FOCUS

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The Calculus Turmoil

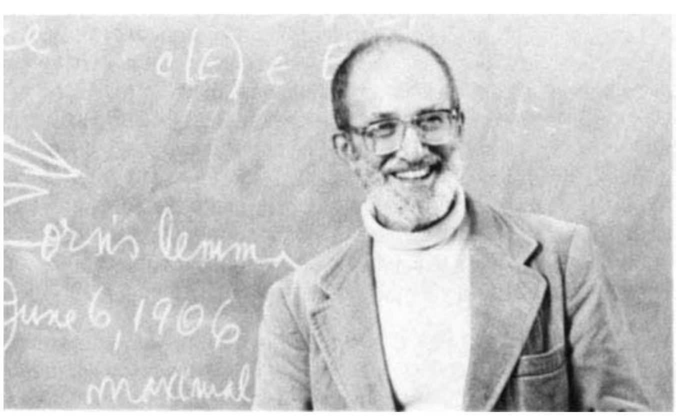
Paul R. Halmos

What's all this calculus turmoil about? Have all mathematicians except me taken leave of their senses? Surely it isn't just a racket—a way of getting money from the NSF? But if it is not, then why are otherwise honest and intelligent people screaming about it? Will somebody please explain to me what's going on?

The two main symptoms (or are they the causes?) of this *new* disease are that lots of students *flunk* calculus and that most *textbooks* are bad.

Well, let's see. "More than a third of the 600,000 or so people who study calculus in college every year fail to complete the course." (Peter D. Lax, "Calculus Reform: A Modest Proposal," *UME Trends,* **2** (Number 2, May 1990). For a different view, see Leonard Gillman's "Two Proposals for Calculus," FOCUS, **7** (Number 4, 1987).) I am prepared to believe that, and I am neither surprised nor horrified. Teachers have been discussing failure rates with one another ever since the concept was invented, and 40% is not unusual among the numbers that they bandy about. The failure rate varies a lot, of course—it depends on the quality of the university in question as a whole, and, derivatively, on the quality of the teachers and the students at the university. A part of the reason is the reason that students take calculus in the first place. The word has a mystique; it's a Mount Everest to climb. It is an essential part of the tool kit of many who wish to study science, and science is a glamorous and well-paid(?) and highly respected

*(**Turmoil** continues on page two.)*



Paul R. Halmos

Challenges for College Mathematics

Lynn Arthur Steen

This issue of FOCUS contains as a special insert the report of a joint Task Force of the MAA and the Association of American Colleges (AAC) entitled "Challenges for College Mathematics: An Agenda for the Next Decade." This report was prepared as part of a multidiscipline AAC project to explore effective approaches to undergraduate majors. Issues arising from the project will be the subject of discussion at the January 1991 AAC meeting of college and university deans and presidents.

In August 1990, the MAA Board of Governors endorsed the Task Force report "as a statement about the major on behalf of MAA" and recommended it to MAA members "as a framework for campus discussion about the undergraduate major in mathematics." The MAA-AAC report follows in the footsteps of the "David II" report "Renewing US Mathematics: A Plan for the 1990s," and precedes by several months the publication of the MS 2000 report "Moving Beyond Myths: Revitalizing Undergraduate Mathematics." Together these three documents will provide an agenda for action to help stimulate departments of mathematics to improve the effectiveness of undergraduate mathematics for all students regardless of background, interests, or intended major.

Lynn Arthur Steen, professor of mathematics at St. Olaf College in Northfield, Minnesota, former MAA president (1985–1986), and chair of the Association's Committee on the Undergraduate Program in Mathematics (CUPM) chaired the joint Task Force of the MAA and AAC that prepared the report described above.

Inside This Issue

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profession. Students who take calculus "because it's there," or for the glamor, the money, or the respect, are taking it for the wrong reason, and their failure is neither surprising nor worrisome.

What is the failure rate for people taking piano lessons? I don't know, but I could make up some statistics as well as the next guy, and I wouldn't be surprised if the true statistics pointed to a situation as "lamentable" as the one in mathematics.

As for how the books are bad: they contain "inert knowledge," including "integration techniques that work only in specially rigged cases," "fake applications," and "tedious calculations."

Are those comments new? And, new or not, are they correct? Yes, they are correct, and no, they are not new: they have been made for many decades (centuries?), and they point to something bad. To something that is all bad? I am not sure.

Very few people advocate epsilons and deltas in elementary calculus, and, for sure, I am not one of them, I confess that when it comes to arithmetic I am still in favor of the multiplication table being drummed into innocent heads (at least through 9 x 9), together with some of the elementary and almost universally needed techniques of estimating orders of magnitude. (If each of 1,000 students writes 10 themes a week for each of the 30 weeks of the academic year, and if, on average, the labor of grading each theme costs 20 cents, then we should be able to decide whether the money we need from the administration is $6,000, $60,000, or $600,000.) Similarly, in calculus, I am still in favor of teaching the classical basic techniques of the differential and integral calculus. I want a student who sees x^3 or sin x to snap back $3x^2$ or cos x, and I want a student who sees $\int \frac{dx}{1+x^2}$ and $\int \frac{dx}{1-x^2}$ to know the difference between them and to be able to come up with the indefinite integral in both cases, even if a pocket calculator can do it more quickly. It is true that much of the classical drill (that I for one went through in the 1930s) can be left to the button pushers—pocket calculators do very well what for a long time we haven't been wanting to do at all. But I am still in favor of teaching some (many?) rigged integration techniques and tedious calculations—they are as rigged and as tedious as finger exercises for the pianist, and just as indispensable.

Is computing to be "an integral part" of calculus? (I can't help wondering whether that's an intentional pun.) Yes, of course, that's just what I've been saying. By computing, to be sure, I mean what calculus teachers have always meant—computing with rational, trigonometric, exponential, and logarithmic functions, and getting used to their behavior and their relations to each other. I do not mean computing in the sense that computing machines are good for—horrors, no!

The greatest "mathematical" invention of the last few centuries is not electronic but notational—I refer to decimals and their relation to arithmetic. An electronic typewriter that can crunch numbers is useful—but I don't want to teach spelling in courses in literature, decimal calculations in courses on number theory, and computing techniques in courses on calculus. I like spelling and decimals and computers—but they should be kept in their places.

