FOCUS

THE NEWSLETTER OF THE MATHEMATICAL ASSOCIATION OF AMERICA

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In this Issue

The Mathematical Association of America
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Mathematicians Elected to National Academy

Five mathematicians and one statistician are among the sixty new members recently elected to the National Academy of Sciences. Election to membership in the Academy is considered one of the highest honors that can be accorded a U.S. scientist or engineer. The newly elected mathematicians are: Persi Diaconis, Harvard University; H. Blain Lawson, Jr., State University of New York at Stony Brook; Charles S. Peskin, Courant Institute of Mathematical Sciences; Calyampudi R. Rao, Center for Multivariate Analysis, Pennsylvania State University; Richard P. Stanley, Massachusetts Institute of Technology; and Daniel W. Stroock, Massachusetts Institute of Technology.

The National Academy of Sciences is a private organization of scientists and engineers dedicated to the furtherance of science and its use for the general welfare. The Academy was established in 1863 by a congressional act of incorporation, signed by Abraham Lincoln, that calls upon the Academy to act as an official advisor to the federal government, upon request, in any matter of science or technology.

NSF-Supported Mathematics on Display

Two mathematics projects were among the exhibits on display at an exhibition and reception to highlight research and education projects supported by the National Science Foundation (NSF) in late March. Held at the Library of Congress, the event was organized by the Coalition for National Science Funding. The two mathematics exhibits were sponsored by the Joint Policy Board for Mathematics.

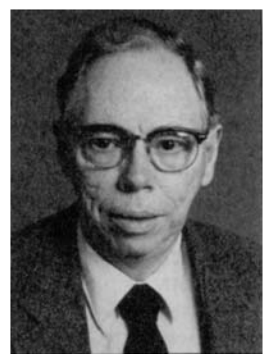
Professor Frank Morgan of Williams College in Massachusetts and two former undergraduate students discussed their research into the mathematics of minimal surfaces. The team sought a basic mathematical understanding of geometric structures and shapes, from soap bubble clusters to metals and crystals. Undergraduate students working under Professor Morgan's direction with funds from the NSF's Research Experiences for Undergraduates program have made original discoveries in this area.

Professor William Yslas Velez of the University of Arizona demonstrated the application of new educational technologies and methods for teaching calculus. This exhibit emphasized the NSF's leadership in the nationwide calculus reform movement through support for innovative curriculum development projects.

The exhibition was aimed at members and staff of the United States Congress, and designed to demonstrate the value of activities supported by the NSF and their importance in meeting the nation's research and education goals.

About 180 participants viewed 23 exhibits on a variety of projects in many scientific and engineering disciplines and interdisciplinary areas.

Donald Lewis Named Director at NSF



Donald J. Lewis of the University of Michigan has accepted appointment as Division Director of the Division of Mathematical Sciences at the National Science Foundation. Lewis, a number theorist, received his doctorate from Michigan in 1950. He was appointed to the Michigan faculty in 1961, and twice served as chair of the department. In 1995 he was a recipient of the AMS Distinguished Public Service Award. Lewis is expected to take up his new position on July 1.

FOCUS

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Editor and Chair of the MAA Newsletter Editorial Committee: Keith J. Devlin, Saint Mary's College of California

Associate Editor: Donald J. Albers, MAA Associate Executive Director and Director of Publications and Programs

Managing Editor: Harry Waldman, MAA

Production Specialist: Amy Stephenson Fabbri, MAA

Proofreader: Meredith Zimmerman, MAA

Copy Editor: Nancy Wilson, Saint Mary's College of California

Letters to the editor should be addressed to: Keith Devlin, Saint Mary's College of California, P.O. Box 3517, Moraga, CA 94575; e-mail: devlin@stmarys-ca.edu

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Editorial

The Supergroup?

Editors are, by nature, never at a loss for the written word, and I am no exception. And yet, this month's editorial is decidedly short. Why? Well, if you look inside the pages of FOCUS, you will see my name as the byline of one of the main feature articles. The topic is popular science writing by professional mathematicians. In preparing for this article, I contacted (by e-mail) a number of acknowledged masters at the craft: John Casti, William Dunham, John Allen Paulos, and Ian Stewart. (At the moment, all those who have been successful in this extremely small field are male.)

My intention was to spice up my article with some nice quotes from each of these people. I should have expected what happened next. Almost by return, each sent me back a reply that was in itself a gem of writing. Without thinking about it, I had cast myself in a role much like one of those pop music impresarios who brings together a group of famous musicians for a 'supergroup' concert. All the impresario has to do is get them together onto one stage. So I did not edit their replies. I simply pasted them together into what I think is one of the most fascinating articles FOCUS has ever published. I hope you do too. To John, William, John, and Ian, thank you—both for your efforts in popular science writing and for your thoughts as expressed in this month's issue.

—Keith Devlin

The above opinions are those of the FOCUS editor and do not necessarily represent the official view of the MAA.

Correction

The April FOCUS had reported incorrectly that Robert Goldrick, a software specialist with Grumman Aerospace, had passed away. The MAA had received erroneous information, and we regret any problems and concerns this caused.

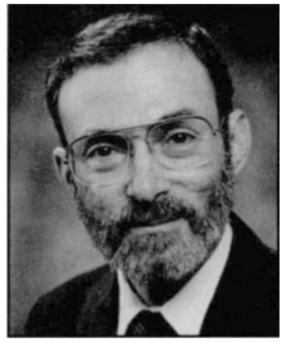
President's Column

The past ten years have seen a remarkable amount of progress in improving mathematics education at all levels. The goal is to enable all students, including those from all racial and ethnic backgrounds and both genders, to master and appreciate mathematics. The emphasis is on understanding mathematics rather than thoughtlessly grinding out answers. For various reasons, there is now an increasing amount of resistance to what is usually called "math reform," which reflects some serious concerns that need to be addressed.

Pre-college math reform, based in large part on the NCTM *Standards*, and college math reform, usually labeled "calculus reform," are compatible in their goals and are now facing similar resistance. I believe that we are all in this together and that we need to work together to maintain momentum and establish better mathematics education for all. Parents, teachers, and the general public need to realize that the new approaches make sense and will empower the young people for the next century.

Unfortunately in the past, much of mathematics has been presented as a bunch of rules—rules for manipulating numbers and symbols. Underlying principles, general problem-solving techniques, and serious quantitative thinking got lost. Certainly much of the interest, beauty, and fun vanished. A major thrust of the current reform movements is to present mathematics in a much broader context. It encompasses ideas and techniques that aren't even seen in traditional treatments of mathematics, and they are interconnected. Mathematics isn't just a sequence of isolated topics that are to be struggled with, learned (or not), and forgotten.

At the college level the emphasis has been on "calculus reform." An excellent overview of calculus reform can be obtained by reading the articles in the January 1995 issue of *UME Trends*. As a starter I especially recommend Alan Schoenfeld's article titled "A Brief Biography of Calculus Reform." A more formal and in-depth report can be found in the just published MAA report *Assessing Calculus Reform Efforts*, edited by J. R. C. Leitzel and Alan Tucker. This is a very readable and inter-



esting account of the history and current status of calculus reform. Where there's hard data, these reform efforts have been largely successful. Many people within the MAA and other mathematical organizations are working hard to improve teacher education programs, develop new curricula, and help collegiate mathematicians get involved in the schools.

The term "calculus reform" is misleadingly restrictive, because the changes at the post-secondary level extend far beyond calculus. A glance through the programs of the past few national mathematics meetings, especially the minicourses and sessions of contributed papers, shows that there are parallel changes in the way abstract algebra, linear algebra, differential equations, and pre-calculus are taught. More specialized courses—such as dynamical systems, Fourier series, and modeling—are also being taught in new, exciting ways that involve taking advantage of the new technology.

The current push for calculus reform got its jump-start from the now famous Tulane Conference in January 1986. During the same period the NCTM *Standards* were being created. They were published in 1989 and have been very widely accepted and used. The current political and sociological climate has led to some backlash. The NCTM is aware of this serious threat and has appointed a task force that will seek appropriate responses. The MAA representative on this task force is Naomi Fisher; her e-mail address is u37158@uicvm.uic.edu.

The goals of the NCTM *Standards*, which address mathematics education for K–12, are to "Create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers...and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields," and "Create a set of standards to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision." The vision calls for changes in the curriculum, including new content such as probability, statistics, and discrete mathematics, as well as for different approaches to some of the topics in the existing curriculum. It is envisioned that students will (1) learn to value mathematics; (2) become confident in their own ability; (3) become mathematical problem solvers; (4) learn to communicate mathematically; and (5) learn to reason mathematically. Each of these goals is elaborated on. For example, (3) states that "students need to work on problems that may take hours, days, and even weeks to solve...some may be relatively simple...others should involve small groups or an entire class working cooperatively. Some problems also should be open-ended with no right answer...."

The most vivid changes in teaching have involved technology, but the real focus has been to improve the learning of students and to make sure that a wider group of students is able to benefit than has in the past. Changes in instructional practice include hands-on experiences using technology, increased focus on conceptual understanding, cooperative learning, student project activity, extensive writing, and less reliance on timed tests in assessment.

It's easy to detect flaws in any movement as broad as the reform movement and to overlook the progress. In February I attended an NSF/DOE conference on systemic reform in science and mathematics titled "Joining Forces: Spreading Successful Strategies." It became clear at this conference that a large number of people across the country provide excellent education in various creative ways. The focus of the conference, as its title suggests, was the daunting but vital task of identifying those programs that really can be duplicated throughout the country, without losing their effectiveness, and then implementing them nationwide.

Statistics from the Department of Education (the *Condition of Education*, 1994) show that we are making progress. For example, substantially more high school graduates in 1992 are taking mathematics courses at the level of algebra I or higher than their counterparts in 1982. Thus in 1992, 56.1% of the high school graduates took algebra II and 70.4% took geometry,

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Prizes and Perils of Popularizing

Keith Devlin

Casti's Tale

John Casti, a mathematician at the Santa Fe Institute and the author of a number of bestselling popular science books, including *Paradigms Lost*, recounts the following recent event.

"I was talking with a colleague at Santa Fe Institute, Brian Arthur, a well-known economist. We were interrupted by a knock on his office door, and when he opened it there was a journalist from Basle here to interview Brian for a Swiss economics magazine. When Brian introduced me, this gentlemen said, 'Finally I meet someone here that I've actually heard about besides Brian.' Well, with people like Murray Gell-mann, Stu Kauffman, Brian, Chris Langton, John Holland, and others around here all the time, you can imagine it was quite a shock to hear that this fellow knew more about me from my pop-science books than about any of these guys and their path-breaking work. Such are the vagaries of information transfer in today's world, I guess."

"Such recognition, along with a steady stream of invitations to give public lectures, talks to student bodies, and after dinner speeches to non-mathematicians are among the rewards of venturing into the world of science popularization," Casti declares. Another benefit is that the process itself is rewarding:

"The choice to turn to popularizations has enabled me to learn a lot about a lot of different fields and problem areas, has brought me into contact with numerous people that I would never have met otherwise, people whose academic specialties range from anthropology to zoology. Furthermore, the research I did for these various books forced me to think about how all the areas of intellectual life relate one to the other. In addition, I have received literally thousands of letters from readers, some of which have developed into ongoing correspondence that has materially expanded my view of the topics I wrote about."

But there are perils too that await the successful popularizer. Casti continues his story this way.

"Lest you think my experiences have been uniformly positive, let me note the one downside of this kind of notoriety, one that I'm sure all of us [popularizers] share. It is simply the fact that the visibility from these books tends to outshine anything else we may do along more traditional academic lines. This, in turn, forces people to stamp us with the label of 'popularizer,' which occasionally works to our disadvantage. For example, when I first came here to SFI three years ago, a meeting of the powers-that-be decided that I should not be a member of the SFI External Faculty since I was really more of a writer than an academic researcher. It took some time and effort on my part to convince a few skeptics that I actually had some ideas besides how to tell a story."

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while only 36.9% and 48.4%, respectively, took these courses in 1982. During the same period, the percentage taking remedial or below-grade-level math dropped from 32.5% to 17.4%. Another table shows that these dramatic shifts are happening for all racial/ethnic groups. We don't hear much about such statistics hidden in dusty government tomes, even when they are positive! With such big changes nationwide in ten years, something right must be happening.

So what are the concerns that are leading to resistance to these changes? One is that the laudable focus on understanding has led to some decline in mathematical skills. Since it is easier to measure and spot deficiencies in skills than understanding, this problem can easily be over-emphasized. On the other hand, this is a serious problem, especially since our future scientists, engineers, and mathematicians must obtain *both* substantial understanding and substantial skills. The reform movements need to address this issue.

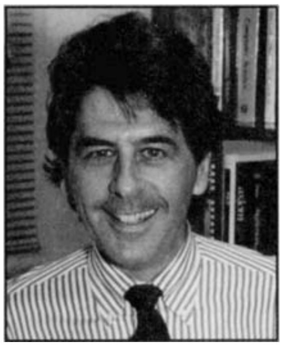
For teachers who are following the NCTM *Standards*, there's no doubt that it is more difficult to determine (or at least quantify) students' knowledge, understanding, and skills. This is now leading to serious challenges as the mathematics community faces assessment issues. Similar challenges are faced by post-secondary faculty as they change their instructional practices. I have no wisdom here except to acknowledge the difficulties tempered with the belief that they can be overcome, though it won't be easy. To steal a quote, "Tests should measure what's worth learning, not just what's easy to measure."

Another concern is largely political. There is a natural American resistance to centralized control. Some fear that the NCTM *Standards* are subverting local control. Widespread reform is hard to accomplish in such an atmosphere. We are still suffering from the bad taste that so-called "New Math" left in America's mouth thirty years ago. That was an effort that focused entirely on the curriculum. An equally serious problem all along has been in the pedagogy. Finally, in the United States, we are suffering from a widespread case of anti-intellectualism wherein all of us are experts on the schools because we once attended schools. We need to continue to learn from the successes and failures of other countries where many of the same problems are being faced.

In the long term, the college and pre-college reform efforts are intimately linked. Students and parents expect pre-college mathematics education to be a preparation for college-level mathematics. This is an important endeavor and everyone needs to be involved. A future column will discuss the role of post-secondary faculty and the MAA in addressing this interface. Students' mathematics learning should be seen as seamless as they progress K–16. We've come a long way in ten years. We have a long way to go.

