delivered more fully elsewhere at the meetings — reporting on a survey of present practice. About

40% of institutions *offer* a "dedicated" bridge course; about a third *require* it of majors. If one includes courses with both content and a clear component devoted to preparing students for the transition to upper level courses, the fraction of institutions offering a course with a significant bridging function rises to more than two-thirds. That means that there is plenty going on out there to be studied. So watch for session announcements and bring something to the conversation in San Diego. See you there!

George Exner teaches at Bucknell University.

JAVA Applets in Teaching Math

A Summer Short Course from the Ohio Section

Interested participants from all sections are invited to take part in the 2007 Ohio Section Summer Short Course, to be held Wednesday to Friday, June 20–22, at Baldwin-Wallace College in Berea, Ohio (near Cleveland). The facilitator for the course will be Dr. Joe Yanik, professor at Emporia State University in Kansas. Dr. Yanik has given several popular JAVA workshops at MAA national and regional meetings. This short course will consist of daily lecture and laboratory sessions in the mornings and afternoons, starting with an introduction to JAVA and the Math Tool Kit and ending with participants creating their own activities for use in the classroom. Evenings will offer opportunities to visit such Cleveland attractions as The Rock and Roll Hall of Fame, The Lake Erie Science Center, The Cleveland Museum of Art, or an Indians game. Registration cost for the course is $150 plus individual costs for housing and meals. For further information and registration see http://activities.ashland.edu/~ohiomaa/shortcourse.html.

Art of Problem Solving: A New Resource for Outstanding Mathematics Students

By Melanie Matchett Wood

There has been a revolution in mathematics training for the top high school students. Five years ago, the best training for outstanding high school mathematics students consisted of local programs in places with a concentration of talented students and summer programs for a limited number of students. *Art of Problem Solving* has changed that with its web site, at http://www.artofproblemsolving.com. It offers classes, vast resources on mathematics problems, and mathematics discussion forums year round — available regardless of location, parental involvement, or teacher availability.

Art of Problem Solving (AoPS) offers online courses ranging from *Introduction to Geometry* to *Intermediate Counting and Probability*. They are aimed at the top 2–3% of high school students. There are some 15 courses per year, with about 50 to 80 students each. In addition, AoPS offers a year long olympiad training program.

AoPS classes take advantage of the online aspect to offer wonderful features unavailable in regular classrooms. Text from the teacher appears line by line in a window. Students can pause to concentrate on something they want to spend more time on and will have the entire transcript to review after class. Students can participate by sending messages that the teacher and assistant can see. The teacher can decide whether and when to pass student comments to the whole class, and can communicate privately with students at any time. This allows the teacher to ask a question and wait until she has received several answers before the students see any of them. Students don't have to worry that their questions will be a waste of time for the whole class. The teacher and assistant can help students who are having trouble, challenge students who are ahead, and answer students' specific questions all without interrupting the class in progress. These features make for a more tailored, easy-to-follow experience. They also make the AoPS online classes much more interactive than regular lectures.

At any given time of day, there are 50 to 100 people on the AoPS forum, a discussion board where middle and high school students can discuss ideas on how to solve problems and share their excitement about mathematics. This gives outstanding students a chance to work on mathematics with other students, even when there is no one of their ability level in their town or city. Talking to other students about mathematics motivates and challenges students who may not be motivated or challenged by their school mathematics classes.

Perhaps the most frequently accessed resource on the AoPS webpage is their introduction to using LaTeX. Students are encouraged to use LaTeX to communicate their mathematics. (LaTeX code is automatically rendered when used in the classrooms and forums.) As a result, some of the top high school mathematics students are entering college not only familiar with LaTeX, but comfortable using it in a real-time online classroom. Around a quarter of the students who participate in the USA Mathematical Talent Search submit their solutions in LaTeX, probably due to the encouragement from AoPS.

AoPS also offers a series of mathematics textbooks. The website actually derives its name from the classic *Art of Problem Solving* books (*Volume 1: the Basics* and *Volume 2: and Beyond*) originally published in 1994 by Sandor Lehoczky and Richard Rusczyk (Richard is the founder of Art of Problem Solving, Inc., which runs the website discussed in this article). These books have long been a staple of preparation for middle and high school mathematics contests and are now in their seventh edition. AoPS has created introductory textbooks on number theory, geometry, and counting and probability, aimed at the top 10–15% of mathematics students. These textbooks are designed to be appropriate for honors classes in middle or high school, or could be used for enrichment outside of regular classes.

Many factors have contributed to the success of AoPS. The staff members are former mathematics olympians and are well qualified to train students at this level. The online format is familiar and inviting to students, who are comfortable taking classes online, chatting online with people about problems, posting to bulletin boards, and blogging about the problems they are working on. The result has been an increasing amount of time and effort put into learning mathematics by many of the best students in the country, and most importantly, an increasing amount of enjoyment coming out of that effort.

AoPS is beginning development on a new project, *Alcumus*, which will combine a database of problems, video lessons, interactive applets, and references to textbooks with machine learning software to provide an automated interactive educational experience. While this project is at least two years from being implemented, AoPS's record in the few years it has been around suggests that Alcumus might just be the next revolution in mathematics education for top students.

Melanie Matchett Wood is a graduate student at Princeton University. She has taught a guest lecture in the AoPS Worldwide Online Olympiad Training program.

FREE MATHEMATICS AND ART WORKSHOP!

Learn from experienced instructors how to blend Math & Art in your classes at Viewpoints 2007, June 10-15, Franklin & Marshall College. The Viewpoints workshops are sponsored by the National Science Foundation. Registration, art supplies, meals, and lodging are all free. Visit php.indiana.edu/~mathart/viewpoints, or contact Dr. Annalisa Crannell, Dept. of Mathematics, Franklin & Marshall College, Lancaster, PA 17603, (717) 291-4222, annalisa.crannell@fandm.edu .

Teaching Time Savers: Choosing Appropriate Derivative Techniques

By Pam Crawford

As many calculus instructors realize, it is one thing for students to learn derivative techniques or rules in class. It is quite another for students to know when to apply — or not to apply — a specific rule. How often has a student told you, "I could follow the in-class discussion of the problems but I got lost as soon as I had to do the problems on my own?"

I have created an assignment to assist my students with internalizing those characteristics of functions that determine which differentiation technique(s) to use. The assignment is based on standard end-of- chapter lists of review exercises.

Instead of giving my students lists of functions to differentiate using various rules, I reverse the instructions. My students must decide which characteristics a function should possess in order to use a specific differentiation rule. From the review exercises, my students must choose two functions to differentiate in each of the following categories: Product Rule, Quotient Rule, Chain Rule, Exponential Rule for base *e*, Exponential Rule for bases other than *e*, Natural Log Rule, and any of the Trigonometric Rules. Then, students must comment on common attributes possessed by the original functions in each category. No function may be used in more than one category and all work must be shown for maximum credit.

The end-of-chapter list of review exercises blends the differentiation rules, giving students no hints as to which rule is appropriate when. Since knowing the derivative of a function often does not give a clue as to the differentiation technique used, technology is not necessarily an advantage.

Students improve their judgment of which differentiation rules to use (or not to use). Typical student reaction to this assignment is "This assignment helped me see characteristics of functions. I now know and understand which rules to use. I can see distinguishing characteristics in functions."

A similar assignment can be designed for integration techniques.

Time spent: about 20 minutes to create the assignment listing your own derivative techniques.
Time saved: about 2 minutes, on average, for every exam or other differentiation assignment you grade.

Pam Crawford is an Associate Professor and Chair of the Mathematics Department at Jacksonville University in Jacksonville, Florida.

Math Awareness Month 2007: Mathematics and the Brain

April is Mathematics Awareness Month! This year, the theme will be *Mathematics and the Brain*. The announcement by JPBM highlights the importance of mathematics in modern neuroscience. "One of the most exciting challenges in modern science is to fully understand the human brain and its mechanisms. Mathematics plays a vital role in this research to understand the mechanisms and function of the human brain from its smallest components to the whole brain. Mathematical models continue to play a central role in understanding brain cells, their interaction, and their function." Information about this year's theme and the MAM poster are available at the MAM web site at http://www.mathaware.org/.

Mathematics Awareness Month is sponsored each year by the Joint Policy Board for Mathematics "to recognize the importance of mathematics through written materials and an accompanying poster that highlight mathematical developments and applications in a particular area." Responsibility for Mathematics Awareness Month rotates among the member associations of JPBM: the American Mathematical Society, the Mathematical Association of America, the American Statistical Association, and the Society for Industrial and Applied Mathematics, which is in charge this year. JPBM encourages colleges and universities to organize events around each year's MAM theme and provides resources for this at the MAM web site.

Spuyten Duyvil II Undergraduate Mathematics Conference

The second Spuyten Duyvil undergraduate mathematics conference will be held on April 14 at the College of Mt. St. Vincent in Riverdale, NY. The keynote speaker will be Peter Winkler of Dartmouth College; his title is "What Can We Learn from Mathematical Puzzles?" The organizers invite interested undergraduates to submit an abstract for presentation at the web site http://www.manhattan.edu/conferences/sdumc by March 15. Presentations are 15 minutes long and may be either expository or accounts of undergraduate research. The conference is funded in part by the MAA.