What then should a calculus course emphasize, if not integral rigging and number crunching? Qualitative understanding of functions?—asymptotics and approximation as implied by the differential equations they satisfy? Fine—sure—why not. Substantial applications—in engineering, in economics, in psychology?—yes, maybe—but our focus should always remain at the mathematical center. We, teachers of calculus, are not usually practicing engineers, economists, or psychologists, and just as we might worry about the competence of those people to teach our stuff, they have the full right to worry about our encroaching on their expertise. But I don't really think there is anything serious to worry about as far as the content is concerned. Most calculus courses (certainly the ones I took, the ones I taught, and the ones I hear colleagues tell about) are pretty much freewheeling—they emphasize the mechanical techniques to the extent that drill is necessary (or a little too much, or not quite enough, but more or less to the right extent), and they contain the illustrations and applications that the teacher is competent to explain and the students are ready to receive.

Do bad books contribute to making the courses bad? Yes and no—mainly no. At the calculus level, in my experience, students simply don't read the book—most of them cannot and none of them has to. They cannot because they just don't know how to read—the formal language of the text is repulsive to them, and boring, and difficult—most of our class time has to be spent in repeating the book in the vernacular. And they don't have to because most of our class time is spent in repeating the book in the vernacular.

A famous (infamous?), old (ancient?), bad (dreadful?) calculus book is Granville, Smith, and Longley—and its badness is irrelevant to using it as a text. It tells some lies, yes, but who reads them?! What matters is that it has a table of contents, to which you can add and from which you can delete—and it has a large collection of numbered problems that you can assign for homework. Being fearful of a largely imaginary and hugely exaggerated disease allegedly caused by the enforced use of uniform texts is a rather silly phobia—any calculus teacher who has taught the subject once can use any book and do as well with it as with any other.

Yes, there *are* good books and they have been with us perhaps even longer than the bad ones that water them down—Landau, Courant, and Apostol may have lived in vain, but they did live.
*(**Turmoil** continues on next page.)*

FOCUS

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(***Turmoil** continued from previous page.*)

No, good books don't catch on—and a large part of the reason is that our calculus students are not ready for them. Most students are more than intelligent enough to cope, but they've been held back—pushed back!—since their early years in grade school and high school, and they arrive at the university not just untrained or badly trained but almost negatively trained. Some of them in their turn become poorly trained grade school teachers and high school teachers and go on blissfully to give their students training at least as bad as they had received.

The results of the bad training are visible everywhere. It shocked me to find, as I was teaching a course on analytic functions, that my students (all of whom had credit in a course called "advanced calculus") not only did not know the sum of a geometric series but maintained that they had never even heard of it—that's bad college calculus. But, in the same course, I was even more shocked to learn that they knew neither the factor theorem (the relation between the roots of a polynomial equation and the factors of the polynomial) nor its name—once again they had never even heard of it—and that's bad high school algebra. And, incidentally, and strongly and relevantly, the same sort of ignorance is visible in other subjects, notably in the use of language. When I require that the solutions of problems be communicated to me not just by scribbled equations but by sentences that begin with a capital letter, end with a period, and have a reasonable sprinkling of commas between, my students groan and tremble—they don't believe me, they think I am unreasonable, and they simple don't (can't) do it.

Did all this start "in the fifties"—is it all a new disease caused largely by "the vast majority of research mathematics?" Balderdash! The problem is not new. Practicing mathematicians have known about it long before the 1930s, when I began to dip my toes into the mathematical ocean, and they will keep talking about it, I predict, long after we are all gone. Old people always say that the world is going to hell in a handbasket, and I'm ready to subscribe to that tenet—things might always have been bad, but maybe, maybe, they are getting worse.

Students know less and less about language (because they are taught less and less), and they know less and less about calculus (because they are given less and less preparation, and, in fact, progressively worse preparation, in high school). Is calculus the only mathematical sufferer? No—absolutely not—and I suppose that the reason calculus is usually singled out is that it's a huge industry—it is the bread and butter of many of us. But precalculus (multiply and divide polynomials and sines and cosines) and postcalculus (add vectors and multiply matrices) are in equally bad shape.

Is the message coming through? I am saying that there is a lot of unjustified turmoil, but I am not saying that there is nothing wrong. I am saying that the calculus turmoil is rather silly—it is both misplaced and exaggerated—but there is something wrong, and what's wrong is much broader and deeper and more serious than calculus students who flunk and textbooks that are dull.

Are most human beings too stupid to learn calculus and grammar? No, the trouble is not genetic—it is, for want of a better word, sociological. In Japan, we are told, students learn calculus, and they are the same kind of human beings as the ones in Manhattan, New York or Manhattan, Kansas. But in Japan there seems to be an almost universal respect for learning, a social background from which children absorb the feeling that intellectual effort can be pleasant and rewarded and applauded—an atmosphere in which scholarship is encouraged as much as (more than?) athletics and commerce, a social structure in which professors are as good as (better than?) baseball pitchers and airplane pilots. Such respect, such a social background and atmosphere, have been eroding in the US for most of the twentieth century—since, as a rough approximation, the time of John Dewey, my favorite villain, one of whose effects appears to have been to cause people to sneer at abstractions and admire the "practical." That, I think is why the US is at or near the bottom of many school achievement lists in which the countries of the world are compared—that is why we have a "calculus crisis" but Japan does not.

Yes, there is a disease, but calculus is neither its cause not its main symptom. We mathematicians can do our small bit to cure it, but not by rewriting calculus books. All that we can do, all that we are professionally able to do, is to insist on raising the quality of primary and secondary education by establishing and maintaining a high quality in college courses, by insisting on and strictly enforcing severe prerequisites, and by encouraging and properly training prospective grade school and high school teachers. That we can do, and I hope we will.

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1991 Summer Mathematics Institutes

- **University of California at Berkeley**
  Intensive Mathematics Program
  for Underrepresented Minority Undergraduates
- **Mills College**
  Intensive Mathematics Program
  for Women Undergraduates

In an effort to increase the number of underrepresented minority and women students seeking careers requiring a PhD degree in the mathematical sciences, the University of California at Berkeley (UCB) and Mills College will each offer a residential summer program for undergraduate students, 17 June–26 July 1991 (thirty students at UCB; twenty-four at Mills). The programs are being organized by Uri Treisman of the Department of Mathematics at Swarthmore College and the Dana Center for Mathematics and Science Eduction at UCB, by Leon A. Henkin of the Departments of Mathematics at UCB and Mills, and by Lenore Blum of the Department of Mathematics at Mills College and the International Computer Science Institute (ICSI). Funding for these programs is being sought from the National Science Foundation.

We are asking faculty members to seek out ethnic minority and women students who could benefit from our programs and to encourage them to apply. The UCB Institute is designed for African-American, Mexican-American/Chicano, Latino, and American Indian undergraduates. The Mills Institute is designed for women of all ethnicities. All applicants must have completed two years of collegiate mathematics with distinction by June of 1991.