Ken Ross

Devlin's Tale

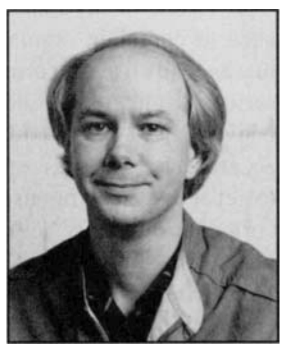


Used to living in a world of precisely defined categories, mathematicians may be more prone than others to regard 'popularizer' and 'researcher' as disjoint sets, a partition that has a tendency to turn the word 'popularizer' into a term of derision in certain circles within the mathematics community. Certainly my own experience bears this out. The general disapproval of my early attempts at popularization on the part of some (not all) of my colleagues at the University of Lancaster, in England, took a more serious turn in 1985. In that year, faced with a government imposed budget cut that could only be met by cutting a third of the faculty within two years, the university brought in a new vice chancellor (president) to steer the institution through the decidedly troubled waters it suddenly found itself in. A former dean at MIT, with a reputation as a pretty tough cookie, he declared that the necessary savings would be achieved by "cutting out the dead wood." What this phrase meant, he explained, were those faculty who were not active in research. An increased emphasis on research was to be the key to our salvation, particularly research that attracted external funding.

With an active research program that had resulted in both a steady string of publications and external funding, I naturally felt that the chill winds of this new agenda would not blow in my direction. So I was completely taken aback a few months later to find myself one of the first mathematics faculty advised to leave. It was some time before I was able to piece together what had led to my name appearing on the cost savings list. One of the problems was, a senior administrator informed me, that I was no longer sufficiently active in research; rather, my interests had turned to, in a phrase I remember to this day, "less academically respectable activities." What those activities amounted to was that, since 1983, I had written a twice-monthly mathematics column in the national newspaper *The Guardian*, I had worked with the BBC on the production of a documentary about mathematics, *A Mathematical Mystery Tour*, and I had started to work on a popular book on mathematics, *Mathematics: The New Golden Age*, which Penguin published in 1987. The fact that my research program had continued to flourish, and had indeed received external funding with an industrial partner, was simply discounted. Having been successful in my attempts at popularization, I had become a 'popularizer,' and therefore could not be, so the logic went, a 'researcher.' Of course, I had tenure, so I did not have to leave. But the writing was on the wall, and so I left for the United States—to take up a position at a research institute at Stanford, as irony would have it.

Not a typical experience, of course, and in many ways a reflection more on the difficulties the entire British university system was facing at the time than on the details of my own particular case. But my experience does highlight the tendency for many in the academic community to regard popularization as exclusive of all other academic activities.

Stewart's Tale



A two-hour drive from Lancaster, at the far more financially secure University of Warwick, my colleague and fellow British mathematics popularizer Ian Stewart (author of *The Problems of Mathematics* and many other mathematics bestsellers) had a much easier time of it.

"I'm lucky, I think: Warwick genuinely approves of activities that take science to the public. You won't get promotion for it, but it won't count against you and it earns some kind of brownie points provided you aren't neglecting your main work (lectures, papers, administration, grants). Our annual report even *asks* for 'public output' to be listed by each member of faculty. [And] my colleagues here generally seem to approve of popularization, under similar conditions!"

Having found, in the United States, an academic institution not only receptive to my popularization work but actively supportive, I am maybe now in an even better environment than Stewart. (I still have a highly active research program as well.) I can certainly agree with his observations on the negatives of popularization even when there is no open hostility: as a regular columnist in *Scientific American* and a fellow contributor to *The Guardian*, Stewart says,

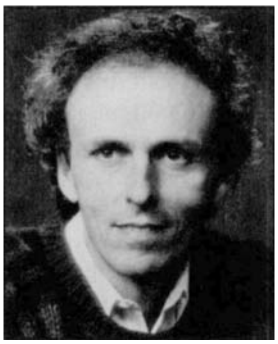
"The main pitfall that I have found is not so much negative comments from colleagues, but a serious failure by some mathematicians to appreciate the difference between journalism and writing a math paper for a journal. There are complaints about due credit not being given (they often think that newspaper articles have lists of references at the end). They have no appreciation that newspapers and magazines have space limitations and house styles. They don't understand that editors and subeditors will hash your work about but leave your name on it. They don't appreciate that with a forty-eight-hour deadline you can't do a total literature search to discover that some obscure Armenian published something vaguely similar in 1922 in *The Korean Journal of Newt-Watching*. They get upset if you tell readers that a manifold is a multidimensional surface but forget to add 'paracompact Hausdorff.' They get *very* upset if you happen to mention that some area—other than their own—might be important. They make sweeping criticisms without doing their homework, as with Marilyn vos Savant and the goats in *Parade*, or people who attack chaos theory on the grounds that it has no applications, when the literature contains little else. And many still do not appreciate that unless somebody tells the public what mathematicians are doing, support for the subject (and I mean appreciation rather than money) will dry up.

"My final moan is about complaints of 'premature publicity.' The story has to go to the public while it's still hot, not after the *Annals* has printed it. If it later turns out that the work fell to bits, that's *another* story, not a cancellation of the first one. In any case such complaints are leveled in very uneven ways: compare Rego-Rourke on the Poincaré Conjecture, Hsiang on

Kepler's sphere packing problem, and Wiles on Fermat.

"This may all sound negative. Not so, at least, not for me. My feeling is that in broad terms the mathematical profession is much more supportive of popularization than it used to be, and appreciates it more. The really top people nearly all do it. It is the next tier down, the brilliant second-raters, who *think* they know what mathematics 'really is' (and want to make sure everybody else does it *their* way), who tend to be the most narrow-minded and ill-informed."

Paulos's Tale



John Allen Paulos of Temple University has probably been the most successful in the pop-math business in recent years. His 1989 book *Innumeracy* was on the *New York Times* bestsellers list for eighteen weeks, his 1992 sequel *Beyond Numeracy* was another great success, and his *A Mathematician Reads the Newspaper* has just hit the bookstores. Paulos characterizes the view of his colleagues toward his success with these words:

"I suspect that the dominant attitude toward academics who write for a popular audience is not snideness, jealousy, or hostility, but rather indifference. This attitude is certainly defensible, but although not a task for everybody, disseminating and vivifying mathematical ideas for a large audience is, I think, quite important. Moreover, it does seem to be undervalued by the profession."

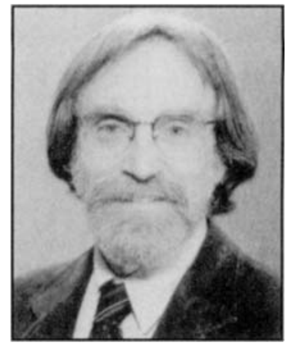
In Paulos's case, the runaway success of *Innumeracy* also led to a somewhat acrimonious exchange with a reviewer in the October 1993 issue of the *American Mathematical Monthly*. The reviewer took Paulos to task for a number of numerical inaccuracies in a book that was, after all, about innumeracy. In his response, Paulos emphasized his goal as follows:

"The aim of *Innumeracy* was to make vivid some of the consequences of mathematical illiteracy—muddled personal decisions, emotional risk assessment, misinformed governmental policies, and, yes, a generalized susceptibility to nonsense. Eschewing calculations for the most part, it attempted to do this via stories, anecdotes, vignettes, and some informal exposition."

Now, as it happens, there were some points in *Innumeracy* where I too thought that a little more accuracy and precision were required, so on that score I can at least see the reviewer's point. But I know from experience that in writing for the popular market, you have to be prepared to be loose with the truth—sometimes very loose. (On television or the radio, the problem is even more acute, as Paulos, who has appeared on programs as diverse as *The David Letterman Show* and the *MacNeil/Lehrer News Hour*, can attest.) You have to paint a big picture using a very broad brush, and to hell with the details and the precision. This requires that you make judgment calls as to just how far you can go. The decision you make depends on the audience you are trying to reach. "Why do you keep using the term 'infinitely long polynomial'?" one of my Lancaster colleagues used to complain regularly. "You could just as easily use the correct term 'infinite series' . . . and maybe add a remark about convergence." My standard answer was accurate, but the message never seemed to get across. I did not use the technically correct terminology because, if I did, my *Guardian* editor would not print my article. He had four-hundred-thousand readers for whom the term 'infinite series' would most likely mean *Coronation Street* or *Eastenders*. Enough of those readers could remember polynomials from high school to allow me to use that term, and the ones that did not remember, or never knew, were in any case not likely to be reading the Devlin column. So 'infinitely long polynomial' it always was. (And they did not 'converge' or 'diverge'; they either gave an answer or they did not.)

Dunham's Tale

William Dunham also achieved significant sales with his book *Journey Through Genius*, published in 1990, and his more recent *The Mathematical Universe* (1994).



He is a professor at Muhlenberg College, a small college in Pennsylvania. As I found when I took a position at a small college in the United States (is 2300 undergraduates small?), such institutions can be very supportive of popularization. Dunham puts it this way:

"Over the years since publication of *Journey Through Genius* and *The Mathematical Universe*, I have worked at a small liberal arts college and (as a visitor) at a large research university. I can report that mathematical colleagues at both institutions have been generous in their comments. I take this as a recognition across the discipline that mathematicians must do a better job reaching out to the wider public. In a number of ways, from the prosaic matter of government funding to the more noble one of cultivating future mathematicians, the health of our discipline depends on it.

"At the small college, writing for the scientifically literate reader actually *counts*. It is regarded as a legitimate professorial enterprise, one that not only serves the institution but also helps the author obtain employment, promotion, and/or tenure. This should not be surprising, for the liberal arts college is a place that devotes a great deal of time to examining, critiquing, and interpreting our intellectual heritage for the non-specialist. Efforts such as mine seem to fit into this mold. But my writing would never be regarded as 'serious' at the research institution. There, a single article on, say, non-cyclic complex k-modules over quasi-affine hypersemigroupoids—even if it boasts a worldwide readership of half of half a dozen—carries infinitely more status than fifty books for the popular audience. The research paper, the previously unproved theorem, the lemma with your name on it, these are the legitimate goals of the mathematician within that environment. All else is a sidebar.

"In some ways, it is difficult to argue with this attitude. Indeed, I suspect that all of us who have been through mathematics graduate school have internalized such a view to a greater or lesser extent. The research institution stands as the arbiter of professional standards, even for those of us far from its gates. Perhaps this is as it should be. Yet higher education has multiple objectives. Not everyone, after all, will be a concert violinist, yet nearly everyone can enjoy and appreciate music. I hope that, for some educated readers, I can be a guide to the greater enjoyment and appreciation of mathematics. And that, it seems to me, is an achievement not without merit."

Dunham also has some advice for anyone tempted to start out along the path of popular mathematics writing:

"While writing *Journey Through Genius*, I was continually made aware of the inverse relationship between a book's *mathematical content* and its *sales potential*. My editors at John Wiley were extremely supportive of my mathematical objective: to examine as faithfully as possible the paths by which great mathematicians proved their great theorems. Yet the editors reminded me time and again that a book resembling a graduate analysis text is unlikely to challenge Stephen King or Danielle Steele at the cash register. In this light, writing for the popular audience is a delicate balancing act, one whose difficulties are fully apparent only to those who have given it a try.

"One clear difference between my kind of writing and the authorship of a highly technical paper on those non-cyclic complex k-modules is that I tend to get a *lot* of mail from readers. Some of my correspondents, I am pleased to say, fall under the heading of 'fans.' Others, I am less pleased to say, fall under the heading of 'cranks.' Underwood Dudley, of course, knows far more about this professional peril than I do, but if anyone wants to see one-paragraph proofs of Fermat's Last Theorem having the Riemann hypothesis as a trivial corollary, drop me a line. I think I have a stack of them in my desk.

"Still, fans tend to be remembered longer than cranks. In my case, a few of these fans have been mathematicians themselves, so a fringe benefit of authoring popular mathematics books has been a string of speaking invitations from across the country. This has been an extremely enjoyable, if totally unexpected development."

Why Do We Do It?

Schooled as we are in a discipline that places an incredibly high demand on accuracy and precision, most mathematicians find they simply cannot—and I mean cannot—give up on enough of that precision to write for the pop market. You can see that by taking a stroll around your local, non-academic bookstore. You can number on the fingers of two hands (sometimes even one) the mathematics popularizers whose books appear on the shelves. There are many more Fields Medalists around than there are mathematicians who have written successful mathematics books for the general reader.

Given that all of us in the pop-math-by-the-professional-mathematician business have met some kind of disapproval from our colleagues, why do we continue to do it? Certainly not because of the money. Even if you 'hit the big time' and find yourself with a regular spot in a newspaper or magazine, or a book on the science bestseller list, you are unlikely to find yourself a millionaire overnight. With very few exceptions (Paulos is probably one of them), there simply is not that kind of money to be made from mathematics books and magazine articles.

I can think of two answers. The first is a personal one, which Ian Stewart sums up with the words, "I write about mathematics because I am a mathematician with a journalistic streak who has a way with words." Amen to that.

The second answer is at the level of the profession. As the sales of popular mathematics books show, there is a market out there for accessible expositions of mathematics. People want to know. Some of them are influential and can affect the financial support the mathematics profession receives from government and private sources; most are not. However, in a field as misunderstood as ours, do we really want to turn our backs on the relative few who show any interest?

So I say three cheers for the math popularizers and more power to their elbows (actually, keyboards these days). But then, I would say that, wouldn't I?

Keith Devlin is the editor of FOCUS. His most recent pop-math books, both of which appeared in 1994, are Mathematics: The Science of Patterns, *published by W.H. Freeman in their Scientific American Library series, and* All the Math That's Fit to Print: Articles from The Manchester Guardian, *published by the MAA in the Spectrum series.*

Mathematica in Calculus Workshop

July 30–August 3, 1996

Bowdoin College

A number of NSF-supported projects have produced *Mathematica* materials for use in reformed calculus courses. This workshop will explore the issues raised by incorporation of *Mathematica* into the calculus curriculum and provide guidance to participants who are considering the use of *Mathematica* in their own calculus programs. Four *Mathematica*-based NSF-supported projects will be featured at the workshop: the Bowdoin College *Mathematica* laboratory development project for Duke University's Project CALC, the Calculus & *Mathematica* program from the University of Illinois and Ohio State University, the Knox College *Mathematica* laboratory development project, and the Calculus using *Mathematica* program from the University of Iowa. All these programs cover topics in both single variable and multivariable calculus. The workshop is supported in part by the National Science Foundation.