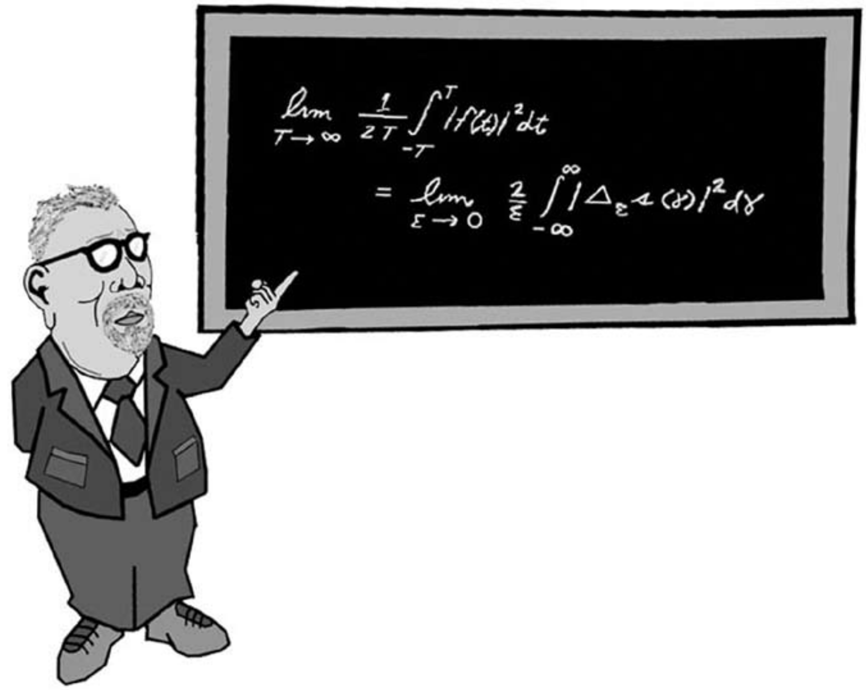
Mathematical Experiences in Business, Industry and Government

By Phil Gustafson

Applications of mathematics to projects in business, industry and government (BIG) offer a wealth of exciting problems for mathematicians. A wonderful sampling of BIG topics was presented at the MAA Contributed Paper Session entitled "Mathematics Experiences in Business, Industry and Government," during the Joint MAA-AMS meetings in New Orleans this past January. This article discusses highlights of the BIG projects presented at the session. The paper session was sponsored by the Business, Industry and Government Special Interest Group of the MAA (BIG SIGMAA), and was organized by Phil Gustafson and Michael Monticino.

Travis Cogdill and Michael Monticino, both of the University of North Texas, gave a presentation on teller staffing in retail banks. Many assumptions concerning the time it takes to serve a customer are made in queuing models used to determine staffing levels. These assumptions include an exponential distribution on service times and that each server has the same performance capabilities. Such assumptions are used to ensure that the problem is tractable and because data on service time has not been publicly available for many industries. This talk presented analysis of a large set of teller service time data obtained from a major national bank to see how well the data supports standard queuing model assumptions and, more importantly, how deviations from these assumptions impact needed staffing levels. This is part of an ongoing project with Argo Data Resources to develop and refine workforce management tools for the retail banking industry. The results suggest that many of the standard assumptions do not hold, and that incorrectly using a model with such an assumption introduces errors significant enough to affect staffing levels even at a small bank branch.

Greg Coxson of Technology Service Corporation spoke on *PSL PDF Generation and Estimation for Binary Codes*, a collaborative project with Matthew Ferrara and Michael Kuperschmid. Low autocorrelation peak sidelobe levels (PSLs) relate to enhanced range resolution for binary-phase-coded radar and communication waveforms. Typical methods to identify the minimum-attainable PSL for a given code length N require exhaustive calculations which grow exponentially with N. In this project, exact PSL histograms were determined for computationally practical lengths. These histograms may lead to ways to estimate PSL distributions for computationally impractical lengths. Plots of the lower four moments for N between 1 and 30 showed that the moments can be approximated closely by aN^k. Histograms for $N = 30$ were compared to a binomially-distributed PSL PDF model based on statistically independent sidelobes. The independent-sidelobe model agreed closely with truth for middle- to high-PSL values, but varies significantly for PSLs one or two units away from the lowest achievable PSL. Future work will examine ways to develop the PDF from the moments accurately enough to estimate minimum PSL for a given N, and ways to account for sidelobe dependence in the probabilistic model.



Norbert Wiener (thanks to the Norbert Wiener Center for Harmonic Analysis and Applications).

David S. Mazel and Greg Coxson of Technology Service Corporation, along with Andy Ilachinski of the CNA Corporation, presented their work on *Fractal Measures to Quantify Agent-based Combat with EINSTein*. The study of agent-based land combat has gained increased attention with the computer program EINSTein (Enhanced ISAAC Neural Simulation Toolkit, developed by Ilachinski). EINSTein is a new approach to modeling warfare that allows users to represent combat as a complex adaptive system. This approach is markedly different from classical Lanchester equations, and allows forces to move, adapt, and employ strategies that in the past were not well-modeled. In EINSTein, users define a red force and a blue force. Each force is composed of individual agents with specific user-assigned personalities, weapons, and goals. Specific squad level personality traits affect the fractal nature of the spatial location of the forces as each force seeks to optimize its goal. The time varying nature of these dimension measurements as correlated to the attrition of agents was illustrated. Plots of real data from Operation Iraqi Freedom were given, and showed that these data also show fractal scaling behavior.

Leigh Noble of the United States Military Academy spoke on characterizing internal stress states in advanced ceramics using fractal analysis. Ceramics often replace metal parts and provide improvements. Currently, the exact relationship between characteristics of advanced ceramic materials and mechanisms of failure is being explored by the research community. Simulations in the literature suggest that fluctuations of the internal stress states are an important step prior to failure. Noble discussed progress in characterizing these stress states by using fractal analysis. Understanding these stress states may lead to better predictions of cracking and mechanical behavior in advanced ceramics.

Joe J. Rushanan of The MITRE Corporation discussed *Number Theory and a New GPS Signal*. Global Positioning System (GPS) modernization involves creating new signals such as the latest L1C signal. A core design component of L1C is a spread spectrum code family of binary sequences with good auto- and cross-correlation properties. The L1C sequences have length 10230, which precludes using well-established families and required some new methods, such as adapting Weil codes. Each Weil code is the shift-and-add of the prime length quadratic residue ("Legendre") sequence and one of its shifts. The sidelobe bound on the Weil codes is comparable to the better sequence families and follows from Weil's Theorem on sums of quadratic residues of polynomials modulo a prime. Selected Weil sequences were padded with a fixed 7-bit pad to yield L1C spreading sequences. Preliminary investigation showed that the correlation properties of candidate spreading sequences are highly dependent on both the specific Weil sequence and the pad insertion point. This initial investigation subsequently guided the search strategy, whose techniques and results were summarized.

John Benedetto and Ioannis Konstantinidis of the Norbert Wiener Center at the University of Maryland described the mission and methods of the Norbert Wiener center. The center has three goals: (1) research activities in harmonic analysis and applications, (2) education in the mathematics of advanced industrial technology (MAIT), and (3) interaction with the international harmonic analysis community. Topics include time-frequency and wavelet analysis, speech and image processing, waveform design and Sigma-Delta quantization for radar and communications, and applied pseudodifferential operators. There is also a strong component in abstract and classical harmonic analysis. The research activities are supported by NSF, NIH, AFOSR, ONR, DARPA, and the Army Corps of Engineers. MAIT is a Professional Master's program along with certificates in mathematical finance, radar, and computational harmonic analysis. Some of our international interaction includes sponsoring conferences such as the annual February Fourier Talks (FFT). For comprehensive information on the Norbert Wiener Center, please visit the website http://www.norbertwiener.umd.edu.

George Heine of the Bureau of Land Management described the use of statistics to detect danger underground. Extracting the gases dissolved in underground coal seams is often described as a new, plentiful and relatively clean energy source. However, it does have potential environmental problems. These include possible underground migration of gases such as methane and hydrogen sulfide to locations where they are hazardous to livestock or humans. For the past 10 years, geologists at the BLM have been collecting soil vapor in a region with a high density of methane wells. Nonparametric statistical methods were used to analyze this data and draw some preliminary conclusions about possible trends in underground migration.

Carla Dee Martin of James Madison University described several projects. One project, for the Department of State, involved building a model to estimate the total amount of land that they owned throughout the world, the results of which were presented before Congress. Another project, for the U.S. Postal Service, required a cost-benefit analysis to justify a new rate structure for first-class and standard mail. An in-depth study for the National Highway Traffic Safety Administration examined how drivers respond to anti-lock brakes. Several other projects included work for the U.S. Treasury Department, fast food chains, and other large corporations. The projects discussed were completed by teams with all levels of training, from Bachelor's degrees to Ph.Ds. As a result, this talk was helpful for undergraduates interested in business, graduate students considering non-academic work, professors advising students on opportunities in mathematics, or other professionals with an interest in business and government.

Paul Coe discussed his experiences as a summer contract employee at a pharmaceutical company. Coe worked as a statistical consultant to a pharmaceutical company for much of his post-baccalaureate career. He started as a summer intern after his second year of graduate school, and continued as a summer contract employee almost every summer since then for the past 20+ years. Coe discussed projects that he worked on, ways in which his academic experiences have been utilized, and ways in which academic statisticians can better prepare our students for consulting in the pharmaceutical industry. Regarding the last point, he had the following suggestions: be flexible and view your potential value broadly; learn to program in SAS; use the services of a "temp" agency such as ManPower; and connect with local professional organizations to network with non-academic mathematicians and statisticians. He also warned that consulting can get in the way of finishing your dissertation.

Paul Schuette of Meredith College described a model of randomized drug testing, which has become increasingly prevalent in the workplace and in sports. Schuette considered a simple binomial model of the efficacy of randomized drug tests as a function of time. This analysis was developed when he served as a consultant for a company employing randomized drug tests under the terms of the Omnibus Transportation Employee Testing Act of 1991.

We learned about exciting applications of mathematics to projects in business, industry and government. In a variety of settings, mathematics is a key component in important projects. Who uses math? The answer includes the mathematicians, scientists and engineers whose projects and products help improve the quality of our everyday lives.

Phil Gustafson is Associate Professor of Mathematics at Mesa State College in Grand Junction, CO, and is Vice Chair for Programs for BIG SIGMAA. He gratefully appreciates the input provided by the speakers for the content appearing in this article, and thanks them for their participation in the paper session.

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Found Math

A game that was controlled by Argentina has now turned 360 degrees!

-- Glenn Davis, ESPN2, Argentina v. Ivory Coast, World Cup 2006

Abstractmath.org: A Web Site for Post-Calculus Math

By Charles Wells

The *Abstractmath* web site at http://www.abstractmath.org/MM/MMIntro.htm is intended for math majors and others who are faced with learning "abstract" or "higher" math, the kind with epsilons and deltas, quotient spaces, proofs by contradiction: all those kinds of abstract things that can knock you sideways even if you got an A in calculus.