Each participating student will explore two areas of mathematics. Part of this exploration will take place in seminars of twelve students each, organized by senior mathematicians. Students will be encouraged to tackle many challenging problems individually, in small groups, and in consultation with graduate student mentors. In addition, there will be weekly colloquia designed to provide participants with a broad view of current work in mathematics as well as of career opportunities for mathematicians. Each student selected will receive room and board, a $2,000 stipend, and travel allowance to and from the summer institute.

Previous Berkeley seminar leaders have included Ani Adhikari, Efraim P. Armendariz, Oscar Moreno, James W. Pitman, Carl Pomerance, Carl L. Prather, Louise A. Raphael, Don P. Rawlings, and Edwin H. Spanier. Previous Berkeley colloquium speakers have included Hajnal Andreka, Richard A. Baker, Lenore Blum, Andrew Casson, Alice Chang, Amy Cohen, Martin D. Davis, Persi Diaconis, F. Alberto Grunbaum, Leon A. Henkin, Morris W. Hirsch, Wu-Yi Hsiang, Irving Kaplansky, Serge Lang, Thomas M. Liggett, Istvan Memeti, Kenneth C. Millett, Brad G. Osgood, Robin A. Pemantle, Diane Resek, James A. Sethian, Stephen Smale, Martha K. Smith, Michael Starbird, and P. Emory Thomas,

Faculty for the 1991 Summer Mathematics Institutes will be drawn from universities and research laboratories nationwide. Mathematicians interested in participating in either institute as seminar leaders or colloquium speakers are invited to contact Professor Uri Treisman (University of California at Berkeley): Department of Mathematics Swarthmore College, Swarthmore, Pennsylvania 19081, (215) 328-8224; or Professor Lenore Blum (Mills College) ICSI, 1947 Center Street, Berkeley, California 94704, (415) 643-9153.

Projected deadline for applications is **22 February 1991.** Further information and application forms for both institutes can be obtained by telephoning: (415) 642-5881; or writing to: 1991 Summer Mathematics Institutes, c/o Dana Center for Mathematics and Science Education, 230 B Stephens Hall, University of California at Berkeley, Berkeley, California 94720.

Regional Technology Centers

Ohio State University invites applications for a National Science Foundation (NSF) project to train school/university teams to establish regional technology centers. Regional team training will be provided through summer inservice sessions and academic year follow-up conferences at Ohio State University. These regional centers, in turn, will train teachers as school technology specialists at their regional sites. Local living expenses with some stipend and travel support are available. In addition, Ohio State University will offer partial first-year support for a few regional sites. Deadline for completed applications: **15 February 1991.** For further information, contact: Frank D. Demana and Bert K. Waits, Ohio State University, Department of Mathematics, 231 West Eighteenth Avenue, Columbus, Ohio 43210.

Calculus, Computers, Concepts, and Cooperative Learning

Purdue University will host a summer workshop partially funded by NSF and West Educational Publishing Company in late May and early June 1991. The two-week, intensive, total-immersion program will focus on the use of computers, research into learning theory, and a cooperative learning environment to help students learn calculus concepts. A three-day preworkshop tutorial on the basic use of the necessary computer systems will be offered as an option. Program participants are expected to return the following summer for two days to discuss their teaching experiences based on the workshop.

The program will provide hands-on experience with computer software (both MS DOS and Macintosh), class materials, and seminars on learning theory, including viewing and discussing videos of Purdue classroom and laboratory experiences in cooperative learning. The program will feature the use of the mathematical programming language **ISETL** and its graphics package, in addition to the symbolic computer systems **Maple** and **Derive.** Participants will be encouraged to consider piloting the calculus course presented in the workshop during academic year 1991–92. Partial support for attending the workshop will be available. For information about the program and application material, contact: Ed Dubinsky or Keith Schwingendorf, Department of Mathematics, Purdue University, West Lafayette, Indiana 47907; bbf@j.cc.purdue.edu or ks@math.purdue.edu. Completed application deadline: **15 February 1991.** Successful applicants will be notified by **15 March 1991.**

In Memoriam

Lamberto Cesari, Professor Emeritus, University of Michigan at Ann Arbor, died 12 March 1990 at the age of 79. He was an MAA member for 36 years.

Lawrence D. Gould, Instructor, North Carolina School of Science and Mathematics, died 9 May 1990 at the age of 58. He was an MAA member for 25 years.

William R. Orton, Jr., Professor Emeritus, University of Arkansas, died 9 May 1990 at the age of 68. He was an MAA member for 37 years.

S. Thomas Parker, Professor Emeritus, Kansas State University, died 14 March 1990 at the age of 76. He was an MAA member for 45 years.

Board Actions in Columbus

Gerald L. Alexanderson, Secretary, MAA

The Mathematical Association of America has had an active year, with a new administrative team in place in the Washington office (June 1990 FOCUS), the dedication of the newly renovated Pólya Building in June (September 1990 FOCUS), and the anniversary celebration this past summer in Columbus (January–February 1990, March–April 1990, June 1990, and this issue of FOCUS). All of these events have been or will be reported on elsewhere, in separate articles in FOCUS and in the annual report of the Executive Director. In these remarks I shall comment briefly on recent actions of the Board of Governors and other related activities.

As I remarked in my report at the Business Meeting in Columbus, we need not fear that the MAA is an organization of the increasingly aged. Our Student Chapters program has been an enormous success. In the most recent list of new members to go before the Board of Governors, 81% were students. This dramatic growth in the number of student members in the Association (mainly undergraduate students, it would seem) is surely in large part due to our student chapters. The Membership Committee is currently discussing ways of encouraging graduate students to join the Association. Along with this growth, we are seeing more activities for students at our national and sectional meetings.

At its Columbus meeting in August, the Board of Governors elected a new editor for the *American Mathematical Monthly,* to replace Herbert S. Wilf of the University of Pennsylvania. The new editor will be John H. Ewing of Indiana University. Professor Ewing has been associate editor of the *Monthly* and editor of *The Mathematical Intelligencer,* which, in the words of the nominating committee, he "transformed . . . into a lively, widely praised mathematical magazine." It is perhaps premature to thank Herb for his fine service as editor of the *Monthly,* since he has over a year remaining in his term, but it is comforting at this point to know that the *Monthly* will remain in capable hands following 1991. The present appointment is an important one, because the new editor will preside over the centennial of the *Monthly* in 1993.