Organizing Committee: William Barker (Bowdoin College), William Davis (Ohio State University), Dennis Schneider (Knox College), and Keith Stroyan (University of Iowa).

Contact William Barker, Dept. of Math, Bowdoin College, Brunswick, ME 04011; (207) 725-3571; fax: (207) 725-3750; e-mail: barker@bowdoin.edu

Explorations in the Chaos Club

Robert L. Devaney

Remarks on receiving the MAA's 1994 Award for Distinguished College or University Teaching of Mathematics, San Francisco, California, January 1995.

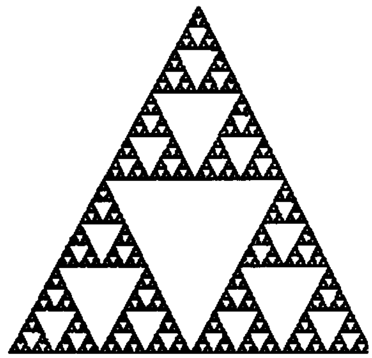
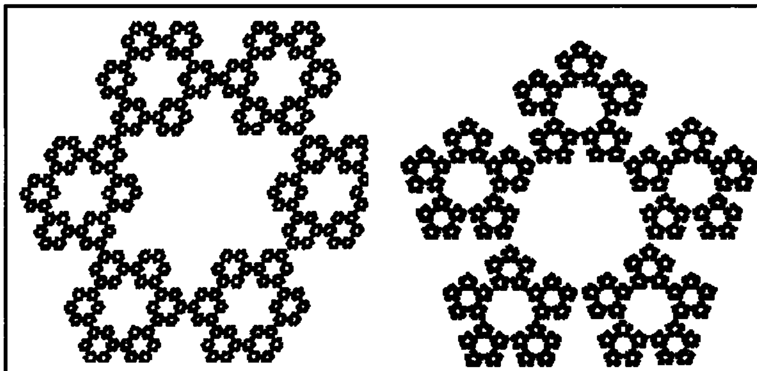


Figure 1 *The Sierpinski triangle*

Over the years, I have been troubled by the fact that we as mathematics educators sometimes shy away from introducing our students to contemporary ideas in mathematics. As a consequence, most of the general public believes that the development of mathematics ended sometime centuries ago with the creation of algebra or the introduction of calculus. It is a shame that most undergraduate students of mathematics never get a glimpse of present-day mathematics during their entire education.

In an effort to change this, Jonathan Choate, Mary Corkery, Beverly Mawn, and I have been involved over the past several years in "chaos clubs," after school mathematics groups that have met in several inner city and suburban school districts in the Boston area. The stated goal of the clubs is to show the students some modern ideas from dynamical systems theory. The hidden agenda is to convince their teachers that such new ideas complement the existing curriculum and serve to motivate and intrigue students.



Figures 2 and 3 *A Sierpinski hexagon and pentagon*

Let me give just one example of the kind of activity that takes place in the chaos clubs. Most mathematicians are by now familiar with the "chaos game," but most students are not. Students are always intrigued when they first see the output of this game. More important, from a pedagogical point of view, this game and its variations are natural vehicles with which to bolster a variety of important mathematical skills as well as to introduce certain other important topics rarely taught in lower level courses.

Before discussing the pedagogical issues involved, we first recall the rules of the chaos game. Begin with three points located at the vertices of a triangle in the plane. Then choose a seed or starting point anywhere within the triangle. To play the game, randomly select one of the vertices and then move the seed half the distance toward that vertex. Then iterate, using the location of the point after the previous move as the seed for the next. After approximately ten such moves, begin to record the location of successive iterations. That is, plot all but the first ten iterations of the chaos game. The goal of the game is to be able to predict the picture that emerges after thousands of such iterations. As is well known, with probability one, this picture is the Sierpinski triangle (see Figure 1).

When students first see this image, they are absolutely amazed. They usually expect the output to be either a random mess or the full triangle, never a fractal image such as this. Nonetheless, once they see this image, it is easy to convince the students why this particular image results.

For later use, note that the Sierpinski triangle consists of three self-similar pieces, each of which is one half the size of the original figure (in terms of linear dimensions). That is, we can recover the number of original vertices and the "compression ratio" 1/2 by simply looking at the image. If we imagine the entire triangle being compressed by a factor of 1/2 toward any of the three corners, we also recover the locations of the vertices as the fixed points of these contractions.

As another example of a chaos game, choose six points at the vertices of a hexagon. Then randomly move a traveling point toward these vertices with a compression ratio of 1/3. That is, at each iteration, move the point 2/3 of the distance toward the selected vertex (compress the distance

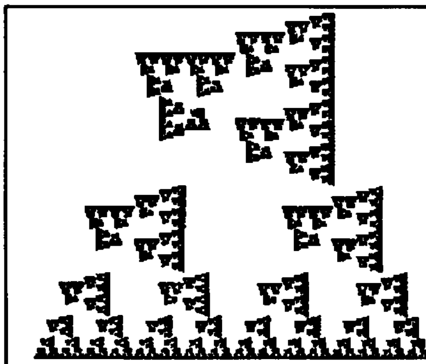


Figure 4 *One 90° rotation*

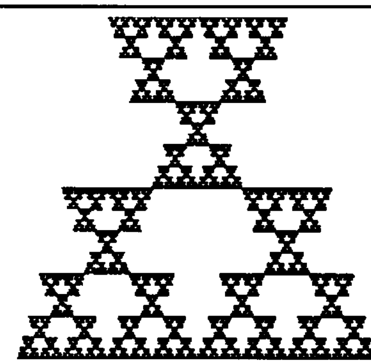
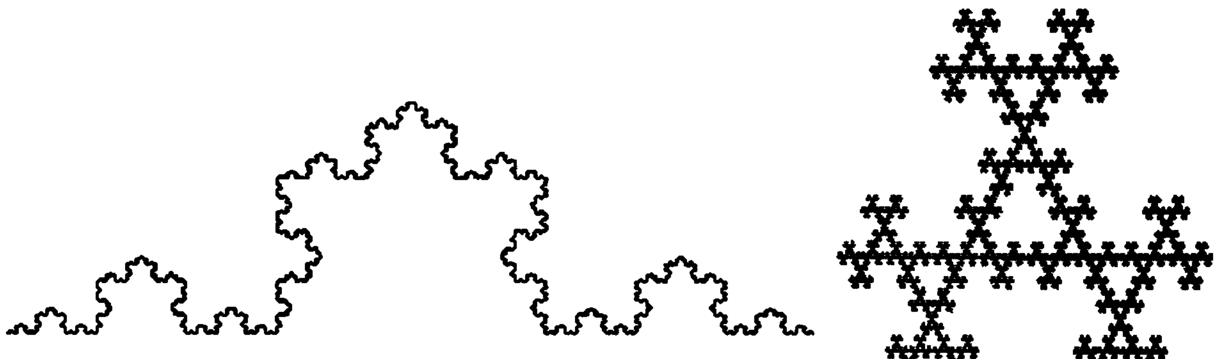


Figure 5 *One 180° rotation*



Figures 6 and 7 *What chaos games were played to produce these images?*

from the traveling point to the vertex by a factor of 3). The resulting image is depicted in Figure 2. Note that this image is comprised of six self-similar copies of itself, each of which is exactly one-third the size of the original. Note also that the internal boundaries of this figure are the well-known curves known as Koch curves. When the chaos game is played using the vertices of a pentagon with a compression ratio of 3/8, the resulting fractal again illustrates these rules (see Figure 3): five self-similar pieces each of which is three-eighths the size of the original.

More interesting geometric images arise when rotations are included in the rules of the chaos game. For example, suppose we again start with three points at the vertices of an equilateral triangle. As before, when either of the lower vertices are chosen, we move the traveling point one half the distance toward the chosen vertex. When the upper vertex is selected, we first move the traveler one half the distance toward that vertex, and then we rotate this point 90° in a counterclockwise direction about the vertex. When this chaos game is played, the resulting image is somewhat different (see Figure 4). We again see three self-similar copies of the entire figure. Each copy is still one half the size of the original. However, the topmost copy is rotated by 90°, while the other two copies are not rotated.

In Figure 5 we display a chaos game where the 90° rotation about the top vertex is replaced by a 180° rotation. Note how the topmost self-similar copy in this figure has been flipped.

In the chaos club, the participants first perform a number of experiments that are designed to familiarize them with the output of the chaos game. Then we ask the students to go backwards. They are asked to identify the rules of the chaos game that were used to produce certain images. As an example, how did we produce the images shown in Figures 6 and 7? That is, how many vertices did we use? Where are they located? What is the compression ratio? And what, if any, rotations were involved?

Once students get the hang of determining the rules of the chaos game from a single image, we ask them to view a movie that has been generated entirely via this method. A printed page is hardly the best format to present a video clip, but Figure 8 is our humble attempt to convey the sense of such a video. How did we create each frame in this movie?

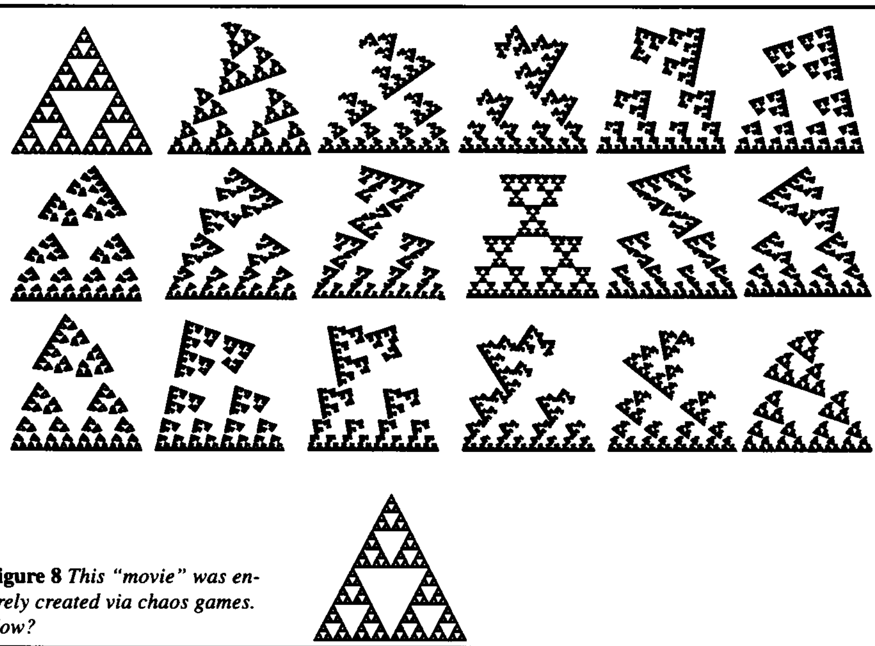


Figure 8 *This "movie" was entirely created via chaos games. How?*

To summarize, experiments such as these tie together a number of ideas from the basic mathematics curriculum, including the geometry and algebra of linear transformations, similarity and self-similarity, and iteration. Other equally important but less often taught topics such as randomness and probability (weighting the vertices differently) emerge too. There is no doubt that these experiments serve to develop students' geometric intuition. These experiments also serve to introduce students to some contemporary ideas such as strange attractors, iterated function systems, and fractal geometry. While fractals have suffered their share of abuse from professional mathematicians over the last few years, it has been my experience that they provide a rich source of important and intriguing mathematical ideas for students to contemplate, as well as a refreshing view of mathematics that they receive in class all to rarely.

An expanded interactive version of this and other papers dealing with activities in the chaos club is available from the Dynamical Systems and Technology project at Boston University via the World-Wide Web at http://math.bu.edu/DYSYS/dysys.html.

Robert Devaney is a professor in the Department of Mathematics at Boston University.

Personal Opinions

The Questionable Probability Theory Behind *The Bell Curve*'s Bell Curve

Miriam Lipschütz-Yevick

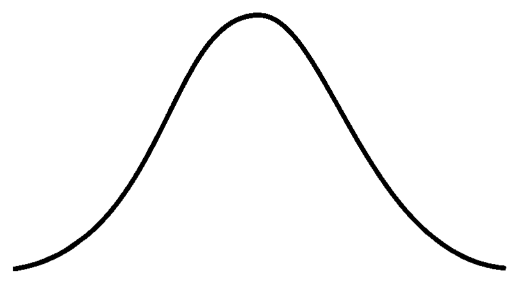
The mathematical underpinnings of Hernstein and Murray's *The Bell Curve* are to be found in the appendices. In the first of these we see a diagram of a few bellshaped (normal) distribution curves with the (scientifically fuzzy) explanation:

"...a common way in which natural phenomena arrange themselves approximately."

The title of the book and the various statistical techniques used do in fact indicate that the authors' interpretation of the observed data assumes that I.Q. is normally distributed in the population. The applicability of many of their statistical methods necessitates that the bellshaped curve prevail. The discussion below explains why a theoretical model based on the conclusions the authors draw from the observed data *will not bring about a bellshaped distribution.*

The normal distribution, even if very prevalent, does not however fall out of the sky. In fact the mathematical criteria needed to produce a normal distribution are not satisfied in the case of the population the authors of *The Bell Curve* hypothesize—a non-homogeneous group in which there is a significant difference between the mean I.Q. of the two groups. The authors cannot have it two ways: either the two population groups—black and white; poor white and middle and upperclass white—are sufficiently homogeneous to generate a bellshaped curve with a common mean, or we are dealing with two distinct populations and the various statistical tests based on the model of a bellshaped curve simply do not apply.