I have been developing *Abstractmath* for a couple of years and now it is time to open it up to the wide world. Not that it is finished. There are gaps and stubs all through it. But enough is completed that it is respectable, and besides, I need help! Some students and math educators have already discovered the site and told me things that helped them and things that made no sense to them, as well as finding many embarrassing errors. The site needs much more help like that, and suggestions for more compelling examples and useful topics.

Abstractmath is personal and opinionated, but it is based on research by many people in mathematics education and cognitive psychology, and on my own lexicographical research. It concentrates on certain types of problems. One web site can't do everything.

Mathematical English: This is a foreign language disguised as English. Many common logical words (notoriously "if… then") don't mean quite the same thing they do in English. Common words are used with technical meanings, leaving the student to be confounded by their everyday connotations.

Proofs: A mathematical proof has both a logical structure and a narrative structure. If you are reading a proof your major problem is to extract the logical structure from the narrative you read. Consider: "**Theorem:** If n is an integer and n^2 is even, then n is even. **Proof:** Suppose n is odd…" How can a proof that n is even start out by assuming it is odd? *Abstractmath* walks you through examples of proofs as a guide to how to understand them.

Images and metaphors: Mathematicians use lots of compelling metaphors to talk and think about their topics and images to give geometric sense to them. These images and metaphors are also dangerous because they may suggest things that are incorrect. ("$x^2 - 9$ vanishes at 3." Does that mean it doesn't exist at 3?) When mathematicians start to prove something about their topic they abandon these images and metaphors and go into a rigorous mode of thinking in which all mathematical objects are inert and unchanging. Does anyone ever tell the students this (as opposed to doing it in front of them)? *Abstractmath* does, with examples.

Mathematical objects: People new to abstract math have a great deal of trouble thinking of mathematical objects as *objects* rather than processes or bunches. A quotient space has elements that are sets (these sets are not subspaces — they are elements!). A function space has elements that are functions (not values of functions). *Abstractmath* discusses many examples of this phenomenon.

I hope you will look into abstractmath.org, whether you are a student or a teacher, and let me know how it can be improved. You can also contribute articles or examples, or publish them on your own web site and ask me to link to them.

Charles Wells is Emeritus Professor of Mathematics at Case Western Reserve University.

Election for Section Governors in 2007

Voting for the 2007 Section Governors is now underway. Ballots were sent out in early February for the following sections:

- Eastern Pennsylvania-Delaware
- Florida
- Illinois
- Intermountain
- Iowa
- Louisiana-Mississippi
- Maryland-District of Columbia-Virginia
- Michigan
- North Central
- Southern California-Nevada
- Texas

Members can vote in two ways: using the reply envelope enclosed in their ballot or online. Please go to www.maa.org/voting/sg to vote on line. All voting must be received no later than 12:00 noon EST, Wednesday, March 14, 2007.

We encourage everyone to vote!

An Illuminating Introduction to the Möbius Function

By James Tanton

The Möbius function, which of course appears in the Möbius inversion formula, can be difficult to motivate in a first-experience number theory course. After teaching an extra-curricular class to motivated high-school students I was surprised to find a natural appearance of this function (and motivation for inversion) through a classic puzzle, the starting point of our course:

Along a school corridor stand one-hundred lockers, numbered 1 through 100, each initially closed. One-hundred students, also numbered 1 through 100, take turns walking down the corridor. Student 1 opens every locker. Student 2 touches every second locker (lockers 2, 4, 6,…) closing each. Student 3 touches every third locker, changing its state to closed if it was open, to open if it was closed, and so on, all the way down until the 100th student walks down and changes the state of every 100th locker (namely, just the final one!) After all one-hundred students have walked, which lockers are left open?

Students enjoy enacting this puzzle (use 25 cups or playing cards) and it is usually a surprise to all to find that lockers 1, 4, 9, 16,…, whose numbers are perfect squares, are the ones left open. It is not difficult to explain why this is the case:

Locker N is touched by student d only if $d|N$. As only the square numbers possess an odd number of factors, the square numbered lockers are left open.

The number of lockers specified in the problem is immaterial. For the sake of convenience let's assume the number of lockers, and the number of students, is infinite and in one-to-one correspondence with the set of natural numbers. (For the case of a finite number of lockers simply truncate the results that follow.)

I based my student course on a wonderful paper by Torrens and Wagon ([3]) which explores the possibility of sending down only a subset of students to obtain a pre-described configuration of open and closed lockers. A first challenge of this type would ask: Which students should be sent down the corridor to set locker 1 open and leave all remaining lockers closed? Considering the states of lockers 1, 2, 3,… in turn leads one to the subset of students $S=\{1,2,3,5,6,7,10,11,13,14,15,17\ldots\}$. This looks like the set of square-free numbers and it is not difficult to prove that this is indeed the case, since each number greater than one possesses an even number of square-free factors. Establishing this, and exploring other issues raised in [3], certainly provided great fodder for short course for beginning students. I was surprised to discover that a natural extension of the locker problem — also contemplated by my students — provides illuminating motivation for material that is a standard part of a college number-theory course.

Enter the Möbius function

Lockers come in two states — open or closed — and alternate between these two when touched. Let's now consider objects that cycle through k different states when touched for some fixed number $k \geq 2$. We shall represent the cycle of states as $0 \to 1 \to 2 \to \cdots \to k-1 \to 0$ or, more compactly, by $i \to i+1 (\bmod k)$. These objects could be light bulbs operated by simple pull strings. (For $k = 4$, imagine a bulb that cycles through the states "off," "dim," "bright," and "very bright.") Consider the problem:

Light bulbs numbered 1, 2, 3,…, all initially off (state 0), line a corridor. A subset of students from a set of students numbered 1, 2, 3,… shall be sent down the corridor. Student r, if called, will pull just once the chord of each bulb with number a multiple of r. Which students should be sent down the corridor so as to set bulb 1 into state 1 and leave all remaining bulbs in state 0? (Students may make repeat trips.)

For $k = 4$, thinking about bulbs 1, 2, 3, … in turn leads one to the "multi-subset" of students: $S = \{1,2,2,2,3,3,3,5,5,5,6,7,7,7,10,11,11,11,13,13,13,14,15,17,17,17,\ldots\}$, where the number of times student r is listed in S corresponds to the number of times we must send student r down the corridor. (It is worth computing this set up to student 30, the first student whose number is a product of three distinct primes.)

Let s_n denote the number of times we must send student n down the corridor. Working mod k it appears that:

$$s_n = \begin{cases} 1 & \text{if } n \text{ is the product of an even number of distinct primes} \\ -1 & \text{if } n \text{ is the product of an odd number of distinct primes} \\ 0 & \text{otherwise} \end{cases}$$

Here, sending a student down the corridor "-1 times" means sending that student down $k-1$ times. We have discovered the Möbius function: $s_n = \mu(n)$ (at least in a mod k setting). Of course we need to prove that this multi-set of students really does do the trick.

Claim: *If student n is sent down the corridor $\mu(n)$ times, then bulb 1 will be in state 1 and all other bulbs in state 0.*

Proof: Bulb 1 will be touched only once and so will be in state 1. For $n > 1$, bulb n will be touched s_d times by student d for $d|n$ and by no other students. If n has prime factorization

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_t^{\alpha_k},$$

then any factor d of n for which s_d is non-zero is a product of distinct primes from the set $\{p_1, p_2, \ldots p_t\}$. There are

$\binom{t}{r}$ factors d that are a product of r of these primes (for $1 \le r \le t$), and since

$$\binom{t}{0}-\binom{t}{1}+\binom{t}{2}-\cdots\pm\binom{t}{t}=0,$$

it follows that $\sum_{d|n} s_d = 0$, and so bulb n will be in state 0.

Comment: We defined μ in a mod k setting, but it is clear that it be defined as a function from $\mathbb{N}$ to $\mathbb{Z}$ via the same formula. In this context, the above proof can be repeated, essentially verbatim, to establish the classic result

$$\sum_{d|n}\mu(d)=\begin{cases}0 & \text{if } n\neq 1\\ 1 & \text{if } n=1\end{cases}.$$

It is also clear from its definition that $\mu(n)$ is a multiplicative function — another classic observation.

Enter the Möbius inversion formula

Rather than require just bulb 1 to be in state 1 and all other bulbs be in state 0 suppose, for each $n \in \mathbb{N}$, we desire bulb n to be in state b_n for $0 \le b_n \le k-1$. This gives the sequence of "bulb states:" $B = \{b_1, b_2, b_3, \ldots\}$

Again let s_n denote the number of times (mod k) that student n must be sent down to accomplish this configuration of states. It is clear we must set: $s_1 = b_1$

For bulb $n = 2$, we must send student 2 down the corridor enough times to counteract the effect of student 1, and then b_2 more times to achieve the desired state. Thus: $s_2 = -s_1 + b_2 = -b_1 + b_2$ (Recall that this is to be interpreted mod k.)

In the same way we must have: $s_3 = -s_1 + b_3 = -b_1 + b_3$.

For bulb $n = 4$, we must counteract the effects of students 1 and 2 and then pull the chord of bulb 4 b_4 more times. This gives: $s_4 = -s_1 - s_2 + b_4 = -b_2 + b_4$

Similarly,

$$s_5 = -s_1 + b_5 = -b_1 + b_5$$
$$s_6 = -s_1 - s_2 - s_3 + b_6 = b_1 - b_2 - b_3 + b_6$$
$$\vdots$$
$$s_{30} = -b_1 + b_2 + b_3 + b_5 - b_6 - b_{10} - b_{15} + b_{30}$$

An astute student might (and in my course there were several) observe that this too indicates the appearance of the Möbius function. We conjecture:

$$s_n = \sum_{pq=n} b_p \mu(q)$$

Claim: *Given a sequence $B = \{b_n\}$ of bulb states, sending student n down $s_n = \sum_{pq=n} b_p\mu(q)$ times produces the sequence of states given by B.*

Proof: Bulb n is touched s_d times by student d for $d|n$ and by no other student. Thus bulb n will be in state:

$$\sum_{d|n} s_d = \sum_{d|n}\sum_{pq=d} b_p\mu(q) = \sum_{pqr=n} b_p\mu(q) = \sum_{pqr=n} b_p \sum_{d|\frac{n}{p}}\mu(d) = b_n,$$

which is what we want.