The Board also elected a nominating committee to develop slates of candidates for the offices of president-elect, first vice-president, and second vice-president. The committee members are Doris W. Schattschneider (chair), Wade Ellis, Jr., Martha J. Siegel, Lynn Arthur Steen, and Alan C. Tucker. Suggestions of names of possible candidates are solicited for the Committee in the September 1990 issue of FOCUS, page four.

The Board approved a report, "Challenges for College Mathematics: An Agenda for the Next Decade," of a joint Task Force of the MAA and the Association of American Colleges (AAS). The center insert in this issue of FOCUS reproduces this report in full.

The Board also considered at some length a draft of a report from the Committee on the Mathematical Education of Teachers (COMET), "A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics." Suggestions were solicited; a final version of the report will be presented for approval at the January 1991 meeting of the Board.

A proposal for restructuring the many committees of the Association into six groupings referred to as "councils" or "areas" has been discussed at previous Board meetings and meetings of committee chairs. In Columbus the Board approved the restructuring plan, with approval of a final version deferred to the January 1991 meeting. Final details involving the assignment of some committees to these "councils" or "areas" remained to be worked out.

For secretaries and program committee chairs in the Sections, there was good news emanating from the Board meeting in Columbus. A new Pólya Lectureship was approved. I quote from the report of the Committee assigned the task of formulating the standards and policies for this Lectureship: "George Pólya embodied the high quality of exposition which the Mathematical Association of America seeks to encourage. To further this goal, the Board of Governors of the Association hereby creates the George Pólya Lectureship. This lectureship will be held by an individual representing the high standards set by George Pólya. Each Section will be entitled to a Pólya Lecture approximately once every five years." This program will provide to Sections speakers of national prominence. When a Section is eligible for a Pólya lecturer, it may also invite a speaker from the list of national officers, editors, etc., as in the past. The Pólya lecturer will not have to substitute for a speaker from the list normally available to Sections.

Summer meetings remain a concern. Attendance at the Columbus meetings ran over 1,300, a large number by recent standards, but that was probably due to the special nature of the anniversary meetings. If attendance were to fall again to the 600–800 level experienced at some recent summer meetings, there will probably be further negotiations with the American Mathematical Society (AMS) to adjust the format of summer meetings or to go on some sort of schedule like that proposed earlier, with summer meetings only in those years without international congresses (ICM or ICME). A joint AMS-MAA committee met in Washington, DC in June 1990 to discuss summer meetings. Some of the ideas that were developed at the meeting will be evident in the plans for the summer meetings in Orono, Maine, in August 1991.

At the Business Meeting and Prize Session in Columbus, awards were made to winners of the Carl B. Allendoerfer, Lester R. Ford, and George Pólya Awards. (Names and citations appear in this issue of FOCUS, page nine.) John W. Kenelly gave a report on the drive for the Building Fund, for which the Association is actively soliciting contributions to cover the renovation of the buildings housing the Washington, DC offices of the Association. Earlier that week a President's Breakfast honored those who had contributed at the "hexahedra" level.

The next meeting of the Board will take place in San Francisco, California, Tuesday, 15 January 1991.

Nominations Sought for Yueh-Gin Gung and Dr. Charles Y. Hu Award

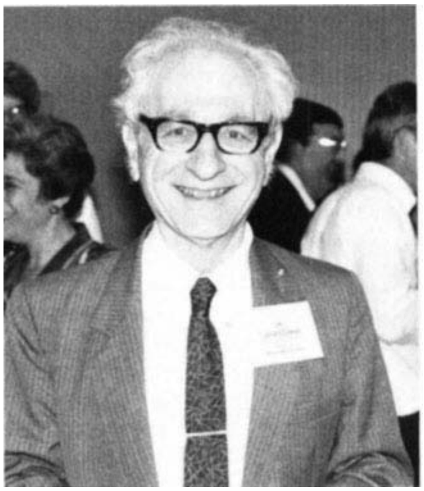
The committee to select a recipient for the Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics seeks nominations for that award. The principal clause in the regulations governing the award states:

"The award is to be made for outstanding service to mathematics. The period of service may be long or short, and the award may be made on the basis of one or several activities. The contribution should be such as to influence significantly the field of mathematics or mathematical education on a national scale. The recipient should be a member of the Association who is a resident of the United States or Canada."

Nominations must include supporting information and should be sent by **26 November 1990** to Professor Deborah Tepper Haimo, Department of Mathematics and Computer Science, University of Missouri at St. Louis, St. Louis, Missouri 63121.

Goodbye, Columbus

All photographs courtesy of Gerald L. Alexanderson

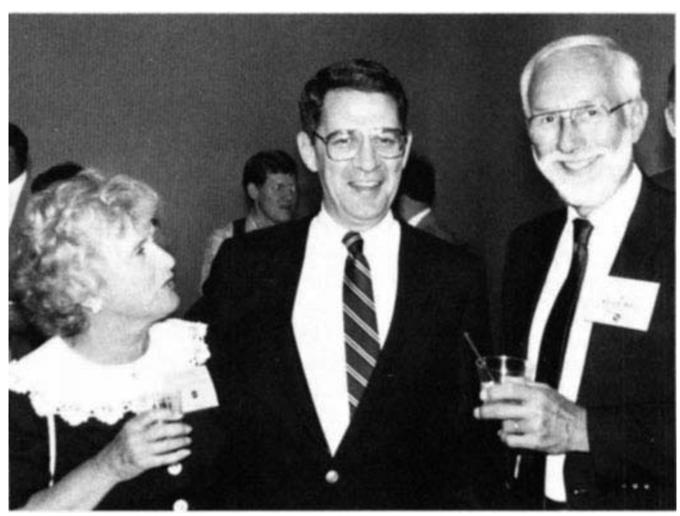


Leonard Gillman, 42nd President of the MAA

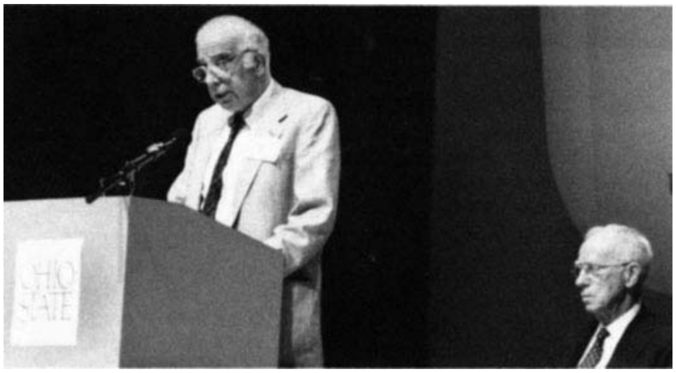
Meeting participants gather in the lobby of Mershon Auditorium on the campus of Ohio State University.