A large number of small, independent , random effects (say, those that combine to generate I.Q.s) may, under certain circumstances, combine to display a collective (statistical) regularity. In particular the sum of a large number of such small random fluctuations may combine into what we call a "stable" limiting distribution law, to which family the bellshaped curve belongs. A good example of when this does happen is the example discussed in *The Bell Curve* of the distribution of the body heights in a class of schoolboys. Similarly a close to bellshaped frequency curve will be observed for the physical sizes in a homogeneous adult population of one gender. There is a reason for this. (For a more detailed discussion see Miriam Lipschütz-Yevick, "Probability and Determinism," *American Journal of Physics*, 1957; a classical and beautiful exposition can be found in the early work *Théorie des Probabilités*, Gauthier-Villars, Paris, 1925, page 103, by the great French mathematician Paul Lévy.)



A Bell Curve.

It so happens that the physical stature of an individual is determined by the sum of the sizes of some two hundred bones making up the skeleton. In a large population of males, say, the small, accidental differences from the mean size—which are caused by a host of environmental and genetic factors—over the whole population for a particular bone fluctuate randomly from individual to individual and quite independently from bone to bone. Some of the bones will be larger than average, some

See *Bell Curve* on page 11

The Strange Story of Newton's Calculus Textbook

Ralph A. Raimi

On an "educational" radio broadcast a while ago, a man billed as a computer expert (hereinafter "the Expert") was explaining about electronic publishing and the Internet. Many books, he explained, will not even appear on paper at all in future years, but will exist catalogued and described in computer network listings, to be sampled or downloaded, maybe to be read at leisure on one's own monitor, or maybe printed on one's home printer. All of which was true, of course, though not exactly news to anyone with experience in computers.

But then he got more interesting. He pointed out that these developments would reduce the role of what today are called publishers, companies like Random House and McGraw–Hill that pay writers and cause their books to be printed, advertised, distributed, and sold. Thinking only of profit, they have often stifled ideas that could change the world, "...and publishers have been making such mistakes ever since one turned down Newton's *Calculus* on the grounds that nobody needed a new calculus book...."

At that point I turned off the radio to record verbatim as much as I could remember of this astonishing thing the Expert had said. These few lines resemble one of those puzzles in the Sunday comics which asks you to "Find all the mistakes in the following picture." Let us count.

Isaac Newton flourished in the last part of the seventeenth century. He was from an early age the most famous scientist in the world. He could publish what he wanted, immediately. He was actually often reluctant to publish his work, and it took a lot of persuasion from his friend Halley to get him to collect and organize the system of mathematical physics he called the

See *Calculus Textbook* on page 11

Calculus Textbook from page 10

Principia, rejected by nobody and welcomed by an expectant scientific world in 1687. Thus

Mistake #1 The Expert's idea that some publisher "turned down" a book of Newton's is ludicrous.

Now what was this famous turndown about? A "calculus book?" In Newton's time there were no calculus books, not even one by Newton. Newton was one of the inventors of a branch of mathematics which these days we call "calculus," but that sort of work was appearing in papers and books—and personal letters—concerned with what the authors actually thought of as geometry or algebra, or scientific matters such as optics, astronomy and alchemy, having a mathematical component.

Ralph A. Raimi

The idea of "a calculus book" is more recent than even Newton's *Principia*, late as that was in Newton's life. The first approximation was that of l'Hopital in 1696, but it was not a textbook in the modern, collegiate sense, and it was one of a kind. A publisher's calculation that "nobody needed another calculus book" could have no meaning before about 1920, when science and engineering students were beginning to grow numerous enough in American colleges to provide a steady market for such texts as a genre. In Europe the notion came even later. Thus

Mistake #2 A sentiment of the form "so-and-so's calculus book was turned down [by some publishers] because they thought

See *Calculus Textbook* on page 12

Bell Curve from page 10

smaller, so that winners are more or less matched equally by losers. Yet even the *largest deviation* from the mean will contribute a negligible part—i.e., be statistically negligible—to the sum of all the individual differences which together determine how physical sizes are statistically distributed over the whole population.

These exactly are the necessary and sufficient conditions—the individual and uniform (collective) smallness of the variations compared to their sum—for the normal distribution to evolve when a large number of small independent random effects, or "errors" conspire together, i.e., sum up, to produce a statistically regular distribution of some "phenotype."

Clearly these conditions would not be satisfied if our population were composed of, say, American males and Japanese females—for the deviations from the mean would not be uniformly small. The result in this case would, most likely, be two-peaked, a bimodal distribution for physical size. And by the same token, *The Bell Curve*'s conclusion that intelligence quotient is distinctly different for the two subpopulations hypothesized, cannot yield a normal distribution with the one subpopulation squeezed into the lower ten percentile. We are, from a theoretical point of view, not in the domain of the normal distribution.

A bellshaped distribution for a phenotype can then be ascribed to a genetic factor only if this factor operates *randomly* and *independently* on each of a large number of genes which conspire together to produce the particular phenotype. And the measure of the factors must be such that the fluctuations in the values of each component are individually and uniformly (i.e., no component deviation is *overwhelmingly* large) negligible against their sum. Once again *The Bell Curve*'s conclusions preclude that these theoretical (mathematical) conditions be satisfied for the distribution of I.Q.s. For *The Bell Curve* concludes that the subpopulation is such that its genotype will systematically land the measure of its intelligence in the lowest ten percentile. The small individual genetically induced components which are summed in this case are neither independent nor randomly distributed in a uniformly negligible manner over the whole population. A bellshaped curve would hence not be statistically generated and empirically observed.

Yet we *do* empirically observe a normal distribution for I.Q.s as well as many other test results. This is compatible with the hypothesis that the normal distribution evolved from a large number of random, independent environmental and genetic fluctuations, whose differences from the mean were individually and uniformly negligible against their total in a single population. Fluctuations whose values lie mainly to the left of the mean (reflecting negative environmental factors) will so sum statistically and similarly for positive variations—collectively producing a bellshaped curve.

The Bell Curve's assumptions (or conclusions as the case may be) could more easily be fitted into another model, that of a non-normal stable distribution. The graphs in the book showing the high values for measurements of achievement for a small group of elite college graduates, etc., are compatible with this model. To wit, when a few of the measures of the component terms contribute a sizable fraction of the sum (so that the components are statistically not uniformly negligible) a highly skewed distribution will evolve. The distribution of the sum will reflect the distribution of its largest term(s) and a sizable part of the total distribution will be concentrated in the upper tail end of the curve. Such, for instance, is the distribution of wealth and income in most present-day societies. Such too is the distribution of scientific, intellectual, or artistic achievements, where a minute fraction of practitioners makes most of the major contributions.

In view of the sloppy theoretical underpinnings of Murray and Hernstein's book, it is doubtful that the measure of these two scholars' achievements would be located at the extreme upper end of such a non-normal stable distribution curve. Let us remember that it has been the hallmark of contemporary authoritarian and racist-theory-inspired governments to eliminate the true intellectual elite (those at the upper end of the distribution) and their creations in short order (vide Nazi Germany, Stalinist Russia, Cambodia, Bosnia, Rwanda...).

Miriam Lipschütz-Yevick, now a retired professor of Rutgers, has published a number of papers related to stable probability limit laws.

Letter to the Editor

Editor,

I could hardly believe my eyes as I read Keynes and Wicklin's description of science museums [FOCUS, February 1995, page 1]. "Just a few years ago...the exhibits conveyed to all visitors that science was rigid, boring, and hardly accessible to the general public. Fortunately times have changed."

Setting up a phony contrast is, of course, the staple of hack journalists, who seem to believe that all important ideas and events have originated in the last few months. One expects better of an MAA publication.

In 1950 (more than "just a few years ago"), when I was a graduate student at Caltech, I visited the Griffith Planetarium and Museum in Los Angeles. One of the exhibits was arranged so that, by pressing a button, a large number of pellets were released down a chute, behind a glass partition; as they fell, the pellets bounced off an array of rounded pegs and finally collected into a number of vertical slots. The heights of the columns of pellets gave a beautiful illustration of the Gaussian distribution. By pressing the button again, the whole apparatus rotated 180 degrees and the pellets fell again, but—surprise—the pegs now had slanted tops, and the heights of the columns formed a skew distribution. The demonstration was so impressive that, as you can see, I remember it vividly 45 years later.

In the last 25 years I have visited the Boston Science Museum many times, and found it full of interactive exhibits, many of them intriguing, quite accessible, and not at all boring.

By badmouthing the museums of yesteryear, the authors have achieved nothing except to exhibit their own ignorance.

Sincerely,

Paul G. Comba
Prescott, Arizona

Editor's note: Three other writers made essentially the same point as the writer of the above letter. Rick Wicklin and Harvey Keynes reply:

Our opening paragraph was intended to describe a typical or average case scenario. Dr. Comba correctly points out that there have been shining examples of interesting interactive museum exhibits for many years. Museums in several large cities have consistently done an excellent job of engaging visitors, and we regret that our remarks were interpreted as being disparaging towards them.

Calculus Textbook from page 11

there were already enough calculus books" describes nothing that could have happened before the twentieth century.

In passing, and in particular, let us include

Mistake #3 It is false to say Newton ever wrote a *Calculus*.

Finally, to put the matter into an even wider context, the whole story as told by the Expert would never have been invented by him had he not himself assumed his audience to believe a popular and mischievous axiom of twentieth century journalism, which is that geniuses are generally misunderstood and not honored in their time. Otherwise he would not so readily have accepted (or invented) the idea that a publisher would turn down a Newton "calculus book." Newton was a genius; geniuses are not recognized in their time; therefore Newton, unrecognized, goes unpublished while some silly nephew of the publisher gets printed instead. Stands to reason.

But this picture of unrecognized genius is false. Sure, there are rare cases of it: Van Gogh and Kafka come to mind. In the world of mathematics, however, only angle-trisectors and circle-squarers seem to suffer from such neglect. Like Archimedes and Michelangelo before him, and Beethoven and Einstein to follow, Newton was from the beginning an intellect to conjure with. Thus

Mistake #4 Contrary to the Expert's unspoken assumption, geniuses are generally known as such during their own times.

The Expert probably didn't worry too much about whether his story was true or not; he figured it could be true, which was good enough. The story illustrated what to him seemed an important truth: that publishers can make mistakes in judging the value of a new manuscript and that electronic publishing will help prevent such mistakes from holding up progress. So the story contributes to real truth as the Expert saw it and that's the only kind of truth that counts.

This is a common excuse. The literal truth, they often think, will be misunderstood anyhow; people will only be confused by it. What would you rather have on a noon-time radio science spot: an unimportant fact about the way mathematical publishing worked in the year 1696 or an important, deep truth about the evils of commercial publishing and how computers will help avoid that? Maybe it wasn't exactly Newton; it was bound to have happened to somebody. Who really cares?

So in pursuit of the higher truth, our expert conjured up a seventeenth century populated by students of "calculus," to whom a publishing industry was evidently supplying a rich choice of textbooks. Within that seventeenth century he conjured up a *Calculus* written in vain by a Newton whose works, like those of most geniuses, were ignored by practical people. It is hard to pack so much misinformation into a few words, but the Expert did it.

Ralph Raimi is a professor in the Department of Mathematics at the University of Rochester in Rochester, New York.

Sign of the times?

The FOCUS desk could not resist passing on the following joke, in the age old spirit of laughter being a good way to help deal with adversity.

A mathematician goes into a fast food restaurant to apply for a job. The manager asks, "What is your education?"

"Well, I have a Master's degree in mathematics."

"Hmph. All of our mathematicians have Ph.D.s."

The MAA Awards Structure: Is It Appropriate?

Henry L. Alder

Does the MAA's awards structure include all activities for which awards should be made? Are there MAA awards for which the criteria should be changed? Does the MAA perhaps offer too many awards?

On these and similar questions, the Coordinating Council on Awards has decided to seek the membership's advice.

The following background is designed to help you to offer an informed opinion: The Strategic Plan approved by the Board of Governors states that "Awards signal to our members and to the public what we value—teaching, writing, student accomplishments—and serve as a platform on which to build the public message of our goals and objectives."

At present the MAA offers the following awards annually.

Teaching Section Awards for Distinguished College or University Teaching of Mathematics, the national Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching of Mathematics, and the Edyth May Sliffe Awards for distinguished High School and Junior High School Mathematics Teaching.

Writing Expository writing awards for articles in each of the MAA's journals (the *American Mathematical Monthly*, the *College Mathematics Journal*, *Mathematics Magazine*, and *Math Horizons*), the Merten M. Hasse Prize for a noteworthy expository paper by a younger mathematician (awarded only in odd-numbered years), the Beckenbach Book Prize awarded for a distinguished, innovative book published by the MAA, and the Chauvenet Prize awarded for an outstanding article on a mathematical topic by a member of the MAA.

Service Certificates of Meritorious Service, which are sectional awards given at the national level.

Student Accomplishments The Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student (awarded jointly with the AMS and SIAM).

In addition, the MAA confers the Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics, the most prestigious award made by the Association.

On special occasions the MAA also awards a Certificate of Merit to individuals having given outstanding service to the mathematical community and/or the MAA, but not fitting any of the awards regularly made by the MAA.

The Coordinating Council on Awards reviewed the MAA's awards structure at its meeting on January 16, 1993 in San Antonio. At that time, the council encouraged the establishment of the Edyth May Sliffe Awards for junior high school mathematics teachers and the awards for outstanding research in mathematics by undergraduates (both of which have since been implemented). It also considered awards for public service and public understanding of mathematics, but decided against both of these as they would duplicate awards already made by the AMS and the Joint Policy Board for Mathematics, respectively.

The council again reviewed the MAA awards structure at its meeting on January 5, 1995 in San Francisco and at that meeting decided to solicit input from the membership on the questions raised at the beginning of this article.

Please let us have your opinion on these and related questions by responding to Henry L. Alder, Chair, Coordinating Council on Awards, Department of Math, University of California, Davis, CA 95616; e-mail: hlalder@ucdavis.edu.

Henry L. Alder is a professor of mathematics at the University of California at Davis and chairs the MAA's Coordinating Council on Awards.

Call for Nominations for the Yueh-Gin Gung & Dr. Charles Y. Hu Award

The MAA's highest award is the Yueh-Gin Gung & Dr. Charles Y. Hu Award for Distinguished Service to Mathematics. The committee which recommends the winner for this award would appreciate nominations from the MAA membership. The award recognizes a variety of forms of service over an extended period of time, or a single form of service in a limited period. Please send recommendations for consideration of the committee to Henry O. Pollak, 40 Edgewood Road, Summit, NJ 07901. Suggestions for the award in January 1997 should be received by October 1, 1995, and should be accompanied by a paragraph outlining the basis for the recommendation.