Conversely, if first given a "student list:" $S = \{s_1, s_2, s_3, \ldots\}$

indicating the number of times student n will be sent down the corridor (still mod k) the resulting state of bulb n will be:

$$b_n = \sum_{d|n} s_d .$$

We have, in fact, established the Möbius inversion formula (in the mod k context for any value of k):

$$b_n = \sum_{d|n} s_d \Leftrightarrow s_n = \sum_{d|n} b_d \mu\left(\frac{n}{d}\right)$$

The proof of the Möbius inversion formula in the general setting follows verbatim if we imagine choosing a value k that is larger than any of the values $b_1, b_2 \ldots b_n, s_1, s_2, \ldots s_n$ for a fixed value n.

Final comments

Letting go of our mod k thinking, the Möbius function appears in the problem of finding multiplicative inverses of Dirichlet series. (See [2] and [1] for instance.) For example, students are now poised to prove:

$$\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right)\left(\sum_{m=1}^{\infty}\frac{\mu(m)}{m^2}\right)=1$$

The quantity $\sum_{m=1}^{\infty}\frac{\mu(m)}{m^2}$ can be interpreted as the probability of selecting two natural numbers at random that happen to be relatively prime. (See [1].) It has value $\frac{6}{\pi^2}$.

James Tanton is founding director of the St. Mark's Institute of Mathematics.

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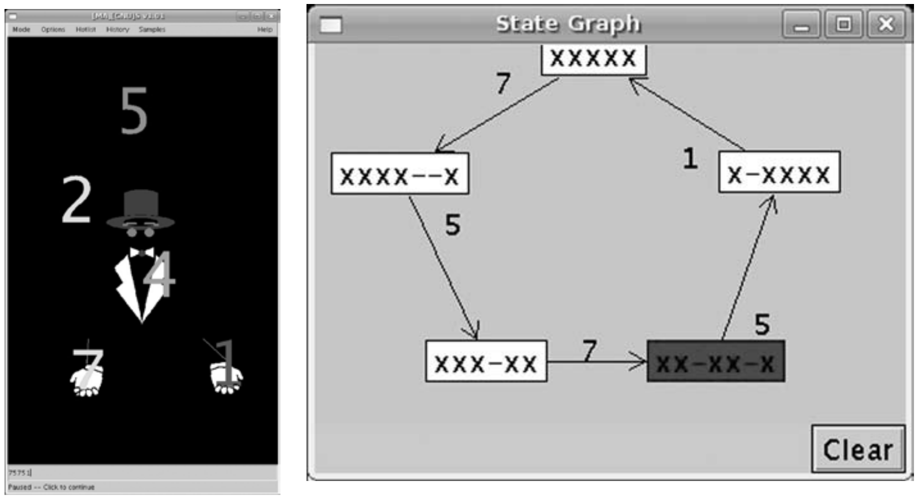
[1] M. Bridger and A. Zelevinsky, "Visibles Revisited," *College Mathematics Journal* **36** (2005) 269-300.
[2] A. Cuoco, "Searching for Möbius," *College Mathematics Journal* **37** (2006) 137-142.
[3] B. Torrence and S. Wagon, "The Locker Problem," *Crux Mathematicorum*, To appear.

Mathematically Entertained

By Tim Chartier

Laughter sounds from a ballroom in a New Orleans hotel. From the hallway, it is difficult to see inside the room as the doorway is filled with people. What is so entertaining? Math! Throughout the session "Entertaining with Math" at the 2007 Joint Mathematics Meetings, a standing room only crowd laughed, sat in attentive silence, and was awed by memorable acts of skill and agility in a session that presented mathematical ideas through the performing arts. From dance to magic and from graph theory to probability, the session offered creative ideas that integrate performing arts with underlying mathematical concepts.



(a) (b)

Figure 1: MAGNUS juggles five balls in which each ball is represented by the number of seconds until it is next thrown. A graph gives a path of juggling patterns (b) where the notation xxx--x *represents the state in which balls land at 1,2,3 and 6 seconds in*

Talks tossed around mathematical ideas — quite literally for both Greg Warrington of Wake Forest and Akihiro Matsuura of Tokyo Denki University. The audience followed a path in a digraph that navigated Warrington through a variety of five ball juggling patterns. Warrington's skillful juggling was accompanied electronically by MAGNUS, a digitized juggler that is controlled through free software available on Warrington's homepage. As seen in Figure 1, this software can assist those who, unlike Warrington, have not progressed beyond the one-ball toss or the two-ball drop.

Matsuura discussed the use of a slinky (as opposed to the more common image of dominoes) as an analogy to proof by induction. He held the child's toy in one hand and propelled it in the air which induced the slinky's distinctive end-over-end style walk across the room as it repeatedly landed in Matsuura's quick-moving hands. Juggling cigar boxes is an old trick that Matsuura connected to permutations as he reordered his numbered boxes with a quick toss in the air. Finally, four crystal balls were held and maneuvered to illustrate symmetry and dynamic aspects of groups.

Math took a magical turn in a number of talks. John Harris of Furman University demonstrated his skills as a mathematical mentalist by deducing such things as the sum of digits appearing on a spectator's calculator. He also mentally found the fifth root of a nine digit perfect fifth power called out by a spectator. At the end of his talk, Harris discussed the mathematics behind these feats. Two of the things he mentioned were the cyclic nature of the number 1/7 and a special property of the ones digits of perfect fifth powers.

Art Benjamin demonstrated and taught the mathematics behind "An Amazing Mathematical Card Trick." In his trick, a spectator shuffled twenty cards, then the cards were mixed together with some cards being face up and some face down. Afterwards the cards were dealt in four rows of five cards and "folded" to produce a reassembled deck (Figure 2). All of the cards were face down except for the 10, J, Q, K, and Ace of Hearts. He then explained the method and the mathematics behind it.

J. Alfredo Jimenez of Penn State Hazleton presented mathematical puzzles and tricks that he uses to engage children and college students. One such puzzle required counting the number of handshakes that takes place when Harry Potter, Ron, Hermione and Hagrid meet in Diagon Alley. A favorite from Jimenez' bag of tricks required rolling dice and summing four products of the dice. Jimenez could predict the answer both when the dice were cubes or in the shape of other regular polyhedra.

"Card Colm" Mulcahy of Spelman College showed how to control the winning (and losing) hands in two-person poker involving 10 cards randomly chosen by a participant. He used reasoning borrowed from the famous Birthday Paradox to guarantee an interesting poker hand at least 98% of the time. Mulcahy also demonstrated two different ways to control which person gets which cards, using a 1964 magic principle of Bill Simon's and a more recent method pointed out by Martin Gardner.

Mathematical ideas also took physical form through movement. Tim Chartier of Davidson College presented his work in introducing and teaching mathematical ideas through mime and mask work. He opened his talk by suddenly embodying a wide-eyed, fearful "math-phobic" (Figure 3); this drew a knowing burst of laughter from the audience. Chartier shared a mime sketch in which he struggles with a never-ending, invisible rope. He uses such a sketch to motivate questions regarding sizes of infinity and limits in presentations in schools and college classrooms.

(a) (b) (c) (d)

Figure 2: Art Benjamin performs an "amazing card trick" where (a) the cards are mixed and later "folded." In (b), the row closest to Art has already been folded onto the row next to it, and Art is folding his rightmost column onto its neighboring column. Folding is repeated (c) until one stack of cards remains. When the cards are distributed, the result could make you flush.

Karl Schaffer of De Anza College and the Dr. Schaffer and Mr. Stern Dance Ensemble showed video clips from the ensemble's extensive repertory. In one clip, Schaffer and Erik Stern perform a vaudevillian hand-shaking routine, and Schaffer discussed classroom activities in which students typically find unusual ways of counting combinations of handshakes; for example: how many ways for two people to shake hands? (See Figure 4.) Schaffer was joined by sarah marie belcastro and Tom Hull to demonstrate a rhythmic, hand-clapping "tessellation" and the use of a loop of rope to form a few platonic solids.

Figure 3: Session organizer Tim Chartier mimes a "math phobic" during his presentation.

Dramatic presentations also covered mathematical ground. Mike Martin of Johnson County Community College showed clips from his award-winning concept videos that encapsulate calculus lessons and set mathematical problems into dramatic situations. In one episode, the President of the United States is informed of a zombie virus that is infecting the country. The news results in a call from the White House to Martin, who hosts the videos, which leads to a mathematical solution that saves the world.

Mark John Meyer an undergraduate from American University discussed the use of a play about infinity and a Jeopardy style game show in mathematics courses.

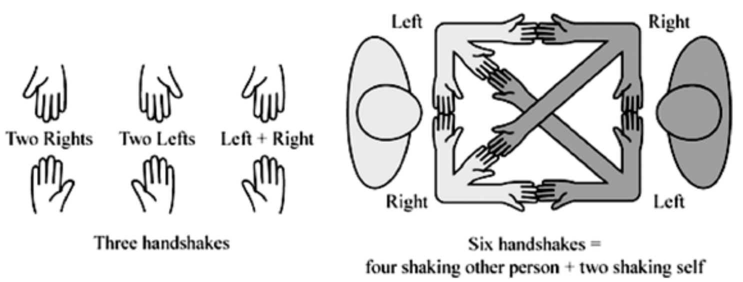


Figure 4: How many ways are there for two people to shake hands? It depends how you count!

Meyer also presented results from a survey study that reflects evidence that such entertainment raised interest and offered new perspectives on mathematical content for the students.

Colin Adams of Williams College and Mikhail Chkhenkeli of Western New England College entertained the audience with the tale of Dirk Mangum P.I. (Principle Investigator, that is) who comes face-to-face with Berkeley Math Department chair Walter P. Parsnipski. The hardboiled P.I. attempts to use multihyperpseudouppersemitudinal fluxions to solve the Canooby Conjecture, perhaps the greatest open problem in all of Pinched Rumanian Monofield Theory.

Have you heard the claim that mathematics is not fun? The session "Entertaining with Math" presented numerous, creative counterexamples to such a claim and many adaptations suitable for classroom use by teachers and professors without a performance background.