Thomas W. Tucker, current First Vice-President, Alan C. Tucker, First Vice-President, 1988-1989, Andrew M. Gleason, and Alice C. Beckenbach

Ronald L. Graham, Danalee Buhler, Peter Frankl, Brad W. Jackson, and Joe P. Buhler demonstrate their juggling talents at the Mathematical Circus.

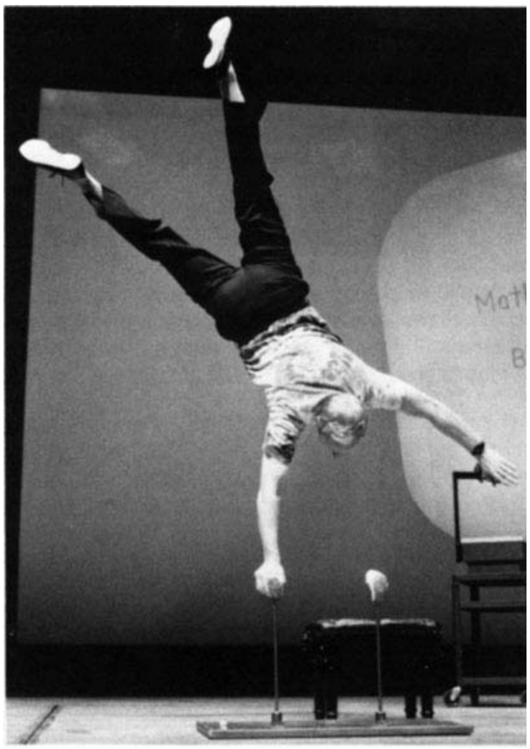


Shirley Willcox, David P. Roselle, MAA Secretary, 1975–1983, and Alfred B. Willcox, MAA Executive Director, 1969–1989



Henry L. Alder, 37th President of the MAA, introduces G. Baley Price, the Association's 27th President.

Wade Ellis, Jr., Speaker, MAA Day



Ronald L. Graham, MAA First Vice-President, 1982–1983

Donald J. Albers, Second Vice-President, 1983–1984, and Saunders Mac Lane, 24th President of the MAA

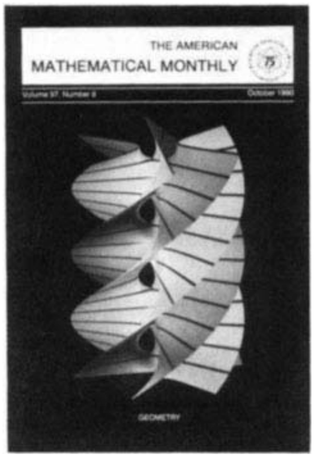
Eileen L. Poiani and
Henry O. Pollak, 36th President of the MAA

Ivan Niven, 40th President of the MAA, and Stan Wagon

FOCUS on MAA Journals

The American Mathematical Monthly

Herbert S. Wilf, Editor



The October 1990 edition of the *American Mathematical Monthly* realizes a landmark achievement in the nearly century-long span of the journal's publication. This issue, a special, thematic collection devoted entirely to geometry, features color on both its cover and its pages (the first time the *Monthly* has ever done so), to emphasize the subject's visual beauty. The cover illustration (above) reproduces a minimal surface David A. Hoffman of the University of Massachusetts at Amherst and Hermann Karcher of the University of Bonn discovered. James Riordan and Jim Hoffman of the Geometry, Analysis, Numerics, and Graphics Center (GANG) at the University of Massachusetts at Amherst created the image. It is singly-periodic and behaves at infinity much like the union of helicoids.

Furthermore, this addition of color on the cover and inside plates complements the journal's superb miscellaney of essays from several distinguished authors. Together, their words and pictures interact to shape a spectacular and historic issue.

- S. S. Chern of the Mathematical Sciences Research Institute at the University of California at Berkeley asks, "What is Geometry?" His essay, the issue's keynote selection, together with his poem in the original Chinese and in English translation, expresses the unity of geometry and physics.
- Marcel Berger of the Institut des Hautes Etudes Scientifiques, Bures-sur-Yvette, presents a delightful discourse on convexity with numerous examples—some of which he locates in art and anatomy.
- Ronald L. Graham of AT&T Bell Laboratories and Frances Yao of Xerox Palo Alto Research Center (PARC) lead a "Whirlwind Tour of Computational Geometry" and discuss the major problems, both solved and unsolved, in this young, dynamic field.
- David A. Hoffman and William H. Meeks, III, both of the University of Massachusetts at Amherst, deliver an especially "colorful" contribution featuring dazzling, computer-generated, color prints of minimal surfaces. Their article will excite both the eye and the mind.
- Robert Osserman of Stanford University explores recent developments in another branch of geometry: curvature. Indeed, everyone interested in curvature should read his examination of this field's progress and agenda during the eighties.
- Finally, William P. Thurston of Princeton University considers the possibility of tiling a portion of a plane lattice that is cut out by a simple closed curve. He also discusses some engaging examples he worked out with John Conway.

Chaos in the College Mathematics Journal

Ann E. Watkins and William E. Watkins, Editors

The January 1991 *College Mathematics Journal* will be a special issue on how the study of dynamical systems can enliven traditional courses. Authors include Robert L. Devaney, Gilbert Strang, and James T. Sandefur. Devaney, this issue's coordinator, writes, "The mathematics of chaos and fractals is at once accessible, alluring, and exciting. Fractal geometry offers a wonderful arena for combining computer experimentation and geometric insight." In the new section on research projects for students, Paul C. Matthews and Steven H. Strogatz suggest problems to get students started exploring the connection between chaotic mappings and probability distributions. The popular "Fallacies, Flaws, and Flimflam" presents a gentle series of theorems and proofs that culminates with the proof of a theorem, which may be new to you, that the set of real numbers is countable.

Some Advice for the Next Editor Although we are only about a year and a half into our term as coeditors of the *CMJ*, we have some advice to give to our successor(s). Some of this advice concerns authors, so if you are a potential author, please stop reading now.

1. ***The quickest way to predict whether a manuscript will eventually be accepted:*** Count the number of authors. Of the first roughly three hundred manuscripts submitted as main articles to the *CMJ* during our term as editors, about 6% of manuscripts with a single author but 35% of manuscripts with multiple authors were accepted for publication.
2. ***The second quickest way to predict whether a manuscript will be accepted:*** Count the number of references. Referees like authors who have done their homework and papers that have historical perspective and that give proper attribution to those who have already published on the topic.
3. ***What to do when the trisector comes:*** Don't try to point out the mistake. Otherwise, by return mail you will receive a substantially longer revision in which the "carelessness" has been rectified.
4. ***What to do if you (or the printer) makes a mistake:*** Don't worry. No one will notice. Has anyone looked at the cover of the May 1989 *CMJ?*
5. ***How to deal with irate authors who think that their manuscripts should have been accepted and that the referees are out-to-lunch:*** There is no way to deal with irate authors.
6. ***How to find good referees:*** Thorough and helpful referees are gold, and editors hoard them and cherish them. Occasionally, after reading an article in FOCUS, a potential referee writes to the editor volunteering his or her services.