Carnegie Corporation Awards MAA Third SUMMA Grant

The SUMMA program continues to encourage college and university mathematics faculty to initiate or replicate intervention projects for minority middle and high school students. The Carnegie Corporation of New York has now funded a third two-year grant for this project in the amount of $376,000. Since May 1991, SUMMA has awarded sixty-three small planning grants totalling $250,000. Thirty-five mathematics-based intervention projects have been established on both two- and four-year college and university campuses in twenty-two states. These now serve 1300 students, including 1100 minority students. With SUMMA serving as a catalyst and reviewer of proposals, the projects have been able to raise more than $8.2 million in additional public and private support.

Advising Undergraduates About Graduate School in Mathematics: Methods that Work

Diane Herrmann and David Lutzer

This is the second in a series of articles growing out of the work of an MAA committee on various aspects of advising undergraduate mathematics students. The previous article, "Career Advising for Mathematical Sciences Majors," appeared in the October 1994 issue of FOCUS (page 3). Please send comments to David Lutzer at djlutz@mail.wm.edu.

In this article, the question is not whether too many mathematics students go to graduate school or whether sending students to graduate school is a mark of a good undergraduate mathematical sciences program. Instead, as part of the report from the MAA's Committee on Advising, we pose a more operational question: "What can advisors do to support those undergraduates who might be interested in graduate study?"

Information—Early and Often

Providing adequate information is the first component in advising graduate-school-oriented students. Freshmen and sophomores need to know the range of options available to mathematical sciences majors after graduation, including both post-baccalaureate employment and graduate study. In addition they need to know what to do today in order to keep open the graduate school option, just in case they subsequently decide that further mathematics study is their cup of tea. (For information about what today's graduate schools want from entering students, see "Survey of Preparation for Graduate School," by Richard Neidinger, in the September 1988 issue of FOCUS.) Freshmen and sophomores also need to hear that financing mathematics graduate school is not likely to be a problem because teaching assistantships and fellowships are widely available.

Juniors and seniors need more specific information that is easily accessible. Departments having a mathematics library or coffee room can set aside a corner of it as a graduate school resource area. Starting a collection of graduate school information is not hard. The American Mathematical Society's annual publication on fellowships and assistantships is easy to obtain—it's free, provided one pays one's AMS dues early. In addition, this year's seniors will apply to many graduate schools and will eventually choose just one. They are usually happy to donate their surplus graduate catalogs and brochures to a departmental library. Other easy sources of information for the department's graduate information corner include the university's own career office and the department's visiting speakers—if asked, visitors are usually happy to bring along information about their own graduate programs.

Personalized Information

Information is only the first step, and successful advising programs add personal contact as a second ingredient in their support for graduate-school-oriented students. For example, in addition to bringing information about their own graduate programs, visiting speakers can be asked to spend an hour having coffee and cookies with undergraduate mathematics majors, talking about graduate school options at their home departments. A department can also ask its own junior faculty members to share their recent graduate school experiences with undergraduates. In addition, one's own bachelor's graduates from the last few years who went to various graduate schools can be called upon to discuss their graduate school experiences. Perhaps these alumni can return at homecoming to talk to this year's mathematics club, or perhaps the department can offer to pay for a few long distance calls from groups of today's seniors to recent graduates. If that isn't possible, then perhaps e-mail contacts between this year's seniors and previous bachelor's graduates can be arranged.

Mathematics in Other Disciplines

Mathematics is an excellent foundation for graduate study in other areas, but making those options known to mathematics students is a special advising challenge. Some mathematics departments have found that faculty from other disciplines are willing to talk to mathematics majors about graduate school options in such fields as biology, economics, physics, chemistry, electrical engineering, computer science, etc. Partially as a result of such cross-departmental advising, one prominent U.S. mathematics department sends a quarter of its large baccalaureate class to graduate schools in other arts and sciences disciplines.

Undergraduate Research to Prime the Pump

Some departmental advisors have begun to use Research Experiences for Undergraduates (REU) as a hands-on supplement for advising graduate-school-oriented students. Participation in one of the National Science Foundation's REU programs often confirms the student's desire to attend graduate school and can provide helpful reference letters, too. In some cases, colleges organize their own summer REUs, particularly if they have a large number of international students (who are not eligible to participate in NSF-funded REU programs). And if formal REUs are not available, then mathematically-oriented summer jobs and internships can serve many of the same functions, particularly for students with strong interest in applied mathematics.

Some departments have found that they can call upon their own bachelor's graduates who are now faculty members at research universities or members of corporate research teams to support the department's current undergraduates in summer research settings. This possibility alone makes it worthwhile to maintain contact with one's mathematics majors, whether they go on to graduate school or into industry.

Peer Support Groups

In a given year, seniors who want to apply for graduate school can act as a support group for each other, but bringing that support group into being often requires departmental intervention. The department might start by offering preparation sessions for the GRE advanced mathematics examination or for some of the actuarial examinations. Old tests are available commercially and can serve as the basis for review sessions, and a cumulative review of undergraduate mathematics is worthwhile even if a student does not, in the end, take actuarial exams or the GRE.

If the department has a graduate program, then its own graduate students can provide support for the department's current crop of seniors, but only if the department can break down the barriers between its seniors and its graduate students. Departmental teas to which both groups are invited is one way to start.

Spring term of a student's senior year is an anxious time for graduate school applicants, and other students can help an applicant with the stress of waiting and decision making. In addition the department can use its contacts with former students who went to graduate school elsewhere to assist students in choosing between admissions offers.

Tracks Within the Major

The final component of a successful advising program for graduate-school-oriented students is the care with which the department designs the track system (if there is one) in its mathematical sciences major. Track systems are inherently advising tools and, unfortunately, some departments have created systems that encourage undergraduates to decide far too early whether or not they are interested in graduate study in the mathematical sciences. If a department's track system encourages freshmen and sophomores to make that kind of decision, then it is time for a curriculum change.

Conclusion

Today some departments see it as their responsibility to actively encourage students to pursue graduate study in mathematics, while others take a more passive view of graduate school preparation, aiming primarily toward post-baccalaureate employment for their majors. Whatever one's perspective on this issue, there are easy, concrete steps that advisors can take to assist graduate-school-oriented students in exploring the options that are open to them. Surely these steps toward training the next generation of mathematicians are part of what a recent CUPM report described as "a special shared responsibility of all undergraduate and graduate teachers and advisors" (*The Undergraduate Major in the Mathematical Sciences, a CUPM Report*, MAA, 1991).

Diane Herrmann teaches at the University of Chicago and is a member of the Committee on Advising. David Lutzer is Dean of Arts and Sciences at the College of William and Mary and chair of the Committee on Advising.

Graphing Calculator Short Course

July 10–14, 1995

The Columbus State Community College Mathematics Department in cooperation with the Ohio Mathematics Association of Two-Year Colleges (Ohio MATYC) will again host a graphing calculator short course organized by Bert Waits and Frank Demana from the Ohio State University. College and high school mathematics faculty are invited to attend.

Themes include precalculus and calculus using the TI-85, developmental mathematics through college algebra using the TI-82, using the CBL to collect and analyze data, and a demonstration of the TI-XX (the next generation super calculator).

For information, contact Ed Laughbaum at Columbus State Community College, 550 East Spring Street, Columbus, OH 43215; (614) 227-5308; e-mail: elaughba@cougar.colstate.cc.oh.us.

Texas Instruments will offer attendees reduced prices on the TI-85 ($65) and TI-82 ($60).

Revitalizing Introductory Mathematics

Project Kaleidoscope Workshop

Mount Holyoke College

August 11–13, 1995

In the past decade remarkable changes have occurred in college mathematics. A national effort to reform the teaching of calculus has been tremendously successful and influential. The availability of fast and inexpensive computers has changed the nature of instruction in linear algebra, probability, and statistics. The mathematics laboratory, almost unheard of a few years ago, is quickly becoming as significant for mathematics instruction as it is for the natural sciences. Writing is also becoming an increasingly important part of the curriculum. The Project Kaleidoscope Workshop at Mount Holyoke College will delineate and summarize some of these changes and offer mathematics faculty guidance in revitalizing introductory curricula to be accessible, engaging, and intellectually rigorous. Through case studies, plenary presentations, and small group sessions, participants will learn about proven successful approaches, programs that work, and how they can be adapted and adopted in other settings. (Project Kaleidoscope is an informal alliance of individuals, institutions, and organizations committed to strengthening the nation's undergraduate science and mathematics community. Now in its fifth year, Project Kaleidoscope continues to identify creative people and strong programs in science and mathematics departments across the country.) Three-person institutional teams are invited to apply to the workshop. For more information, see the PKAL Gopher at bbs.augsburg.edu or contact Project Kaleidoscope, 1730 Rhode Island Ave, NW, Suite 1205, Washington, DC 20036; voice: (202) 232-1300; fax: (202) 331-1283; e-mail: p00274@psilink.com.

Events at DIMACS, Rutgers University

June 7–9: MiniWorkshop on Groups and Computation - II, Rutgers University, Piscataway, NJ

August 2 & 3: DIMACS Special Year in Mathematical Support for Molecular Biology: Mini Workshop on Geometrical Methods for Conformational Modeling, Rutgers University, Piscataway, NJ

August 14–18: DIMACS Special Year on Logic and Algorithms: Tutorial on Computer-Aided Verification, Rutgers University, Piscataway, NJ

August 20–25: DIMACS Special Year on Logic and Algorithms: Tutorial on Finite Model Theory, Rutgers University, Piscataway, NJ

August 28–September 1: DIMACS Special Year on Logic and Algorithms: Tutorial on Proof Complexity, Rutgers University, Piscataway, NJ

September 13–15: DIMACS Special Year in Mathematical Support for Molecular Biology: Algorithm Implementation Challenge Workshop, (DNA Sequence Determination from Shotgun Sequence Data), Rutgers University, Piscataway, NJ

September 21–22: DIMACS Special Year in Mathematical Support for Molecular Biology: Mini Workshop on the Mathematics of Drug Discovery, Rutgers University, Piscataway, NJ.

October 13–15: DIMACS Special Year in Mathematical Support for Molecular Biology: MiniWorkshop on Gene-Finding and Gene Structure Prediction (to be held at the Genome Center at the University of Pennsylvania)

October 15–17: DIMACS Special Year on Logics and Algorithms: Workshop on Verification and Control of Hybrid Systems, Rutgers University, Piscataway, NJ

Workshops held at DIMACS, Rutgers University. For more information, contact Pat Toci; (908) 445-5930; e-mail: toci@dimacs.rutgers.edu; or DIMACS Center at (908) 445-5928; e-mail: center@dimacs.rutgers.edu; WWW: http://dimacs.rutgers.edu/; TELNET: telnet info.rutgers.edu 90.

Updates for Mathfest 1995

Burlington, Vermont, University of Vermont

August 6–8, 1995

The Joint Meetings Committee which bears the financial responsibility for these meetings has made decisions to reduce costs by eliminating some traditional services at Mathfests and making substitutions for some others. We hope that participants will support these changes as a way to make the Mathfest financially viable.

Updates to Program

Association for Women in Mathematics: The title of the panel discussion is *Do women and men have different career trajectories?*

National Association of Mathematicians: The David Blackwell Lecture will be given by Donald F. St. Mary, University of Massachusetts, Amherst.The title of the lecture is *Computational Ocean Acoustics.*

Miscellaneous

Badge/Program Distribution: Those participants who register in advance and elect **not** to receive their badges and programs by mail before the Mathfest will be given their badges and programs upon check-in at the university residence hall. All other participants who do not receive these items in the mail must come to the Mathfest Registration Desk in Billings Hall to pick them up.

University Housing: Participants who desire university residence hall accommodations are advised to register for them in advance. If there are any rooms available on site for walk-ins, they will be sold as room only with no meal package. Breakfast and lunch will be available on a very limited cash basis in the dining halls; several eating establishments are within walking distance. Prices for on-site dormitory rooms (if available) are $22/single; $18/double (per person).

Audio-visual equipment: There will be one overhead projector and screen in session rooms. Unfortunately, no extra equipment will be available. Speakers should plan to bring any special equipment with them.

Dinner/Cruise: Unfortunately there was an error in the prices listed on the advance registration form for this event in the April issue. This issue includes the advance registration form with the correct prices. We regret the error.

Registration for MAA Student Workshop on the Internet: Students interested in this session should send e-mail to jheckler@maa.org to reserve a space. There is no charge.

Transparencies: There will be no facility for preparing or purchasing transparencies on site. Speakers should prepare them in advance and bring them to the meeting.

The Visiting Lecturers Brochure, 1995-96 Academic Year, is now available on the MAA Gopher.

To access the MAA Gopher with a gopher client, type: gopher gopher.maa.org

Foundation Seeks Proposals for Programs for Women and Girls in Mathematics

The Tensor Foundation has announced a new program to encourage women and girls in mathematics. Five grants of $5000 each will be made in August 1995 for student-centered projects conducted by high school or college mathematics faculty, to begin in the academic year 1995–96. The deadline for proposals is July 26, 1995. A Request for Proposals containing guidelines for proposal preparation and submission is available on the MAA Gopher or from the Mathematical Association of America, 1529 18th St, NW, Washington, DC 20036; (800) 331-1MAA; e-mail: maahq@maa.org.

Burlington Mathfest

Advance Registration

Personal Information

Name ______________________________

Mailing Address ______________________________

____

Telephone ______________ e-mail ______________

Badge Information

Affiliation ______________________________

(Please limit affiliation to 35 characters – one line only)

Name to appear on badge ______________________________

Guest Badge ______________________________

Membership ✓ all that apply

- AMS ☐
- CMS ☐
- MAA ☐
- PME ☐
- AWM ☐
- MR field of interest ____

If you do not wish your program and badge to be mailed to you on July 12, place a check in the box. ☐

Registration Fees

Mathfest	**by July 14**	**at meeting**
☐ Member AMS, CMS, MAA, PME	$125	$163
☐ Nonmember	$194	$252
☐ Graduate Student	$35	$45
☐ Undergraduate	$20	$26
☐ High School Student	$2	$5
☐ Unemployed	$35	$45
☐ Temporarily Employed	$95	$120
☐ Third World Fee	$35	$45
☐ Emeritus Member of AMS or MAA	$35	$45
☐ High School Teacher	$35	$45
☐ Librarian	$35	$45
☐ One-day Member	—	$89
☐ One-day Nonmember	—	$139
☐ Exhibitor	—	$0

Event Tickets

Event	**# Tix**	**Price Per**	**Total**
Opening Banquet			
Regular	____	$25	____
Vegetarian	____	$25	____
Dinner and Cruise			
Adult	____	$32	____
Children 6–11 yrs	____	$26	____
PME Banquet			
Members and Families	____	$10	____
Nonmembers	____	$17	____
MAA 25-year Banquet			
Regular	____	$25	____
Vegetarian	____	$25	____

TOTAL for Event Tickets $ ____

See separate tour form in this issue.