Tim Chartier, who organized the "Entertaining with Math" session at the Joint Meetings, teaches at Davidson College. He thanks the presenters in the session for their contributions and for refinements to this article.

Communicating Mathematics: July 16-19, 2007

In 1977, Joseph Gallian defied conventional wisdom and founded an undergraduate research program in mathematics. Thirty years later, dozens of such programs flourish at institutions throughout the nation, providing students with a taste of mathematical research. To some degree, the popularity of these programs is due to the success of Gallian's original.

In July 2007, the National Security Agency and the University of Minnesota Duluth are sponsoring the conference "Communicating Mathematics" on the occasion of Gallian's sixty-fifth birthday to recognize the thirtieth anniversary of his research program and his many other achievements. The conference will be held at the University of Minnesota Duluth, Gallian's home institution and the site of his undergraduate research program.

The purpose of the conference is to inspire research productivity, collaborations, and enthusiasm among mathematicians in all areas, at all stages of their careers. Many of the invited speakers are former student participants in Gallian's program who have gone on to a variety of successful mathematical careers. In addition to a series of colloquium-style invited lectures that will be accessible to a broad mathematical audience, there will be parallel sessions for contributed talks. It is hoped that attendees will gain an understanding of and appreciation of the current problems, techniques, and trends in a wide variety of mathematical fields. Some funding will be available to support participants' travel, with preference given to graduate students and recipients of recent doctoral degrees.

After attending Slippery Rock University and the University of Kansas, Joe Gallian earned a Ph.D. at the University of Notre Dame with a thesis on finite group theory. Gallian then joined the faculty of University of Minnesota Duluth, where he remains today. In addition to organizing his undergraduate research program, Gallian has worked with Project NExT for many years. Through Project NExT, he has inspired a generation of young mathematics professors. Gallian has a long record of service to the Mathematical Association of America, including speaking at numerous MAA meetings around the nation. In January of this year, Gallian became president of the MAA.

For more information about this conference, visit http://events.olin.edu/CommunicatingMathematics/.

MAA Member Maria Klawe Inaugurated as Fifth President of Harvey Mudd College

Maria Klawe was inaugurated as the first woman President of Harvey Mudd College on February 2, 2007. For the past three years, Klawe was the dean of Princeton University's School of Engineering and Applied Science.

Previously, Klawe was at the University of British Columbia where she held posts including dean of science, vice president of student and academic services, and head of the Department of Computer Science. Klawe's rich career also includes eight years with IBM Research in California. She received her PhD and B.S. in mathematics from the University of Alberta

Klawe has made significant research contributions in several areas of mathematics and computer science. She was the founder and director of the Electronic Games for Education in Math and Science, a project which explores the use of computer games in enhancing mathematics education for grades 4 through 9 and studies the effect of gender in technology-based learning environments, and made seminal developments in educational software.

Klawe is a fellow of the Association of Computing Machinery, chair of the Board of Trustees of the Anita Borg Institute for Women and Technology, and a trustee of the Institute for Pure and Applied Mathematics in Los Angeles and the Mathematical Sciences Research Institute in Berkeley. She is a member of the Mathematical Association of America, the American Mathematical Society, Society for Industrial and Applied Mathematics, the Association for Women in Mathematics and the Canadian Mathematical Association. She has received many awards for her research in science and mathematics and for her efforts to encourage women to pursue careers in science and engineering.

Harvey Mudd College is a liberal arts college focusing on engineering, science and mathematics education founded in 1955 as one of the The Claremont Colleges in Claremont, California. In 2006, the Department of Mathematics received the AMS Award for an Exemplary Program in Achievement in a Mathematics Department.

Short Takes

By Fernando Q. Gouvêa

"What Works?" Not much…

The federal *What Works Clearinghouse* is intended to help educators and administrators assess the evidence for the effectiveness of various educational ideas, methods, and curricula. WWC has already reviewed a wide range of middle school mathematics curricula. Four new reviews, focusing on mathematics programs for elementary school, were posted between September and December, 2006. The four programs reviewed were *Everyday Mathematics, Houghton Mifflin Math, Saxon Elementary School Math,* and *Scott Foresman Addison Wesley Mathematics*. The reports survey available studies on each program, evaluate the studies to see if they meet evidentiary standards, and provide a summary assessment. Of the four programs, only *Everyday Mathematics* received a positive judgment: "The WWC found *Everyday Mathematics* to have potentially positive effects on mathematics achievement." The other three programs were found to have "no discernible effects on mathematics achievement." The full reports and a list of all the studies considered can be found at http://www.whatworks.ed.gov/.

A Play about Alexandre Grothendieck

The Grothendieck Circle web site is dedicated "to make publicly available (and in some cases translate) the material written by and about Alexandre Grothendieck as well as to provide biographical material on Grothendieck's life and his origins." One of the latest additions to the site is a play by Adrian Heathcote, *Grothendieck's Dream of the Rising Sea*. The play is inspired by Grothendieck's *Récoltes et Semailles* ("Reapings and Sowings"), a rambling and opinionated memoir. The play can be found in the "biographical texts" section of http://www.grothendieckcircle.org.

Another Successful Math Major

In its December 17, 2006 issue, the *New York Times* profiled Reed Hastings, the founder and CEO of *Netflix*. One of the details mentioned in the profile is that Hastings graduated from Bowdoin College, a liberal arts college in Maine, with a major in mathematics. Hastings says that "I majored in math because I found the abstractions beautiful and engaging." After a stint with the Peace Corps in Africa, Hastings went to graduate school in computer science, started and sold off a software company, and then went on to create *Netflix*.

NSB to Call for Better Science and Mathematics Education

According to a report in the February 2 issue of the *Chronicle of Higher Education*, the National Science Board's *Commission on 21st Century Education in Science, Technology, Engineering, and Mathematics* is preparing a report for Congress that calls for dramatic improvements in the teaching of science and mathematics. The report includes a "call for a national coordinating council, which would work with federal agencies and the states to improve the consistency of science teaching across the country as well as the preparation of schoolteachers by universities." The report is also said to call for the development of national standards, both for the certification of mathematics and science teachers and for school curricula. Finally, the report will address various federal programs designed to create incentives for young men and women to choose careers in science and mathematics education.

The Comission, chaired by Shirley M. Malcom of the AAAS and Leon M. Lederman, emeritus director of Fermilab and a Nobel Prize winning physicist, was created in 2006 by the National Science Board to study science and mathematics education. NSB was encouraged to create the report by members of Congress of both parties who sit on the appropriations subcommittee that oversees the National Science Foundation budget. The final report, which requires NSB approval, is to be presented in May of 2007.

Headlines and Deadlines for Students

The American Mathematical Society has long had a "Headlines and Deadlines" email list intended to alert AMS members to interesting developments and opportunities. More recently, the AMS has created a new service, "Headlines and Deadlines for Students," aimed at undergraduate mathematics students. The emails are issued about once a month and include both news and information about deadlines for such things as the Mathematical Contest in Modeling. Much of this information is also available online at http://www.ams.org/news-for-students/. Advisors of MAA Student Chapters may want to consider to signing up for the new service and to encourage students to do the same; signing up is done online at http://www.ams.org/news-for-students/signup.

***Involve*: A New Journal for Research Involving Undergraduates**

One of the more unusual exhibits at the Joint Mathematics Meetings was a little booth announcing a new journal, to be called *Involve*. The new journal wants to occupy a space somewhere in between two extremes: journals dedicated entirely to undergraduate research on the one hand, and mainstream research journals on the other. Articles for *Involve* should be publishable in research journals but must "include a minimum of 1/3 student authorship." For more information, visit the journal's web site at http://www.involvemath.org.

New Orleans Meetings Raise $10,000 for Second Harvest

The raffle and t-shirt sale at the Joint Mathematics Meetings raised $10,000 for Second Harvest Food Bank, a New Orleans charity. These funds were used for the Second Harvest *BackPack Program*. The BackPack program provides food to children who would otherwise go hungry, by issuing them a backpack filled with food to take home for the weekend and out-of-school times. BackPacks are stocked with nutritious, child-friendly food. AMS and MAA are pleased that they could help Second Harvest by making this donation, and would like to extend thanks to everyone who bought a t-shirt or a raffle ticket or who gave a separate donation.

Letters to the Editor

It has been a while since we last had space to run a large number of letters. The letters below comment on material in several of our recent issues. Letters are always welcome; ideally, use email and send letters to fqgouvea@colby.edu. *We cannot guarantee to print all the letters we get, and please note that letters may be edited for space and clarity.*

On "Alice in NUMB3Rland"

In Alice Silverberg's article "Alice in NUMB3Rland" [November 2006], she writes that she was told by Cheryl Heuton, the co-creator of the CBS series *NUMB3RS*, that "getting the math right and getting it to fit with the plot are not priorities." (The words are Silverberg's.) Unfortunately, this assertion of Heuton's contradicts numerous previous statements made behind the scenes by Heuton and others, including co-creator Nick Falacci, to the effect that their primary goal was to produce an accurate representation of mathematics and mathematicians, to educate as well as entertain (as opposed to the teams behind series such as *ER* and *Law & Order* which have made no claims regarding an intent to educate). It also is contrary to the amount of time and money the *NUMB3RS* team spends on mathematical consultants and advisors, despite the rising number of mathematical errors, misstatements, and mispronunciations that appear on the weekly series.

Keith Devlin writes in his February 2005 *Devlin's Angle* column, "Their starting point, they told me, was to develop a prime-time television series that featured mathematicians and scientists… Their solution… was to fit their idea into a tried and tested formula: the police procedural detective series." In the January 30, 2006 issue of Ivars Peterson's *MathTrek*, Falacci says, "For the mathematicians out there, we want to be as engaging and accurate as possible." Peterson writes in the same article, "The series aims for mathematical correctness and scientific accuracy, says Andrew Black, who is a researcher and writer for the show." Lead actor David Krumholtz, in a February 2006 interview for *Zap2it.com*, said, "Not only do we put the math in context, but we're able to visualize it for the audience [through special effects]. We do these visual metaphors that are really quite powerful, and they're really well done. They make the math make just that much more sense." The production staff employs numerous mathematical consultants in addition to Lorden, including researchers at Wolfram Research. Heuton and Falacci cited "the educational value in our show" in the press release announcing the launch of the We All Use Math Every Day activity program paid for by Texas Instruments and the National Council of Teachers of Mathematics.