(***Journals** continues on page nine.)*

Challenges for College Mathematics:

An Agenda for the Next Decade

Report of a Joint Task Force

of the

MATHEMATICAL ASSOCIATION OF AMERICA

and the

ASSOCIATION OF AMERICAN COLLEGES

LYNN ARTHUR STEEN, CHAIR, St. Olaf College
JEROME A. GOLDSTEIN, Tulane University
ELEANOR GREEN JONES, Norfolk State University
DAVID LUTZER, College of William and Mary
PHILIP URI TREISMAN, University of California, Berkeley
ALAN C. TUCKER, SUNY at Stony Brook

This report was completed in cooperation with a national review of arts and sciences majors initiated by the Association of American Colleges as part of its continuing commitment to advance and strengthen undergraduate liberal learning. The Mathematical Association of America was one of twelve learned societies contributing to this review. Each participating learned society convened a Task Force charged to address a common set of questions about purposes and practices in liberal arts majors; individual task forces further explored issues important in their particular fields. Reprints of this report on mathematics are available from the Mathematical Association of America, 1529 Eighteenth Street, NW, Washington, DC 20036.

In 1991, the Association of American Colleges published a single volume edition of all twelve learned society reports with a companion volume containing a separate report on "Liberal Learning and Arts and Science Majors." Inquiries about these two publications may be sent to Reports on the Arts and Sciences Major, Box R, Association of American Colleges, 1818 R Street, NW, Washington, DC 20009.

Generous funding for the project and dissemination of the reports was provided by the Fund for the Improvement of Postsecondary Education (FIPSE) and the Ford Foundation.

Contents

Preface

In response to a request from the Association of American Colleges (AAC), the Mathematical Association of America (MAA) convened a special Task Force to address a range of issues concerning the undergraduate major as a sequel to the 1985 AAC report *Integrity in the College Curriculum.* Whereas the 1985 *Integrity* study examined the effectiveness of general education, the new AAC study addresses the contribution that "study in depth" makes to liberal education. Each of the several AAC disciplinary task forces examined how study in depth relates to goals for the major, assurance of intellectual development, and relations with other fields.

The MAA-AAC Task Force operated in the context of two other simultaneous MAA reviews of the mathematics major. One was conducted by a subcommittee of CUPM, the Committee on the Undergraduate Program in Mathematics; the other by COMET, the Committee on the Mathematical Education of Teachers. Since these other committees are charged by MAA to provide specific advice to the mathematical community about requirements for the mathematics major, the MAA-AAC Task Force dealt only with broader questions that are central to the AAC study. Hence this report is not intended as a detailed statement about curricular content, but as a statement of issues and priorities that determine the context of undergraduate mathematics majors.

In preparing the present draft, the Task Force held several open hearings on issues concerning the undergraduate major at national meetings of the Mathematical Association of America and the American Mathematical Society. A draft of this report was mailed for review to several hundred persons including every Governor, Chair, and Secretary of the 29 sections of the Mathematical Association of America; department heads, deans, and provosts at a variety of institutions; and many leaders of mathematical professional societies. This draft was also reviewed at meetings of AAC and MAA in January 1990. At the latter meeting, the document was discussed extensively at an invitational three-hour roundtable meeting that involved about forty experienced college and university mathematicians, including many who have been working actively to improve opportunities for women and minorities in the mathematical sciences.

Sectional Governors of the MAA were asked to nominate exemplary departments whose programs illustrate issues discussed in the report. Descriptions of some of these programs have been adapted as illustrations of promising practices in the final draft. During the spring of 1989 graduating seniors on several campuses were surveyed as part of a multi-disciplinary AAC effort to gather student opinion. Subsequently, leaders of several student chapters of the Mathematical Association of America were asked to review a draft of the report. Their careful and thoughtful responses underscore our belief in the value to students of the recommendations contained in this report.

The report has benefited enormously from these many external reviews. We believe that it now represents a consensus of the informed mathematical community concerning urgent issues of importance to the undergraduate mathematics major. In August, 1990 the report was unanimously approved by the MAA Board of Governors as an official MAA statement concerning the undergraduate major. We hope that widespread discussion of this report will help focus the efforts at reform that are already underway on many campuses.

LYNN ARTHUR STEEN,
Task Force Chair

Challenges for College Mathematics: An Agenda for the Next Decade

Mathematics in Liberal Education

The 1985 AAC report *Integrity in the College Curriculum* [6] sets forth a vision of undergraduate education steeped in the tradition of liberal education. It describes study in depth in terms of the capacity to master complexity, to undertake independent work, and to achieve critical sophistication. To achieve the kind of depth envisioned by the authors of this report, students must grapple with connections, progress through sequential learning experiences, and enhance their capacity to discern patterns, coherence, and significance in their learning. Study in depth should enhance students' abilities to apply the approaches of their majors to a broad spectrum of problems and issues, and at the same time develop a critical perspective on inherent limitations of these approaches.

We are especially concerned in this study with how the experience in the major contributes to the education of the great majority of students who do not pursue advanced study in the field of their undergraduate major. Hence we focus more on the *quality* of students' engagement with their collegiate major than on curricular content which may be required for subsequent study or careers. This emphasis on general benefits of the major rather than on specific things learned gives the AAC undertaking a distinctive perspective that is not often emphasized in discussions of the mathematics major by mathematicians.

Historical Perspective

For over 35 years the Committee on the Undergraduate Program in Mathematics (CUPM) has helped provide coherence to the mathematics major by monitoring practice, advocating goals, and suggesting model curricula. Until the 1950s, most mathematics departments functioned primarily as service departments for science and engineering. CUPM was established in 1953 to "modernize and upgrade" the mathematics curriculum and "to halt the pessimistic retreat to remedial mathematics." At that time total enrollment in mathematics courses in the United States was approximately 800,000; each year about 4,000 students received a bachelor's degree in mathematics, and about 200 received Ph.D. degrees.