Deadlines

Advance Registration/Residence Hall Reser.	June 15, 1995
Hotel changes/cancellation thru MMSB	July 5, 1995
Final Advance Registration (no housing)	July 14, 1995
90% Refund on Residence Hall Package	July 25, 1995*
50% Refund on Events	July 31, 1995*
50% Refund on Registration Cancellation	August 4, 1995*

*no refunds after this date

Total Payment

Category	**Total**
Registration Fee(s)	____
Event Tickets	____
Dorm Payment or Hotel Deposit	____
TOTAL Amount Due	$ ____

Make checks payable to the AMS. Canadian checks must be marked "U.S. Funds". You may charge this total to your VISA or MasterCard.

Card Number: ______________________________

Card Type: ______________ Expiration Date: ______________

Signature: ______________________________

Name on Card: ______________________________

Please complete this form and return it to:

Mathematical Meetings Service Bureau (MMSB)
P. O. Box 6887
Providence, Rhode Island 02940
U.S.A
401-455-4143 or 1-800-321-4267 x 4143

For Office Use Only

Codes: ______________

Options: ______________

Hotel: ______________

Dates: ______________

Dorm: ______________

Room Type: ______________

Hotel Deposit: ______________

Dorm Payment: ______________

TOTAL Amt. Paid: ______________

Room/Board Paid: $ ______________

Room/Board Due: $ ______________

Remarks: ______________

Burlington Mathfest

Accommodations

General Information

Where applicable, please check off one of the following

☐ I will be making my own reservations. Name of hotel or motel: ______________________

☐ I live in the area or will be staying privately with family or friends.

☐ I plan to share a room with ______________________, who is making our reservations.

University Reservations

Full prepayment for room and board is required. *Purchase of a room and board package (breakfast and lunch) is required, and it is included in the rates listed below.* All rates are per person. Mathfest participants may occupy the residence halls from Saturday, August 5 to Wednesday, August 9 only. Acknowledgment of your residence hall reservations will be sent to the address indicated on the reverse side of this application. Please mark applicable rates listed below and enter the totals where applicable. There is no children's rate, but sleeping bags are allowed.

Special Requests:

____

____

____

____

____

____

Description	# Staying	At	# of Days	Total
Single	______	$36 x	______	______
Double	______	$32 x	______	______
TOTAL				**$** ______

Rates are per person.

Please ✓ all that apply

- Male ☐
- Female ☐
- Smoker ☐
- Nonsmoker ☐

Date and Time of Arrival: ______________________

Date and Time of Departure: ______________________

Names of Other Occupants Arrival Date Departure Date

______________________ Child? (give age) ______

______________________ Child? (give age) ______

______________________ Child? (give age) ______

Majority of rooms will be assigned in the Living/Learning Complex; Marsh/Austin/Tupper will be used for overflow.

Hotel Reservations

Please indicate type of room:

☐ Single $89
☐ Double $89
☐ Triple $89
☐ Quad $89

Special Requests:

____

____

____

To guarantee a room, please Include $89 by check or provide a credit card number.

☐ Deposit enclosed ☐ Hold with my credit card Card Number ______________________ Exp. Date ______

Date and Time of Arrival: ______________________

Date and Time of Departure: ______________________

Names of Other Occupants Arrival Date Departure Date

______________________ Child? (give age) ______

______________________ Child? (give age) ______

______________________ Child? (give age) ______

All hotel reservations are for the Sheraton Burlington at 870 Williston Road, Burlington, VT.

MAA Contributed Papers in Orlando

The Mathematical Association of America and the American Mathematical Society will hold the Joint Annual Meetings Wednesday, January 10, 1996 through Saturday, January 13, 1996 in Orlando, Florida. The complete meetings program will appear in the October 1995 issues of FOCUS and the AMS *Notices*. This preliminary announcement is designed to alert participants about the MAA's contributed papers sessions and their deadlines.

Please note that the days scheduled for these sessions remain tentative. The organizers listed below solicit contributed papers pertinent to their sessions; proposals should be directed to the organizer whose name is followed by an asterisk (*). For additional instructions, see the Submission Procedures column on page 22.

Sessions generally must limit presentations to ten minutes, but selected participants may extend their contributions up to twenty minutes. Each session room contains an overhead projector and screen; blackboards will not be available. You may request one additional overhead projector, a 35mm slide projector, or a 1/2-inch or 3/4-inch VHS VCR with one color monitor. Persons needing additional equipment should contact, as soon as possible, and prior to October 23, 1995: Donovan H. Van Osdol, Department of Math, University of New Hampshire, Durham, NH 03824; e-mail: dvanosdo maa.org.

My Favorite ODE Solver and Why

Friday morning and Saturday afternoon

Courtney Coleman*
Harvey Mudd College
301 East 12th St
Claremont CA 91711-5990
(909) 621-8023
e-mail: coleman@hmc.edu
fax: (909) 621-8366
Robert Borrelli, Harvey Mudd College

This session will accept suitable papers from people who have used ODE solvers in introductory courses in ordinary differential equations. The focus will be on solvers that are available to college teachers and students either for free or for a reasonable fee. The speakers will say why they like the solver, what it can do for the introductory ODE course, something about the minimal hardware requirements, and give examples of experiments in ODEs that use the solver to advantage.

Assessment of Student Learning for Improving the Undergraduate Major in Mathematics

Friday morning and Saturday afternoon

William Marion*
Valparaiso University
Dept of Math and Comp Sci
Valparaiso IN 46383
(219) 464-5422
e-mail: bmarion@exodus.valpo.edu
fax: (219) 464-5065
Barbara T. Faires, Westminster College

In January 1995 the MAA's Committee on the Undergraduate Program in Mathematics (CUPM) adopted a document prepared by its Subcommittee on Assessment titled "Assessment of Student Learning for Improving the Undergraduate Major in Mathematics." A draft of the document had circulated within the mathematics community during 1994. The Subcommittee on Assessment now wants to collect specific examples of assessment programs in undergraduate mathematics departments. We invite papers on such programs—ones that are still in the planning stage, have just begun, or are ongoing.

Standards for Introductory College Mathematics Courses Before Calculus

Wednesday morning and Thursday afternoon

Gregory D. Foley*
Sam Houston State University
Div of Math and Info Sci
Huntsville TX 77341-2206
(409) 294-3708
e-mail: mth_gdf@shsu.edu
fax: (409) 294-1882
Jon Wilkin, Northern Virginia Community College

In the autumn of 1995 the American Mathematical Association of Two-Year Colleges will publish *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus.* This document is aimed at breathing new life into a too often neglected part of the undergraduate mathematics program—freshman or pre-freshman courses that either provide a foundation for further mathematical study or serve as terminal courses for students in liberal arts, humanities, social science, teacher preparation, or two-year technical programs. We invite papers that offer innovative content and approaches for courses at this level.

Planning Reformed Calculus Programs: Experiences and Advice

Thursday morning and Friday morning

Martin E. Flashman*
Humboldt State University
Dept of Math
Arcata CA 95521
(707) 826-4950
e-mail: flashman@axe.humboldt.edu
fax: (707) 826-3140

Changing calculus programs involves more than adopting textbooks or technology. The CUPM subcommittee Calculus Reform and the First Two Years (CRAFTY) is sponsoring this session so departments planning changes can share experiences and advice. We invite papers reporting on the process or results of planning calculus reform. Possible topics include connections with client disciplines and administration; articulating goals; pedagogy, technology, and topic selection; resource considerations; preparations for change; and continuing program assessment.

Innovations in Teaching Linear Algebra

Wednesday afternoon, Thursday evening, and Friday afternoon

Donald LaTorre*
Clemson University
Dept of Math Sci
Clemson SC 29634-1907
(803) 656-3437
e-mail: latorrd@clemson.edu
fax: (803) 656-5230
Steven J. Leon (ATLAST), University of Massachusetts at Dartmouth; David C. Lay (LACSG), University of Maryland

The teaching of undergraduate linear algebra is undergoing substantial change. This session invites papers on personal experiences with innovations in teaching linear algebra, including: (1) the creative use of computer algebra systems, supercalculators, or computer software; (2) experiences with the NSF-funded ATLAST summer workshops; (3) experiences with the core curriculum recommended by the Linear Algebra Curriculum Study Group (LACSG); (4) "gems" of exposition in linear algebra; and (5) other innovative teaching initiatives in undergraduate linear algebra.

Active Learning Strategies for Statistics and Probability

Wednesday morning and Thursday afternoon

Allan J. Rossman*
Dickinson College
P.O. Box 1773
Dept of Math
Carlisle PA 17013-2896
(717) 245-1690
e-mail: rossman@dickinson.edu
fax: (717) 245-1690
Mary R. Parker, Austin (Texas) Community College

We invite presentations on the active engagement of students in the process of learning about statistics and probability. These learning strategies may include projects, hands-on activities, experiments, writing, computer exercises, and open-ended questions about real data. Issues might include the incorporation of these into the course, effects on students' achievements and attitudes, and their use in courses other than the introductory course, such as mathematics for liberal arts, precalculus, and mathematical statistics.

The Scholarship of Humanistic Mathematics

Thursday morning and Saturday afternoon

Alvin White*
Harvey Mudd College
301 East 12th St
Claremont CA 91711-5990
(909) 621-8867
e-mail: awhite@hmc.edu
fax: (909) 621-8366
Joan Countryman, The Lincoln School; Harald Ness, University of Wisconsin Center

The idea that teaching, learning, and creating mathematics issues from the same psyche as literature, aesthetics, and music has attracted, since 1986, a worldwide network of people who share that idea. The existence of the Humanistic Math Network has encouraged individuals and groups to try new approaches to teaching, doing, and publishing. We invite papers that describe these new approaches including projects, poetry, and stories.

Chaotic Dynamics and Fractal Geometry

Wednesday morning and Thursday afternoon

Denny Gulick*
University of Maryland
Dept of Math
College Park MD 20742
(301) 650-1443
e-mail: dng@math.umd.edu
fax: (301) 314-0827
Jon Scott, Montgomery College

During the past few years, chaotic dynamics and fractal geometry have gained prominence in mathematics and in applications. The goal of this special session is to promote these fascinating subjects. We invite papers on topics related to either chaotic dynamics or fractal geometry. The papers need to have an expository flavor.

Constructivism Across the Curriculum

Wednesday morning and Thursday afternoon

David M. Mathews*
Longwood College
201 High St
Farmville VA 23909
(804) 395-2184
e-mail: dmathews@math.purdue.edu
fax: (804) 395-2635
Keith E. Schwingendorf, Purdue University North Central

We invite papers describing undergraduate courses in mathematics or statistics that reflect the use of constructivist learning theory. Papers may address courses for elementary education majors, liberal arts or other special groups of students, or courses for mathematics majors. Content areas might include algebra and trigonometry through calculus to linear algebra, analysis, abstract algebra and beyond. The goal of this session is to disseminate innovative, successfully class-tested pedagogical techniques and classroom strategies that reflect the constructivist theoretical view of how mathematics is learned.

Creating an Active Learning Environment: Preparing Pre-Service Teachers

Thursday morning and Saturday morning

Hubert J. Ludwig*
Ball State University
Dept of Math
Muncie IN 47306-0490
(317) 285-8680
e-mail: 00hjludwig@bsuvc.bsu.edu
fax: (317) 285-1721
Kay Meeks Roebuck, Ball State University

Pre-service teachers must be prepared to create classroom environments in which students are active participants in the process of learning mathematics. Active learning techniques include the utilization of appropriate technology, discovery learning, and using concrete models. We invite papers related to the preparation of pre-service teachers for the utilization of active learning techniques in their teaching, particularly those describing courses and/or projects designed to prepare pre-service teachers to use computers as a teaching tool.

Interactive Mathematics Texts in the Classroom—A MathKit Perspective

Saturday afternoon

James E. White*
Institute for Academic Technology
University of North Carolina, Chapel Hill
2525 Meridian Pkwy, Ste 400
Durham NC 27713
(919) 405-1926
e-mail: jimw@iat.mhs.unc.edu

Starting in 1992 the MAA sponsored the Interactive Mathematics Text Project, a project jointly funded by IBM and the NSF. During the project, an authoring environment called MathKit was developed in collaboration with scores of mathematics and science teachers. MathKit includes computer algebra and graphics in an intuitive scripting language called MathScript. To date, teachers (and their students) have created over three hundred interactive mathematics and science "WorkBooks" in MathKit. In this session, we invite teachers to report on various aspects of student use of MathKit WorkBooks, both in classrooms and at home.

Teaching Mathematics by Blind Instructors or to Blind Students

Wednesday afternoon

Norberto Salinas*
University of Kansas
Dept of Math
Lawrence KS 66045-2142
(913) 864-3909
e-mail: norberto@kuhub.cc.ukans.edu
fax: (913) 864-5255

The theme of this session is the use of alternative techniques by blind instructors to teach mathematics to sighted students, and the use of adaptive equipment and high technology in the learning process. Presentations will include demonstrations of these techniques.

Innovations in Teaching Precollege Algebra Courses

Friday morning

Mohammad H. Ahmadi*
University of Wisconsin
Dept of Math and Comp Sci
Whitewater WI 53190
(414) 472-5175
e-mail: ahmadim@uwwvax.uww.edu
fax: (414) 472-5238

This session focuses on courses ranging from basic to college algebra. Presentations are invited on personal experiences in teaching such courses using innovative instructional techniques such as (but not limited to) cooperative learning, computer software, and use of laboratories. The paper should describe the methods of teaching, student assessment techniques, and effectiveness of the approach on variables such as student performance.