More information is on my webpage "Analysis of NUMB3RS and the We All Use Math Every Day (WAUMED) worksheet program" at http://homepage.smc.edu/nestler_andrew/numb3rs.htm.

Andrew Nestler
Santa Monica College

Fooling the Calculator I

Regarding the "Curve-sketching" item on pages 20, 21 of the January 2007 FOCUS, the following may be of interest. I assign the problem below in second semester calculus. A graphing calculator is unlikely to be helpful to the student confronting this problem. A version of this problem can also be found online at http://www.amherst.edu/~nstarr/san06/root.pdf.

An exercise on the use of rate of growth

One of our alums in a Ph.D. program in material science, sent me the following question via e-mail: "How does one solve

$$\frac{3620}{x} + \ln x = 16.82\ ?$$

We know the answer is 328, but we're not sure what the way to solve this is. Thanks."

My response included the following remarks. "My first reaction is that the equation is 'transcendental' and has no reasonable algebraic solution. (Moreover, your 328 is only an approximate solution, though it might suffice for your purposes.) Consider the function defined by

$$y = \frac{3620}{x} + \ln x\,.$$

There must be at least two solutions"

She wrote back, "It was quite helpful" and added that her lab partner had said, "*ask him for the other answer*!", remarking that her partner's "calculator (one of those fancy ones!) keeps giving 411."

There are a number of effective ways to pin down the solution which is close to 328, but your tasks are, instead, the following.

A. Explain why there must be exactly one other solution.
B. Try to give at least a crude location for the other solution.
C. Find an x for which

$$\frac{3620}{x} + \ln x$$

differs from 16.82 by no more than .001 and explain why your result satisfies the specified tolerance. Note that the task can be dramatically simplified if you make effective use of the rate of growth of the logarithm function.

Norton Starr
Amherst College

Fooling the Calculator II

In the January issue of FOCUS, you write, "Can we come up with examples in which the calculator's graph is misleading in one way or another? Can we show students examples in which the calculus-enlightened mind has the advantage over brute force?"

Dave Rusin has collected many, many such examples, and put them up at http://www.math-atlas.org/99/calc_errors.

Gerry Myerson
Macquarie University

More on Learning from Technology

In the January 2007 FOCUS, Pisheng Ding's letter asks whether technology use helps or hinders students' understanding of mathematics. Some readers may be unaware how the use (or even pitfalls) of technology can actually be intentionally utilized as a pedagogical vehicle for students to exercise more rigorous thinking and explore important underlying mathematical concepts. Examples appear in my articles in the January 2007 and December 1999 issues of *Mathematics Teacher*.

Lawrence M. Lesser
The University of Texas at El Paso

Euler and Extracting Roots by Hand

In the latest FOCUS, Fernando Gouvêa expresses doubt that Euler would have computed 20 digits of the square root of a multidigit numeral, working by hand. This is actually not a terribly difficult thing to do, using the "traditional" algorithm (which is a variant on the long-division process). A few years ago, on a flight to New Orleans for the JMM, I realized that one of the slides in my talk made reference to Briggs' extracting over thirty digits of the square root of 10, and while I was reasonably certain he had used the long-division-esque algorithm, I had no sense of how difficult that would be to achieve. So I tried it.

I got the first 40 digits in 90 minutes, and checked the answer by squaring its first 15 places (via lattice multiplication) in under 15 minutes. Computing the square root of 6(1.66493…), rather than of 10, does not make the process at all more difficult.

Flight attendants and others sitting around you on a plane pretty much ignore you when you're filling a sheet of paper with silly arithmetic scrawl. Not sure what the reactions would be in these more perilous days in the air.

Mark McKinzie
St. John Fisher Colleg

I'm afraid my comment was an attempt at levity that fell flat. Yes, of course I know that it is fairly easy to do this computation by hand. I'm even old enough to have been taught it in school. I was trying to capture what I imagined might be the typical reaction of a modern reader. Apologies for underestimating you all!

More on Mathematics and Gender

Regarding the December 2006 article "Perception and research: mathematics, gender, and the SAT" by Cathy Kessel, it is surprising that the ratios of scores over 700 on the SAT test were not weighted by the number of students of each sex. For example, in the bottom row of the table the M/F Ratio for scores over 700 changes from 5.6 to 6.0 when weighted by the number of males and females writing. It's a good thing these studies weren't about SAT ratios for blacks and whites or for residents of the USA and Antarctica.

Erik Talvila
University College of the Fraser Valley

A Conjecture on the Gold Icosahedron

After reading Harry Waldman's article regarding the Islamic gold icosahedral box in the December 2006 FOCUS, I would like to offer this conjecture regarding the markings on each side of the box

Because the Arabic language reads right-to-left, I believe that the numerals may have been intended to be ordered in reverse of how they were printed in the article; so, the revised ordering of numerals would yield the strings 02, 11, 12, 13, 14, 15, 16, 17, 18, 19, 101, 102, 103, 104, 105, 106, 107, 108, 109, 202. In this case, the numerals may actually be a combination of a place descriptor (1 or 10) followed by a unit digit, linguistically similar to how (for example) nineteen in Spanish, "diecinueve," is a contraction of "diez y nueve." This yields the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, leaving unknown only the translations of "02" and "202" (presumably to the numbers 10 and 20, or possibly vice-versa). The result would be that the numbers simply label each side, much like the 20-sided dice used for role-playing games.

John Carlsen
Syncopated Systems

Found Math

So much climbing, on a spherical world;
had Newton not been a mere beginner at gravity
he might have asked how the apple got up there
in the first place. And so might have discerned
an ampler physics.

— *Les Murray, "Quintets for Robert Morley"*

FOCUS Deadlines

	August/September	October	November
Editorial Copy	July 9		September 14
Display Ads	July 10	August 1	September 21
Employment Ads	June 11	August 13	September 7

What I Learned From... Giving Oral Examinations

By Russell E. Goodman

I think most mathematics teachers have joked, at some point in their careers, about giving students an oral exam instead of a standard paper-and-pencil exam. I am convinced there are situations where the pencil-and-paper exam should not be the only method used to assess students' mathematical abilities. As a result, I decided to test out oral examinations in "Mathematical Concepts," our mathematics course for elementary education majors.

I have taught this course three times (roughly 20 students per class) and enjoyed doing so each time. Despite the lack of mathematical prowess of many of the students, they have good attitudes, work hard and respond well to an energetic professor, which I try to be. In addition to building these students' understanding and skills in mathematics, I make it a priority to help these future teachers understand what it means to communicate mathematics accurately and effectively. Ta-da! The perfect place to use oral exams! I include an oral exam component to each of the three semester exams (not the final exam).

Logistics

I organize the course so that each exam is held on a Thursday evening. The following day, Friday, I cancel our regular class period and have an appointment (set up earlier in the week) with each student to do their oral exam. Thus, the written and oral exams are in close proximity to one another.

The review sheet I produce for each exam has an additional section giving a pool of five oral exam topics/questions. Therefore, the students know what the possible questions are and can study and prepare appropriately. When a student shows up for their oral exam appointment, they roll a six-sided die and the number appearing determines the topic/question to which they must respond. (If they roll a six, they are considered "lucky" and can choose their topic.)

A complete list of topics can be found at http://tinyurl.com/23yan9. Here are some examples:

- Describe the system for naming numbers in our Hindu-Arabic numeration system.
- Explain why $7 \div 0$ and $0 \div 0$ are both undefined (for somewhat different reasons) by discussing the missing-factor method to division.
- Please describe the tests for divisibility by 4, 8, 9 and 11. Explain **why/how** the test for divisibility by 4 works. I will then give you a number and ask you to test it for divisibility by 11.

The student then has seven minutes to address the question/topic. Students typically address their topic using the "mini-lecture" format, but some have been creative in the use of manipulatives, handouts, etc. Regardless of the method of delivery, I am strict with the seven minute time limit. Only once do I recall needing to cut a student off, so it seems my pool of topics/questions is appropriate.

I instruct the students to speak as if I were an elementary student (i.e. not the instructor or someone who knows all about elementary mathematics) and to proceed as if they are my teacher. They are expected to dress and act appropriately and to consider this a professional presentation. For example, chewing gum or not making good eye contact with me during an oral exam is considered poor form and results in a lower grade.

Once the student decides they are finished responding to the topic/question (I do not inform them when they are done, rather it is up to them to know) I thank them for their presentation, excuse them from the seminar room where I hold the exams, and then quickly jot down initial thoughts regarding the student's grade.

A student's oral exam is graded in three broad categories: mathematical content and communication (9 points), mathematical vocabulary (3 points) and professionalism (3 points). Thus, the oral exams amount to 15 of the 100 points for each exam during the semester.

I take copious notes during each student's presentation. This allows me to produce fairly thorough (typed) feedback for each student and to be very specific with my praise and/or criticism. I attempt to keep the feedback on-topic and to find a balance between praise and criticism. I know these are students who have literally no experience teaching in a classroom, so I use the feedback as an opportunity to be as constructive as possible and to help them in developing their (mathematical) communication skills.

Most comments start with "Maggie, I liked how you…" or "Chad you should strive to improve the way you…" but sometimes are more frank in beginning "Ashten, you were mistaken when you wrote πr^2 …" I feel the students appreciate the constructive feedback and take my advice and criticisms to heart. The improvements I typically see from the first oral exam to the second and the third have convinced me the students are indeed working to improve their mathematical communication abilities.

What Did I Learn?

There are a handful of things I have learned through giving these oral exams. Some are logistical and some are more general, but they are all valuable lessons I have learned from this experience.

1. I always hold high expectations for what my students are capable of and many rise to the occasion. I saw significant improvement in many students' mathematical communication skills.

2. Give the students several days to prepare for the oral exam, especially if it almost coincides with a regular exam.

3. Allow each student to have some form of notes with them during the oral exam. I use notes in class as a professor, so why should I deny them the same opportunity? It will still be apparent whether or not a student has mastered his or her topic.