Following an unsuccessful initial effort to introduce a "universal" first-year course in college mathematics, CUPM concentrated in its second decade on proposals to strengthen undergraduate preparation for Ph.D. study in mathematics [15]. Spurred on by Sputnik and assisted by significant support from the National Science Foundation, interest in mathematics rose to levels never seen before (or since) in the United States. By 1970 total undergraduate mathematics enrollments had expanded to over three million students; U.S. mathematics departments produced 24,000 bachelors and 1,200 doctoral degrees a year.

But then the bubble burst. As student interest shifted from personal goals to financial security, and as computer science began to attract increasing numbers of students who in earlier years might have studied mathematics, the numbers of mathematics bachelor's degrees dropped by over 50% in ten years, as did the number of U.S. students who went on to a Ph.D. in mathematics. However, total undergraduate mathematics enrollments continued to climb as students shifted from studying mathematics as a major to enrolling in selected courses that provided tools necessary for other majors. In 1981, at the nadir of B.A. productivity, CUPM published its second major comprehensive report on the undergraduate curriculum [16] which advocated a broad innovative program in *mathematical sciences.*

During the 1980s the number of undergraduate majors rebounded, although both the number and the percentage of U.S. mathematics students who persist to the Ph.D. degree continued to decline [1, 21]. As a consequence, one of the most urgent problems now facing American mathematics is to restore an adequate flow in the pipeline from college study to Ph.D. attainment. Three recent reports from the National Academy of Sciences [39, 9, 44] document this crisis of renewal now confronting U.S. mathematics.

The precipitous decline in the 1970s and early

1980s in the number of undergraduate mathematics majors paralleled both a significant decrease in mathematical accomplishment of secondary school graduates and an erosion of financial support for mathematical research. Strangely, all this occurred at precisely the same time that the full scope of mathematical power was unfolding to an unprecedented degree in scientific research and business policy. These conflicting forces produced a spate of self-studies within the mathematical community, some devoted to school education [13, 19, 40, 41], some to college education [2, 50, 59], a few to graduate education [8, 12], and others to research issues [9, 24, 29]. The crisis of confidence among mathematicians reflected broad public concern about the quality and effectiveness of mathematics education [24, 45, 32]. Even research mathematicians are now taking seriously the crisis in school mathematics [27, 62]. A recent report [43] by the National Research Council recommends sweeping action based on an emerging consensus with broad support that extends well beyond the mathematical community.

Market-driven demand for undergraduate mathematics majors is strong. There is a shortage of mathematical scientists with doctor's degrees; there is consistent demand (with high salaries) for those who hold master's degrees in the mathematical sciences; there is steady demand for undergraduate mathematics majors in entry-level positions in business, finance, and technology-intensive industry; and there is increasing need for well-qualified high school teachers of mathematics, especially as a consequence of the national effort to raise standards for school mathematics. Moreover, the increasingly analytical nature of other professions (business, law, medicine) as well as the continued mathematization of science and engineering provide strong indicators of the long-range value of an undergraduate mathematics major for students who are entering other professions.

Today, mathematics is the second largest discipline in higher education. Indeed, over 10% of college and university faculty and student enrollments are in departments of mathematics. However, more than half of this enrollment is in high school-level courses, and most of the rest is devoted to elementary service courses. Less than 10% of the total post-secondary mathematics enrollment is in post-calculus courses that are part of the mathematics major. Even in these advanced courses many students are not mathematics majors—they are enrolled to learn mathematical techniques used in other fields. As a consequence, the major has suffered from neglect brought about in part by the overwhelming pressure of elementary service courses.

In spite of the 1965 and 1981 reports from CUPM and dozens of collateral studies published in the last decade, there is no national consensus about the undergraduate mathematics curriculum. However, examination of practices at many campuses reveals common threads that are highly compatible with goals of the AAC *Integrity* study. These include a multiple track system that addresses diverse student objectives, emphasis on breadth of study in the major, and requirements for depth that help students achieve critical sophistication. In this report we build on these common threads to suggest new goals for study in depth that will both enhance the mathematics major and better suit students who will live and work in the twenty-first century.

Common Goals

Mathematics shares with many disciplines a fundamental dichotomy of instructional purpose: mathematics as an object of study, and mathematics as a tool for application. These different perspectives yield two quite different paradigms for a mathematics major, both of which are reflected in today's college and university curricula. The first, reminiscent of the CUPM recommendations of the 1960s, focuses on a core curriculum of basic theory that prepares students for graduate study in mathematics. The second, reflecting the broader objectives of CUPM's 1981 mathematical sciences report, focuses on a variety of mathematical tools needed for a "life-long series of different jobs." Typically faculty interests favor the former, whereas student interests are usually inclined towards the latter. Most campuses support a mixed model representing a locally devised compromise between the two standards, although accurate descriptions of present practice are virtually non-existent. (Partly in response to the paucity of information about degree programs in mathematics, the National Research Council through its project Mathematical Sciences in the Year 2000 is seeking to stimulate a comprehensive examination of degree patterns in mathematics.)

The 1985 AAC *Integrity* study, echoing themes reflected in similar studies [10, 38, 42], provides a broad context in which to examine practices of individual disciplines. The AAC report highlights "study in depth" as an essential component of liberal education, but not for the reasons commonly advanced by students and their parents—preparation for vocation, for professional study, or for graduate school. *Integrity* views study in depth as a means to master complexity, to grasp coherence, and to explore subtlety. "Depth cannot be reached merely by cumulative exposure to more and more ... subject matter." The AAC goals for study in depth are framed by twin concerns for intellectual coherence intrinsic to the discipline and for development of students' capacity to make connections, both within their major and with other fields.

Both previous models advanced by CUPM for the undergraduate major reflect these same potentially conflicting concerns. The earlier major, which emphasized a traditional core mathematics curriculum as preparation for graduate study, was motivated principally by the internal coherence of mathematics; the more recent proposal stressed an interplay of problem solving and theory across both the broad spectrum of the mathematical sciences and the boundaries that mathematics shares with other disciplines. Although many believe that the latter model, emphasizing broadly applicable mathematical methods, is better geared to graduates' future employment, others—including many liberal educators—believe that broader abilities such as the art of reasoning and the disposition for questioning are of greater long-term practical value. However, both types of majors—and the many mixtures that exist on today's campuses—help students discern patterns, formulate and solve problems, and cope with complexity. In this sense, present practice of mathematical science majors in U.S. colleges and universities matches well the overall objectives of *Integrity in the College Curriculum.*

Certain principles articulated by CUPM in 1981 (reprinted in a recent compendium [17] of CUPM curriculum recommendations from the past decade) make explicit areas where *Integrity's* objectives and those of the mathematical community align:

- The primary goal of a mathematical sciences major should be to develop a student's capacity to undertake intellectually demanding mathematical reasoning.
- A mathematical sciences curriculum should be designed for all students with an interest in mathematics, with both appropriate opportunities for average mathematics majors and appropriate challenges for more advanced students.
- Every student who majors in the mathematical sciences should complete a year-long course sequence at the upper-division level that builds on two years of lower-division mathematics.
- Instructional strategies should encourage students to develop new ideas and discover new mathematics for themselves, rather than merely master the results of concise, polished theories.
- Every topic in every course should be well motivated, most often through an interplay of applications, problem-solving, and theory. Applications and interconnections should motivate theory so that theory is seen by students as useful and enlightening.
- Students majoring in mathematics should undertake some real-world mathematical modelling project.
- Mathematics majors should complete a minor in a discipline that makes significant use of mathematics.