Interdisciplinary Programs with Undergraduate Mathematics

Friday afternoon

Jerry Johnson*
University of Nevada, Reno
Mathematics Dept (084)
Reno NV 89557
(702) 784-4433
e-mail: jjohnson@math.unr.edu
fax: (702) 784-1080
Louis Gross, University of Tennessee

There has been an increasing call for undergraduate courses to have more applications and more contact with other disciplines, and the NSF has regularly funded interdisciplinary projects with mathematics as their centerpiece. Papers are sought that describe undergraduate interdisciplinary courses, activities, or programs which involve a significant mathematics component and are well along in their development. Special consideration will be given to programs that are innovative and can be readily disseminated. Incorporation of technology is also desirable.

Research in Undergraduate Mathematics Education

Wednesday afternoon and Thursday morning

Annie Selden*
Tennessee Technological University
Dept of Math
Cookeville TN 38505
(615) 372-3441
e-mail: js9484@tntech.edu
fax: (615) 372-6172
after August 15:
Center for Research in Mathematics and Science Education
San Diego State University
6475 Alvardo Rd, Ste 206
San Diego CA 92120
John Selden, Mathematics Education Resources Company

This session is sponsored by the AMS—MAA Committee on Research in Undergraduate Mathematics Education (CRUME). We solicit research papers which address questions concerning the teaching and learning of undergraduate mathematics. Both theoretical and empirical investigations utilizing qualitative or quantitative methodologies are welcome. Whenever possible, these should be set within established theoretical frameworks and further existing work. We are especially interested in reports on completed studies.

Submission Procedures for Contributed Paper Proposals

After you have selected a session to which you wish to contribute a paper, forward the following directly to the organizer (indicated with an asterisk (*)):

- the name(s) and address(es) of the author(s); and
- a one-page summary of your paper.

The summary should enable the organizer(s) to evaluate the appropriateness of your paper for the selected session. Consequently, you should include as much detailed information as possible within the one-page limitation. **Your summary must reach the designated organizer by Friday, August 25, 1995.** The organizer will acknowledge receipt of all paper summaries. If the organizer accepts your paper, you will receive a standardized abstract form. Use this form to prepare a brief abstract. Please return the completed form as instructed by the organizer by Friday, September 8, 1995. Abstracts received after the deadline will not be published in the 1996 *Joint Annual Meetings Abstracts Journal.* This journal will be available in the meetings registration area during the conference in Orlando.

DO NOT FORWARD COMPLETED SUMMARIES TO THE MAA OR THE AMS. THEY ARE TO BE SENT TO THE SESSION ORGANIZER.

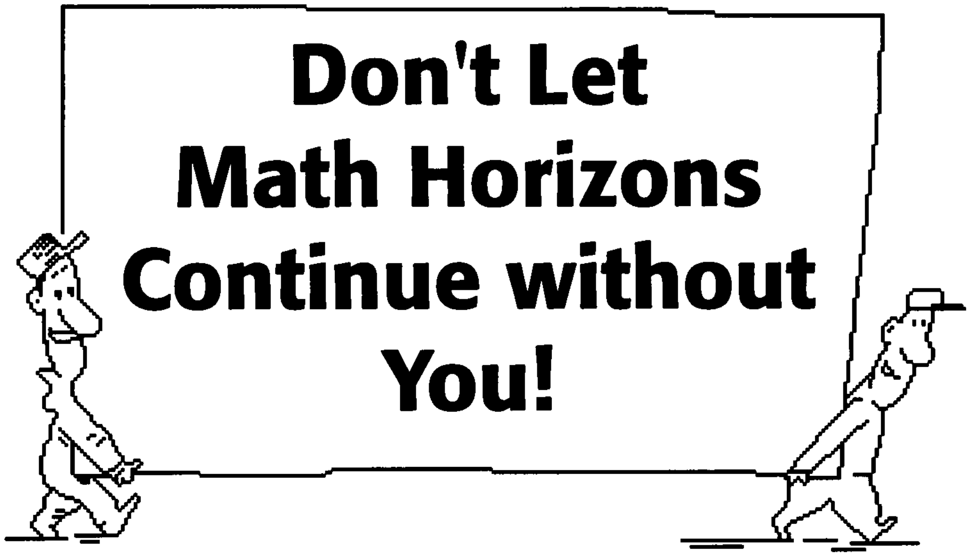
Call for Papers

Fourth Annual MAA Student Chapters Special Paper Session Orlando, January 10–13, 1996

The MAA Student Chapter Session serves as a forum for the exchange of ideas among advisors to individual chapters and section coordinators. Each fifteen-minute talk will focus on one or several activities implemented by a campus chapter or by a section, or on activities supported by the Exxon grants.

If you are interested in presenting a paper on your student chapter activities, please submit the title and an abstract of the proposed talk. Be sure to include your name, affiliation, address, telephone number, e-mail address, and whether you are a student chapter advisor or a section coordinator.

Submissions should be sent to: Karen Schroeder, Mathematical Sciences Department, Bentley College, 175 Forest Street, Waltham, MA 02154-4705; (617) 891-2267; fax: (617) 891-2457; e-mail: kschroed@bentley.edu. Deadline for consideration is September 15, 1995.



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Assessment of Student Learning for Improving the Undergraduate Major in Mathematics

Approved by CUPM at the San Francisco meeting, January 4, 1995. Prepared by the Mathematical Association of America Subcommittee on Assessment of the Committee on the Undergraduate Program In Mathematics.

Preface

Recently there has been a series of reports and recommendations about all aspects of the undergraduate mathematics program. In response, both curriculum and instruction are changing amidst increasing dialogue among faculty about what those changes should be. Many of the changes suggested are abrupt breaks with traditional practice; others are variations of what has gone on for many decades. Mathematics faculty need to determine the effectiveness of any change and institutionalize those that show the most promise for improving the quality of the program available to mathematics majors. In deciding which changes hold the greatest promise, student learning assessment provides invaluable information. That assessment can also help departments formulate responses for program reviews or other assessments mandated by external groups.

The Committee on the Undergraduate Program in Mathematics established the Subcommittee on Assessment in 1990. This document, approved by CUPM in January 1995, arises from requests from departments across the country struggling to find answers to the important new questions in undergraduate mathematics education. This report to the community is suggestive rather than prescriptive. It provides samples of various principles, goals, areas of assessment, and measurement methods and techniques. These samples are intended to seed thoughtful discussions and should not be considered as recommended for adoption in a particular program, certainly not in totality and not exclusively.

Departments anticipating program review or preparing to launch the assessment cycle described in this report should pay careful attention to the MAA Guidelines for Programs and Departments in Undergraduate Mathematical Sciences [1]. In particular, Section B.2 of that report and step 1 of the assessment cycle described in this document emphasize the need for departments to have:

a. A clearly defined statement of program mission; and b. A delineation of the educational goals of the program.

The Committee on the Undergraduate Program in Mathematics urges departments to consider carefully the issues raised in this report. After all, our programs should have clear guidelines about what we expect students to learn and have a mechanism for us to know if in fact that learning is taking place.

—James R. C. Leitzel
Chair, Committee on the Undergraduate
Program in Mathematics, 1995

Membership of the Subcommittee on Assessment, 1995

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Assessment of Student Learning for Improving the Undergraduate Major in Mathematics

I. Introduction

The most important indicators of effectiveness of mathematics degree programs are what students learn and how well they are able to use that learning. To gauge these indicators, assessment — the process of gathering and interpreting information about student learning — must be implemented. This report seeks to engage faculty directly in the use of assessment of student learning, with the goal of improving undergraduate mathematics programs.

Assessment determines whether what students have learned in a degree program is in accord with program objectives. Mathematics departments must design and implement a cycle of assessment activity that answers the following three questions:

- What should our students learn?
- How well are they learning?
- What should we change so that future students will learn more and understand it better?

Each step of an ongoing assessment cycle broadens the knowledge of the department in judging the effectiveness of its programs and in preparing mathematics majors. This knowledge can also be used for other purposes. For example, information gleaned from an assessment cycle can be used to respond to demands for greater accountability from state governments, accrediting agencies, and university administrations. It can also be the basis for creating a shared vision of educational goals in mathematics, thereby helping to justify requests for funds and other resources.

This report provides samples of various principles, goals, areas of assessment, and measurement methods and techniques. Many of the items in these lists are extracted from actual assessment documents at various institutions or from reports of professional organizations. These samples are intended to stimulate thoughtful discussion and should not be considered as recommended for adoption in a particular program, certainly not in totality and not exclusively. Local considerations should guide selection from these samples as well as from others not listed.

II. Guiding Principles

An essential prerequisite to constructing an assessment cycle is agreement on a set of basic principles that will guide the process, both operationally and ethically. These principles should anticipate possible problems as well as ensure sound and effective educational practices. Principles and standards from several sources (see references 2, 3, 4, 5, and 6) were considered in the preparation of this document, yielding the following for consideration:

a. Objectives should be realistically matched to institutional goals as well as to student backgrounds, abilities, aspirations, and professional needs.

b. The major focus of assessment (by mathematics departments) should be the mathematics curriculum.

c. Assessment should be an integral part of the academic program and of program review.

d. Assessment should be used to improve teaching and learning for all students, not to filter students out of educational opportunities.

e. Students and faculty should be involved in and informed about the assessment process, from the planning stages throughout implementation.

f. Data should be collected for specific purposes determined in advance, and the results should be reported promptly.

III. The Assessment Cycle

Once the guiding principles are formulated and understood, an assessment cycle can be developed:

1. Articulate the learning goals of the mathematics curriculum and a set of objectives that should lead to the accomplishment of those goals.

2. Design strategies (e.g., curriculum and instructional methods) that will accomplish the objectives, taking into account student learning experiences and diverse learning styles, as well as research results on how students learn.

3. Determine the areas of student activities and accomplishments in which quality will be judged. Select assessment methods designed to measure student progress toward completion of objectives and goals.

4. Gather assessment data; summarize and interpret the results.

5. Use the results of the assessment to improve the mathematics major.

Steps 1 and 2 answer the first question in the introduction — what should the students learn? Steps 3 and 4, which answer the second question about how well they are learning, constitute the assessment. Step 5 answers the third question on what improvements are possible.

Step 1. Set the Learning Goals and Objectives

There are four factors to consider in setting the learning goals of the mathematics major: institutional mission, background of students and faculty, facilities, and degree program goals. Once these are well understood, then the goals and objectives of the major can be established. These goals and objectives of the major must be aligned with the institutional mission and general education goals and take into account the information obtained about students, faculty, and facilities.

Institutional Mission and Goals

The starting point for establishing goals and objectives is the mission statement of the institution. Appropriate learning requirements from a mission statement should be incorporated in the department's goals. For example, if graduates are expected to write with precision, clarity, and organization within their major, this objective will need to be incorporated in the major's goals. Or, if students are expected to gain skills appropriate for jobs, then that must be a goal of the academic program for mathematics majors.

Information on Faculty, Students, and Facilities

Each institution is unique, so each mathematics department should reflect those special features of the institutional environment. Consequently, the nature of the faculty, students, courses, and facilities should be studied in order to understand special opportunities or constraints on the goals of the mathematics major. Questions to be considered include the following: What are the expectations and special needs of our students? Why and how do our students learn? Why and how do the faculty teach? What are the special talents of the faculty? What facilities and materials are available? Are mathematics majors representative of the general student population, and if not, why not?

Goals and Objectives of Mathematics Degree Program

A degree program in mathematics includes general education courses as well as courses in mathematics. General education goals should be articulated and well-understood before the goals and objectives of the mathematics curriculum are formulated. Of course, the general education goals and the mathematics learning goals must be complementary and consistent [6, pp. 183-223]. Some examples of general education goals that will affect the goals of the degree program and what learning is assessed include the following:

Graduates are expected to speak and write with precision, clarity, and organization; to acquire basic scientific and technological literacy; and to be able to apply their knowledge.

Degree programs should prepare students for immediate employment, graduate schools, professional schools, or meaningful and enjoyable lives.

Degree programs should be designed for all students with an interest in the major subject and encourage women and minorities, support the study of science, build student self-esteem, ensure a common core of learning, and encourage lifelong learning.

Deciding what students should know and be able to do as mathematics majors ideally is approached by setting the learning goals and then designing a curriculum that will achieve those goals. However, since most curricula are already structured and in place, assessment provides an opportunity to review curricula, discern the goals intended, and rethink them. Curricula and goals should be constructed or reviewed in light of recommendations on the mathematics major as contained in the 1991 CUPM report on the Undergraduate Major in the Mathematical Sciences [6, pp. 225-247]. Goal setting should move from general to specific, from program goals to course goals to assessment goals. Goals for student learning can be statements of knowledge students should gain, skills they should possess, attitudes they should develop, or requirements of careers for which they are preparing. The logical starting place for discerning goals for an existing curriculum is to examine course syllabi, final examinations, and other student work.

Some samples of learning goals are

Mathematical Reasoning - Students should be able to perform complex tasks; explore subtlety; discern patterns, coherence, and significance; undertake intellectually demanding mathematical reasoning; and reason rigorously in mathematical arguments.

Personal Potential - Students should be able to undertake independent work, develop new ideas, and discover new mathematics. Students should possess an advanced level of critical sophistication; knowledge and skills needed for further study; personal motivation and enthusiasm for studying and applying mathematics; and attitudes of mind and analytical skills required for efficient use, appreciation, and understanding of mathematics.

Nature of Mathematics - Students should possess an understanding of the breadth of the mathematical sciences and their deep interconnecting principles; substantial knowledge of a discipline that makes significant use of mathematics; understanding of interplay among applications, problem-solving, and theory; understanding and appreciation of connections between different areas of mathematics and with other disciplines; awareness of the abstract nature of theoretical mathematics and the ability to write proofs; awareness of historical and contemporary contexts in which mathematics is practiced; understanding of the fundamental dichotomy of mathematics as an object of study and a tool for application; and critical perspectives on inherent limitations of the discipline.

Mathematical Modeling - Students should be able to apply mathematics to a broad spectrum of complex problems and issues; formulate and solve problems; undertake some real-world mathematical modeling project; solve multi-step problems; recognize and express mathematical ideas imbedded in other contexts; use the computer for simulation and visualization of mathematical ideas and processes; and use the process by which mathematical and scientific facts and principles are applied to serve society.