4. They will be extremely nervous for their exam, especially the first one, so spend a few seconds just chatting and settling them down before "rolling the die."

5. Emphasize, even to the low performing students, that improvement matters! A little encouragement goes a long way in seeing a student get more and more effective at communicating elementary mathematics.

6. Be honest with them. If a student seemed unprepared for their topic, then they were indeed unprepared. Let them know that it was obvious to you (tactfully!) but that they will have a chance to redeem themselves at the next oral exam.

7. The students see the value in this experience. The feedback I have received (through surveys) from my classes indicates that the students indeed viewed these oral exams as positive, formative learning opportunities. One student responded to the question about whether oral exams were beneficial saying "Yes, it made me realize how much I didn't use vocabulary correctly but need to." Another student commented "…although every time I was intimidated, I became more confident."

Overall, I soundly endorse giving oral exams (in the appropriate setting) and encourage others to consider incorporating oral exams in classes such as their elementary mathematics, and conceivably other, classes.

Russel E. Goodman is Assistant Professor of Mathematics and Assistant Women's Soccer Coach at Central College in Pella, IA. He can be reached at goodmanr@central.edu.

Clemson AP Calculus Institute Honors Don Kreider

This year's 2007 Advanced Placement Summer Institute in Calculus BC, to be held on the campus of Clemson University, is dedicated to the memory of Don Kreider, former president of the MAA (see the obituary notice in our January issue). The announcement describes Kreider as "a friend of Clemson and College Board Calculus Development." The Institute will be held from June 12 to June 30. For more information, visit their web site at http://virtual.clemson.edu/groups/mathsci/ap/calculus.bc/.

Making Mathematical Music

The January 26 issue of *Science* includes a review of a new CD, entitled *Codebook*, by the Rudresh Mahanthappa Quartet (Pi Recordings, 2006). Rudresh Mahanthappa is a jazz saxophonist, and his music for this CD is based on number patterns of various sorts. One piece, for example, is based on the decimal expansion of 1/7 and its well-known cyclical property. Another is a "mapping of the Fibonacci sequence onto the 12-tone musical scale". Asked whether any sequence wouldn't have worked just as well, the composer argued that there is something unique about the Fibonacci sequence: "It sounds right no matter what key the others are comping in. I tried alternative sequences and they didn't have that property." We haven't had a chance to hear the music, but would welcome reports from readers as to what it sounds like.

Ray Johnson Receives AAAS Lifetime Achievement Award

The American Association for the Advancement of Science awarded its 2006 Mentor Award for Lifetime Achievement to Ray Johnson of the University of Maryland. The Mentor Award honors members of the AAAS who have mentored and guided significant numbers of underrepresented students toward a Ph.D. degree in the sciences, as well as scholarship, activism and community-building on behalf of underrepresented groups.

Raymond Johnson, a member of the MAA, was the first African-American to earn a Ph.D. in mathematics from Rice University, in 1969. He went on to become the first African-American mathematics professor at the University of Maryland. The chair of the Maryland mathematics department, Patrick Fitzgerald, says that "the institutional success of our Department in educating underrepresented minorities has been based on the leadership of Ray Johnson." More information on the award can be found at the AAAS web site, at http://www.aaas.org.

The Man Behind the Poincaré Conjecture

By Hal Hellman

One of the most famous mathematicians in the world today is a man who has been dead for almost a hundred years. He's famous because of a conjecture, a math problem that he proposed over a hundred years ago: the Poincaré conjecture. He couldn't solve it himself and several generations of mathematicians have wrestled, unsuccessfully, with it ever since. In 2000 it was listed by the Clay Mathematical Institute as one of the most important, unsolved, math problems of our day.

In 2002 a shy Russian mathematician named Grigory Perelman posted three papers on a mathematical website that several teams who studied the work say is the required proof. This work earned Perelman the coveted Fields medal and a monetary reward. He refused both. Not interested.

But another group, led by the prestigious Chinese mathematician Shing-Tung Yau, was less impressed than the Fields people. Yau calls the proof messy. In 2003 he warned of a fatal flaw, and argued that until mathematicians have had a chance to review the proof thoroughly, "it's not math — it's religion." Yau is a professor at Harvard and director of math institutes in Beijing and Hong Kong.

After a couple of years' work on the proof, two of Yau's Chinese students published a paper in a journal edited by Yau; its title began with the words: "A Complete Proof…" In it they claimed to have substituted several key parts of Perelman's proof with their own more detailed arguments. Chinese newspapers both here and in China described the Chinese mathematicians as the group who finally provided the much-sought proof.

Shortly thereafter, *The New Yorker* published a long article that chided Yau for trying to grab credit that properly belongs to Perelman for himself and his institute.

Yau, not surprisingly, is furious. Within weeks, he had his lawyer send a 12-page letter to *The New Yorker*, but addressed to the two authors and the fact checker at the magazine, detailing what it refers to as the article's "false and defamatory content." It demands that *The New Yorker* issue a printed apology to Yau, and retract "all parts of the article about him which are incorrect…" It goes on, "If you [the letter's three recipients] are unwilling to assist in this effort, then you will leave Dr. Yau no choice but to consider other options."

On December 22, 2006, Science named the Poincaré Conjecture the "Scientific Breakthrough of the Year."

Of course any story about a battle in mathematics is newsworthy. What makes this story especially interesting is, first, the strange behavior of Dr. Perelman. If his proof stands up to a full two years of vetting, he will be eligible for a million-dollar award set up by the Clay Mathematics Institute. But Perelman may very well refuse this award as well. Then there is the fact that the proof has strong implications in both topology and cosmology. Among these implications is a better understanding of the shape of our universe.

Almost entirely absent from press coverage of the ongoing tussle, however, is anything about Henri Poincaré himself. He is still considered one of the great mathematicians of all time. More to the point, he would most likely have been tickled by the continuing fracas.

Although rather shy and modest personally, he didn't hesitate to criticize in print. In his very popular book, *The Foundations of Science* (1913), he had no hesitation in directing a sarcastic jibe at Cesare Burali-Forti's attempt to create a logical definition of the number 1. Poincaré suggested that Burali-Forti's definition was very well suited for giving the definition of the number 1 to people who had never heard of it. He accused fellow mathematician Louis Couturat of having naïve illusions. He wrote disparaging things about the work of the Italian logician Giuseppe Peano. In 1881, he feuded (though much more politely), with Felix Klein about a class of mathematical functions. He had called them "Fuchsian," even though he had done the major work himself. Klein felt the credit should go elsewhere and wanted to give the functions a different name. Poincaré also made fun of Georg Cantor's work on set theory.

But he devoted considerably more of his time and energy to a major battle with Bertrand Russell. Russell was the founder of the "logicist" school in philosophy of mathematics; in time, he came to be one of the world's most prominent intellectuals. Toward the end of the nineteenth century, Russell had begun thinking that it would be possible to create a mathematics based on a small number of fundamental logical concepts. These were the beginnings of Russell's logicism.

Poincaré was not happy with this idea at all, but he held his pen until 1906, when the publication of an article by Russell in the *Proceedings of the London Mathematical Society* goaded him into action. It was at this point that he decided to mount a general attack on Russell's logicism. Poincaré began with an article in the French journal *Revue de Métaphysique et de Morale*. (This journal's objective was to bring philosophy and the various sciences (moral as well as physical) into mutual understanding.) Because logicism depends heavily on Georg Cantor's set theory, Poincaré began with an attack that led back to Cantor. Then he questioned the use of logic as Russell had proposed it: "This method," he wrote, "is evidently contrary to all sane psychology…"

In one attack against Russell and his followers, he wrote: "I do not hope to convince them: for they have lived too long in this atmosphere. Besides, when one of their demonstrations has been refuted, we are sure to see it resurrected with insignificant alterations, and some of them have already risen several times from their ashes." The attacks and counterattacks went on for six years, and only ended with the untimely death of Poincaré in 1912.

In an article in *The New York Times*, Dr. John Morgan, a Columbia University mathematician working on Perelman's proof, is quoted as pointing out that the excitement surrounding Perelman's work came not so much from the actual proof of the conjecture, but rather from the method. Perelman, following Hamilton, had used differential geometry and differential equations to prove a conjecture that is purely topological. Thus, the proof highlights deep connections between different fields.

What's interesting here is that Poincaré was one of the few broad-based mathematical scientists in an era of growing specialization. In 1887, at the tender age of 32, he was elected to membership in the exalted French Academy of Sciences, and this seemed to create a turning point in his career. Already well published in the math field, he began to pay more attention to basic questions about the nature and philosophy of the field, and its connection to the broader world of science. By the turn of the twentieth century he had already built a reputation in several fields, including number theory, probability, various areas of mathematical physics, celestial mechanics, and, most relevantly to our story, topology. He even did pioneering work in the special theory of relativity. He knew that his conjecture had important implications for topology, and maybe he understood the implications for cosmology as well.

By the time Poincaré put his hypothesis forth in 1904, proof had already been accomplished for manifolds of lower order (one-dimensional and two-dimensional spaces). By the early 1980s, Poincaré's conjecture had also been proved in the n-dimensional case, as long as n was not equal to three. So only three-dimensional space — the one in which we live! — was left, which is one of the reasons Perelman's proof has had such resounding effects.

Thus far, *The New Yorke*r is holding fast to its stand. It remains to be seen what further steps Professor Yau will take. But whatever happens, my feeling is that Poincaré will be there in spirit, chuckling all the way.

Hal Hellman is a free lance writer who lives in Leonia, NJ. He is the author of several books, including, most recently, Great Feuds in Mathematics. *See the web site for the "Great Feuds" series, at* http://www.greatfeuds.com.

Administration Seeks 6.8 % Increase in NSF Budget

By Ivars Peterson

On Feb. 5, President Bush presented his fiscal year 2008 budget request to Congress for the federal government. In the proposed budget for FY 2008, funding for the National Science Foundation (NSF) rises to $6.43 billion, an increase of $409 million (or 6.8 percent) above the 2007 budget request. At the same time, the U.S. Congress hasn't yet completed action on NSF's 2007 request. The FY 2007 funding bill passed by the House, which is now awaiting Senate consideration, would provide $5.92 billion. Until the measure is enacted, NSF and other federal agencies are operating under a continuing resolution, with flat budgets pegged to 2006 levels. FY 2006 funding for NSF totaled $5.65 billion.

The new funding request signals that the Bush administration remains committed to the American Competitiveness Initiative, announced last year, which called for a doubling of funding for NSF, the Department of Energy's Office of Science, and the National Institute of Standards and Technology over the next decade.

Within the Mathematical and Physical Sciences (MPS) directorate, the budget request for the Division of Mathematical Sciences (DMS) is $223.47 million, an increase of $17.73 million (or 8.6 percent) over the FY 2007 request.

Within the Education and Human Resources (EHR) directorate, the budget request for the Division of Undergraduate Education (DUE) is $210.22 million, an increase of $13.42 million (or 6.8 percent) over the 2007 request of $196.80 million. However, the 2007 funding level was significantly lower than actual funding for FY 2006 of $211.86, and the new request doesn't bring funding all the way back to 2006 levels. DUE includes funding for the National (STEM) Education Digital Library (NSDL), which is roughly even with the 2007 request, and the STEM Talent Expansion Program (STEP), which increases by 12.1 percent.

EHR funding that went to mathematics as a result of the recent NSF-wide priority on the mathematical sciences that ends this year (about $1.09 million in FY 2007 and more in previous years) is now largely back in the core program budgets, with no advantage for mathematics-related proposals. Overall, within EHR, the number of competitive awards is unlikely to increase.

Details of the NSF FY 2008 budget request are available at http://www.nsf.gov/about/budget/fy2008/toc.jsp .

George Anderson Roberts. 1940–2006: A Tribute

By Jacqueline Brannon Giles

There is a term in Hebrew, "tikkun olam," which means "repairing the world." It may be that the memories of the accomplishments and influence of one man can make a significant contribution to the repair of the world, just as the flutter of a butterfly's wings can cause a storm far away. The life and accomplishments of George A. Roberts are an example of a life that took part in "the repairing of the world of mathematics."

Roberts was the first African American man to receive a doctorate in mathematics from Texas A & M University – College Station. Realistically speaking, we know that some environments are more difficult to negotiate than others. The fact that George Anderson Roberts succeeded in obtaining his Ph.D. in mathematics, focusing on Approximation Theory, in 1979, is a historical fact worth repeating and sharing with the next generation. A quiet and focused lifestyle enhanced by confidence and hard work is, perhaps, one of the many attributes Roberts displayed to his students, colleagues, family and acquaintances.

The memories of his outstanding career as a mathematician are somehow related to the Jewish concept "tikkun olam," for it is evident that his life and service to others have immeasurably impacted the lives of those who came in contact with him. It is important for us to record the history of men like Roberts, who accomplished what some might have thought was impossible.

A heritage in mathematical accomplishment can be transmitted through family, friends and students. Numerous students were touched by Roberts' accomplishment and wisdom. There are students who have succeeded in graduate level mathematics because Roberts was their mentor. G. Donald Allen, Professor of Mathematics at Texas A & M College Station, said of him that "George has developed a strong interest in mathematics education issues, and had done some good work in the area of teacher professional development."

Roberts was a family man, a father and grandfather, encouraging his progeny to go forward and persist in doing whatever they desired to do, never giving up, and always valuing the merit of hard work.

A man's gift makes room for him and brings him before great men, a wise book says. Roberts did not speak of his own accomplishments in national or international publications; rather his service to others speaks to the element of greatness in his life. His son, Jason, was so inspired by the life and accomplishments of his father that he has committed to reading his father's work in order to get a grasp of the depth of his father's mathematical thought. It is not impossible that Jason will learn something that enables him to make a mathematical contribution to the area of risk management and risk control in his job in the corporate sector. This dream is so worthy of mention: the accomplishments of the past generations ought to be a foundation for the accomplishments of future generations.

The memorial service program summarized some of Roberts' accomplishments:

Dr. George A. Roberts was a distinguished professor and great teacher who inspired all who worked with him or who worked under his tutelage. His tenure at Prairie View A & M University began in 1983, as Associate Professor of Mathematics. In 1992, he was promoted to the rank, Professor. As a colleague, he was dependable, timely, devoted, honorable and capable in his numerous roles –-chair of over 30 departmental, college, university committees, and served as Vice President of the Faculty Senate. He was Graduate Advisor and Coordinator of Graduate Programs in Mathematics since 1988.

He advanced from the rank of Instructor in 1966, to Professor and head of the Department of Mathematics at Wiley College. During that time he completed his Ph.D. Degree in Mathematics at Texas A & M University, in 1979. The title of his dissertation was "Uniqueness and Interpolation of Entire Harmonic Functions."

Dr. Roberts' contributions to education included teaching, research and service. In addition to his teaching career spanning forty years, he has published in *the Journal of Approximation Theory, presented research papers at national meetings, and made contributions to funded projects.*

Dr. Roberts has received numerous awards, that include: Teacher of the Year, Educational Achievement, Omega Man of the Year, Outstanding Turner Graduate, and Personalities of the South, as well as inducted into the Science Hall of Fame. He served as Adult Leader for the Boy Scouts of America Troop 141, Chair of the Scholarship Committee of Nu Iota Chapter of Omega Psi Phi Fraternity, Adult Sunday School Teacher and Chair of the Deacon Board and the Building Fund Committee at Walnut Grove Baptist Church in Carthage, Texas.

Dr. Roberts was the fifth of eleven children. He was born July 1, 1940 to Barker and Thelma Hicks Roberts. He attended public school in Carthage, Texas. As a young man, George was determined to get an education. He graduated from Wiley College and later received his Masters degree from the University of Arizona. However, his greatest academic accomplishment was becoming the first Black to earn a Ph.D. in mathematics from Texas A & M University – College Station, in 1979.

Dr. Roberts was a devoted husband of forty-one years to the love of his life, Mary, who affectionately called him "Sweetie." He was the loving father of three sons: Michael, Jason, and John. Jason and his wife, Tiesha, gave Dr. Roberts the pride and joy of his life –two grandchildren, Jalen and Karis.

Professor Togba Sapolucia who recognized Dr. Roberts as his teacher and colleague said that, "During my studies in three different countries (Liberia, Guinea, and the United States), Dr. Roberts is the professor who impressed me the most because of his high level of intelligence and understanding of mathematics." Sapolucia is currently a professor at Houston Community College – Northeast Campus.

We should recognize George A. Roberts as a man who managed to achieve much while remaining a man of stability and compassion. He served more than fifty-five years at the same church, and demonstrated his passion and love for "truth and excellence" by teaching Sunday School and serving as Chairman of the Deacon Board at Walnut Grove Baptist Church. We salute a man who enriched our heritage and whose accomplishments will elevate our expectations of the generations to come.

Jackie Giles is a member of the FOCUS editorial board. She teaches at Central College, part of the Houston Community College System.

Teaching Time Savers: Grading Quickly and Consistently Using Equivalence Classes

By Susan E. Martonosi

Grading consistently is perhaps the most time-consuming aspect of grading for me. I can go through the papers and correct them easily and quickly enough, but I am often bogged down by the task of deciding how many points to award to a particular combination of errors and insights, especially when I can't remember how I graded similar solutions earlier in the stack. Moreover, the grade I think a solution might be worth when I start grading is not necessarily the grade I think it's worth after I've examined every paper, so students making similar mistakes don't always receive similar grades.

The end result is probably 15-30 seconds of hemming-and-hawing about the grade on each paper *per problem*, as well as the occasional confrontations by pairs of students wondering why they received different scores for similar solutions. Even when the students don't confront me, I often worry that I haven't graded the papers consistently.

Grading rubrics can help, but only to a limited extent. For papers with multiple right answers or when a student makes a quirky combination of errors, rubrics don't often provide enough guidance. However, I've found a trick that saves time *and* yields more consistent grades.

For each problem on an assignment or exam, I sort the students' papers into equivalence classes: rather than assigning a grade upon the first pass through the stack of papers, I treat the first pass as a chance to read each solution, mark it up and then put it into a pile corresponding to the grade that I *think* it merits based on the mistakes made. This way, whenever I come across a particular type of mistake or combination of mistakes, I can put the paper into a pile of similar papers. I usually end up with approximately ten piles. At the end, I then go through the papers a second time by pile, usually starting with the best pile. I assign grades to the piles, occasionally bumping individual papers up or down if it's clear that my grading scale shifted over time.

This second pass does add some additional time but the benefits outweigh the costs. The second pass applies only to the ten or so equivalence classes, rather than to the n individual papers, so I hem-and-haw about grades only ten, rather than n, times per problem. Furthermore, I no longer worry that I have penalized one student more than another for a similar type of mistake; my grades are consistent. Another perk: by sorting papers based on similar errors, I have been able to identify cases of academic dishonesty, even in a large class.

Time spent: 15-30 seconds to assign a grade to *each equivalence class, per problem*.

Time saved: 15-30 seconds to assign a grade to *each student, per problem*; and fewer meetings with students about grade inconsistencies.

Susan E. Martonosi teaches at Harvey Mudd College.

Employment Opportunities

ALABAMA

University of Alabama in Huntsville

The Department of Mathematical Sciences at the University of Alabama in Huntsville invites applications for a visiting position at the rank of Assistant Professor/Associate Professor, beginning August 2007. A Ph.D. degree in mathematics or applied mathematics is required. Applicants must show evidence of excellent research potential in an area that matches the interests of the department. Applicants must also have a strong commitment to teaching and show evidence of excellent teaching ability. Preference will be given to applicants whose research area is partial differential equations, mathematical modeling, or mathematical biology.

Applicants should send a curriculum vita with the AMS standard cover sheet and three letters of recommendation (with at least one letter addressing teaching) to

Chairman
Department of Mathematical Sciences
University of Alabama in Huntsville
Huntsville, AL 35899.

For more information about the department, visit our web site at http://www.math.uah.edu.

Review of applicants will begin March 1, 2007, and will continue until the position is filled. Women and minorities are encouraged to apply. The University of Alabama in Huntsville is an Affirmative Action, Equal Opportunity Institution.

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