Communication and Resourcefulness - Students should be able to read, write, listen, and speak mathematically; read and understand technically-based materials; contribute effectively to group efforts; communicate mathematics clearly in ways appropriate to career goals; conduct research and make oral and written presentations on various topics; locate, analyze, synthesize, and evaluate information; create and document algorithms; think creatively at a level commensurate with career goals; and make effective use of the library. Students should possess skill in expository mathematical writing, have a disposition for questioning, and be aware of the ethical issues in mathematics.

Content Specific Goals - Students should understand theory and applications of calculus and the basic techniques of discrete mathematics and abstract algebra. Students should be able to write computer programs in a high level language using appropriate data structures (or to use appropriate software) to solve mathematical problems.

Topic or thematic threads through the curriculum are valuable in articulating measurable objectives for achieving goals. Threads also give the curriculum direction and unity, with courses having common purposes and reinforcing one another. Each course or activity can be assessed in relation to the progress achieved along the threads. Possible threads or themes are numerous and varied, even for the mathematics major. Examples include mathematical reasoning, communication, scientific computing, mathematical modeling, and the nature of mathematics. The example of a learning goal and instructional strategy in the next section gives an idea of how the thread of mathematical reasoning could wind through the undergraduate curriculum.

Step 2. Design Strategies to Accomplish Objectives

Whether constructing a curriculum for predetermined learning goals or discerning goals from an existing curriculum, strategies for accomplishing each learning goal should be designed and identified in the curricular and co-curricular activities. Strategies should respect diverse learning styles while maintaining uniform expectations for all students.

Strategies should allow for measuring progress over time. For each goal, questions such as the following should be considered.

- Which parts of courses are specifically aimed at helping the student reach the goal?
- What student assignments help reach the goal?
- What should students do outside their courses to enable them to reach the goal?
- What should the faculty do to help the students reach the goal?
- What additional facilities are needed?
- What does learning research tell us?

The following example of a goal and strategy can be made more specific by referencing specific courses and activities in a degree program.

Learning goal - Students who have completed a mathematics major should be able to read and understand mathematical statements, make and test conjectures, and be able to construct and write proofs for mathematical assertions using a variety of methods, including direct and indirect deductive proofs, construction of counterexamples, and proofs by mathematical induction. Students should also be able to read arguments as complex as those found in the standard mathematical literature and judge their validity.

Strategy - Students in first year mathematics courses will encounter statements identified as "theorems" which have logical justifications provided by the instructors. Students will verify the need for some of the hypotheses by finding counterexamples for the alternative statements. Students will use the mathematical vocabulary found in their courses in writing about the mathematics they are learning. In the second and third years, students will learn the fundamental logic needed for deductive reasoning and will construct proofs of some elementary theorems using quantifiers, indirect and direct proofs, or mathematical induction as part of the standard homework and examination work in courses. Students will construct proofs for elementary statements, present them in both written and oral form, and have them critiqued by a mathematician. During the third and fourth years, students will formulate conjectures of their own, state them in clear mathematical form, find methods which will prove or disprove the conjectures, and present those arguments in both written and oral form to audiences of their peers and teachers. Students will make rational critiques of the mathematical work of others, including teachers and peers. Students will read some mathematical literature and be able to rewrite, expand upon, and explain the proofs.

Step 3. Determine Areas and Methods of Assessment

Learning goals and strategies should determine the areas of student accomplishments and departmental effectiveness that will be documented in the assessment cycle. These areas should be as broad as can be managed, and may include curriculum (core and major), instructional process, co-curricular activities, retention within major or within institution, and success after graduation. Other areas such as advising and campus environment may be areas in which data on student learning can be gathered.

Responsibility for each chosen area of assessment should be clearly assigned. For example, the mathematics faculty should have responsibility for assessing learning in the mathematics major, and the college may have responsibility for assessment in the core curriculum.

Assessment methods should reflect the type of learning to be measured. For example, the Graduate Record Examination (GRE) may be appropriate for measuring preparation for graduate school. On the other hand, an attitude survey is an appropriate tool for measuring an aptitude for lifelong learning. An objective paper-and-pencil examination may be selected for gauging specific content knowledge.

Eight types of assessment methods are listed below, with indications of how they can be used. Departments will typically use a combination of methods, selected in view of local program needs.

1. Tests. Tests can be objective or subjective, multiple-choice or free-response. They can be written or oral. They can be national and standardized, such as the GRE and Educational Testing Service Major Field Achievement Test, or they can be locally generated. Tests are most effective in measuring specific knowledge and its basic meaning and use.

2. Surveys. These can be written or they can be compiled through interviews. Groups that can be surveyed are students, faculty, employers, and alumni. Students can be surveyed in courses (about the courses), as they graduate (about the major), or as they change majors (about their reasons for changing).

3. Evaluation reports. These are reports in which an individual or group is evaluated through a checklist of skills and abilities. These can be completed by faculty members, peers, or employers of recent graduates. In some cases, self-evaluations may be used, but these tend to be of less value than more objective evaluations. Grades in courses are, of course, fundamental evaluation reports.

4. Portfolios. Portfolios are collections of student work, usually compiled for individual students under faculty supervision following a standard departmental protocol. The contents may be sorted into categories, e.g., freshman or sophomore, and by type, such as homework, formal written papers, or examinations. The work collected in a student's portfolio should reflect the student's progress through the major. Examples of work for portfolios include homework, examination papers, writing samples, independent project reports, and background information on the student. In order to determine what should go in a portfolio, one should review what aspects of the curriculum were intended to contribute to the objectives and what work shows progress along the threads of the curriculum. Students may be given the option of choosing what samples of particular types of work are included in the portfolio.

5. Essays. Essays can reveal writing skills in mathematics as well as knowledge of the subject matter. For example, a student might write an essay on problem-solving techniques. Essays should contribute to learning. For example, students might be required to read four selected articles on mathematics and, following the models of faculty-written summaries of two of them, write summaries of the other two. Essays can be a part of courses and should be candidates for inclusion in portfolios.

6. Summary courses. Such courses are designed to cover and connect ideas from across the mathematics major. These may be specifically designed as summary courses and as such are usually called capstone courses, or they may be less specific, such as senior seminars or research seminars. Assessment of students' performances in these courses provides good summary information about learning in the major.

7. Oral presentations. Oral presentations demonstrate speaking ability, confidence, and knowledge of subject matter. Students might be asked to prepare an oral presentation on a mathematics article. If these presentations are made in a summary course setting, then the discussion by the other students can serve both learning and assessment.

8. Dialogue with students. Student attitudes, expectations, and opinions can be sampled in a variety of ways and can be valuable in assessing learning. Some of the ways are student evaluations of courses, interviews by faculty members or administrators, advising interactions, seminars, student journals, and informal interactions. Also, in-depth interviews of individual students who have participated in academic projects as part of a group can provide insights into learning from the activities.

Student cooperation and involvement are essential to most assessment methods. When selecting methods appropriate to measuring student learning, faculty should exercise care so that all students are provided varied opportunities to show what they know and are able to do. The methods used should allow for alternative ways of presentation and response so that the diverse needs of all students are taken into account, while ensuring that uniform standards are supported. Students need to be aware of the goals and methods of the departmental assessment plan, the goals and objectives of the mathematics major and of each course in which they enroll, and the reason for each assessment measurement. In particular, if a portfolio of student work is collected, students should know what is going to go into those portfolios and why. Ideally, students should be able to articulate their progress toward meeting goals — in each course and in an exit essay at the end of the major.

Since some assessment measures may not affect the progress of individual students, motivation may be a problem. Some non-evaluative rewards may be necessary.

Step 4. Gather Assessment Data

After the assessment areas and methods are determined, the assessment is carried out and data documenting student learning are gathered. These data should provide answers to the second question in the introduction — how well are the students learning?

Careful record keeping is absolutely essential and should be well-planned, attempting to anticipate the future needs of assessment. Additional record storage space may be needed as well as use of a dedicated computer database. The data need to be evaluated relative to the learning goals and objectives. Evaluation of diverse data such as that in a student portfolio may not be easy and will require some inventiveness. Standards and criteria for evaluating data should be set and modified as better information becomes available, including longitudinal data gathered through tracking of majors through the degree program and after graduation. Furthermore, tracking records can provide a base for longitudinal comparison of information gathered in each pass through the assessment cycle.

Consistency in interpreting data, especially over periods of time, may be facilitated by assigning responsibility to a core group of departmental faculty members.

Ways to evaluate data include comparisons with goals and objectives and with preset benchmarks; comparisons over time; comparisons to national or regional norms; comparisons to faculty, student, and employer expectations; comparisons to data at similar institutions; and comparisons to data from other majors within the same institution.

If possible, students should be tracked from the time they apply for admission to long after graduation. Their interests at the time of application, their high school records, their personal expectations of the college years, their curricular and extracurricular records while in college, their advanced degrees, their employment, and their attitudes toward the institution and major should all be recorded. Only with such tracking can the long-term effectiveness of degree programs be documented. Comparisons with national data can be made with information from such sources as Cooperative Institutional Research Program's freshman survey data [7] and American College Testing's College Outcomes Measures project [8].

Step 5. Use the Assessment Results to Improve the Mathematics Major

The payoff of the assessment cycle comes when documentation of student learning and how it was achieved point the way for improvements for future students. Assessment should help guide education, so this final step in the cycle is to use the results of assessment to improve the next cycle. This is answering the third assessment question — what should be changed to improve learning? However, this important step should not be viewed solely as a periodic event. Ways to improve learning may become apparent at any point in the assessment cycle, and improvements should be implemented whenever the need is identified.

The central issue at this point is to determine valid inferences about student performances based on evidence gathered by the assessment. The evidence should show not only what the students have learned but what processes contributed to the learning. The faculty should become better informed because the data should reveal student learning in a multidimensional fashion.

When determining how to use the results of the assessment, faculty should consider a series of questions about the first four steps—setting goals and objectives, identifying learning and instructional strategies, selecting assessment methods, and documenting the results. The most critical questions are those about the learning strategies:

Are the current strategies effective? What should be added to or subtracted from the strategies? What changes in curriculum and instruction are needed?

Secondly, questions should be raised about the assessment methods: Are the assessment methods effectively measuring the important learning of all students? Are more or different methods needed?

Finally, before beginning the assessment cycle again, the assessment process itself should be reviewed: Are the goals and objectives realistic, focused, and well-formulated? Are the results documented so that the valid inferences are clear? What changes in record keeping will enhance the longitudinal aspects of the data?

IV. Conclusion

During an effective assessment cycle, students become more actively engaged in learning, faculty engage in serious dialogue about student learning, interaction between students and faculty increases and becomes more open, and faculty build a stronger sense of responsibility for student learning. All members of the academic community become more conscious of and involved in the way the institution works and meets its mission.

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For more information, or to register, please contact Kim Lemmonds, John Wiley & Sons, Inc., 605 Third Avenue, New York, NY 10158; fax (212) 850-6118; e-mail math@jwiley.com.

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January 10–13, 1996 Seventy-ninth Annual Meeting, Orlando, FL. Board of Governors Meeting January 9, 1996

January 9–7, 1997 Eightieth Annual Meeting, San Diego, CA. Board of Governors Meeting January 8, 1997

January 7–10, 1998 Eighty-first Annual Meeting, Baltimore, MD. Board of Governors Meeting January 6, 1998

Sectional MAA Meetings

ALLEGHENY MOUNTAIN April 12–13, 1996, Indiana University of Pennsylvania, Indiana, PA

ILLINOIS March 1–2 1996, Monmouth College, Monmouth, IL

KANSAS Spring 1996, McPherson College, McPherson, KS

MISSOURI April 1996, Southeast Missouri State Univ., Cape Girardeau, MO

Spring 1997, Missouri Western State College, St. Joseph, MO

Spring 1998, Southwest Missouri State University, Springfield, MO

NEBRASKA-SOUTHEAST SOUTH DAKOTA April 19–20, 1996, Univ. of Nebraska-Kearney, Kearney, NE

NORTH CENTRAL October 20–21, 1995, North Dakota State University, Fargo, ND

April 1996, Hamline University, St. Paul, MN

NORTHEASTERN November 17–18, 1995, Salem State College, Salem, MA

Spring 1996, Hampshire College, Amherst, MA

NORTHERN CALIFORNIA October 21–22, 1995, Cal Polytech State University, San Luis Obispo, CA (joint meeting with S. California Section)

ROCKY MOUNTAIN April 1996, Mesa State College, Grand Junction, CO

SEAWAY November 3–4, 1995, Skidmore College, Saratoga Springs, NY

March 1996, Elmira College, Elmira, NY

SOUTHEASTERN April 12–13, 1996, University of Alabama–Huntsville, Huntsville, AL

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TEXAS March 28–30, 1996, Texas Tech University, Lubbock, TX

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WISCONSIN April 12–13, 1996, University of Wisconsin-Platteville

Other Meetings

June 24–28, 1995 Fourth Annual Interdisciplinary Art and Mathematics Conference (AM95), SUNY at Albany. For information, contact Nat Friedman, Dept. of Math, SUNY at Albany, Albany, NY 12222; (518) 442-4621; fax: (518) 442-4731; e-mail: artmath@math.albany.edu.

August 16–19, 1995 Fifth Conference of the International Algebra Society, Georgia State University, Atlanta, GA. For more information, contact Frank Hall, Dept. of Math & Computer Science, Georgia State University, Atlanta, GA 30303; (404) 651-2253; e-mail: fhall@cs.gsu.edu; or Paul Van Dooren, Universite Catholique de Louvain, CESAME, Batiment Euler A.119, B-1348 Louvain-la-Neuve, Belgium; 32-10-47-8040; e-mail: vdooren@anma.ucl.ac.be.

October 12–24, 1995 Seventeenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA XVII), Columbus, OH. Deadline for Poster Session proposals is June 30,1995. For a copy of the final announcement, contact Doug Owens, 253 Arps Hall, The Ohio State University, 1945 North High Street, Columbus, OH 43210-1172; (614) 292-8021; fax: (614) 292-7695; e-mail: Owens.93@osu.edu.

The Mathematical Association of America
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