Emphasis on coherence, connections, and the intellectual development of all students are evident in these principles from the 1981 CUPM report. At the level of broad goals, the prevailing professional wisdom concerning undergraduate mathematics matches well the intent of AAC's *Integrity.*

Diverse Objectives

Once one moves beyond generalities and into specifics of program development, however, mainstream mathematical practice often diverges from many of the explicit AAC goals. Most students study mathematics in depth not to achieve broad goals of liberal education but for some professional purpose—for example, to support their study of science or to become a systems analyst, teacher, statistician, or computer scientist. Others study mathematics as a liberal art, an enjoyable and challenging major that can serve many ends. It is as true in mathematics as in any other field that the great majority of undergraduate mathematics majors do not pursue advanced study that builds on their major.

Because so few U.S. students pursue graduate study in the mathematical sciences, many mathematicians believe that the mathematics major should be strengthened in ways that will prepare students better for graduate study in mathematics. Whereas formerly this view may have been based on the myopic academic view of undergraduate education as the first step in the reproduction of university professors, today in mathematics this perspective is reinforced by vigorous argument based on an impending shortage of adequately trained mathematics faculty. Some argue for greater depth to ensure that all mathematics majors are capable of pursuing further study; others argue for greater breadth to attract more students to the mathematical sciences. National need has now reinforced self-interest in emphasizing preparation of the next generation of Ph.D. mathematicians as an important priority for many college and university mathematics departments [20, 39].

This said, there remains considerable room for debate about strategies for achieving the several different (but overlapping) objectives that are common among the majors offered by the 2500 mathematics departments in U.S. colleges and universities:

- ADVANCED STUDY. Preparation for graduate study in various mathematical sciences (including statistics and operations research) or in other mathematically based sciences (e.g., physics, computer science, economics).
- PROFESSIONAL PREPARATION. Skills required to pursue a career that requires considerable background in mathematics:
  - *Natural and Social Science.* Background for careers in science or engineering, in biology (including agriculture, medicine, and biotechnology), in many computing fields, and in such emerging interdisciplinary fields as cognitive science or artificial intelligence.
  - *Business and Industry.* Preparation for careers in management, finance, and other business areas that use quantitative, logical, or computer skills normally developed through undergraduate study of mathematics.
  - *School Teaching.* Preparation for teaching secondary school mathematics, or for careers as mathematics-science specialists for elementary school.
- LIBERAL EDUCATION. General background for non-mathematical professions such as law, medicine, theology, or public service, or for other employment which does not directly use mathematical skills.

There is no lack of sources for advice about objectives. Recent documents outline priorities for teacher training [14], applied mathematics [25], and service courses [33]. The diversity of U.S. education ensures that different departments will have different priorities that recognize differences in entering students, in goals of graduates, and in institutional missions.

For many students the link between their undergraduate major and their post-graduate plans is very elastic. Such students study mathematics for the same reason that hikers climb mountains—for challenge, for fun, and for a sense of accomplishment. Often mathematics is paired with another major to provide complementary strengths. At one institution this has helped strengthen the mathematics major while enhancing connections to other fields of study:

> An analysis of the senior class shows that nearly half of the graduates had double majors. In many cases the mathematics major was taken to provide theoretical and computational grounding necessary for a modern approach to primary majors in economics, physics, chemistry, or biology. In other cases availability of a double major gave students who enjoyed mathematics an opportunity to continue their studies in mathematics while also gaining a desired major in an apparently unrelated field—for example, English, philosophy, or music. Many of our majors chose mathematics as a sound liberal arts approach to a general education. The success of the major program in mathematics is due in part to the belief of both faculty and students that the study of mathematics is not just for those intending to pursue a career in the area.

Some argue that the goals of liberal education are best served by a mathematics major designed to prepare students for graduate school. Even though most mathematics majors never undertake further study of mathematics, advocates of pre-graduate preparation argue that the special combination of robust problem solving with rigorous logical thinking achieved by a solid pre-graduate major also serves well the more general objectives of sequential study, intellectual development, and connected learning. This view is substantiated in part by a strong history of mathematics majors being eagerly sought by employers and graduate programs in other areas (e.g., law, economics) for a wide variety of non-mathematical professions.

Others argue that since most of today's college students do not foresee graduate study in mathematics as a desirable goal, it is only through a more general major stressing links to other fields that enough students can be recruited to major in mathematics to serve as a proper basis for the nation's needs in teaching, in science, and in mathematics itself. In this view, a broad major stressing mathematics as part of liberal education is an effective strategy for strengthening the pool of potential mathematical doctoral students as well as students in other mathematically oriented professions. It is rare to find a department of mathematics that would naturally place top priority on a major that specifically serves liberal education, as is common, for example, in many departments of English or philosophy. The needs of society and the constraints of client disciplines in science and engineering do not permit mathematicians this luxury. But in the present climate, departments are rediscovering the strategic value of a broad major, even for those who do continue professionally in mathematics. Here's how one institution expresses its objectives for the mathematics major:

> The mathematics major is designed to include students with a wide variety of goals, tastes, and backgrounds. Mathematics is an excellent preparation for fields from technical to legal, from scientific to managerial, and from computational to philosophic. It is also a source of satisfaction for people in every line of endeavor. Recognizing this, we have constructed a program to welcome interested students of all sorts. Our comparatively unstructured program reflects not only the diversity of interests of our students, but their increasingly diverse backgrounds, and the increasingly diverse nature of what are now being called the "mathematical sciences."

Since department goals must match institutional missions, it would not be right for any committee to recommend uniform goals for individual departments. We can, however, urge increased attention to an important and distinctive feature of undergraduate mathematics: *National need requires greater encouragement for students to continue their study of mathematics beyond the bachelor's level—whether as preparation for school teaching, for university careers, or for employment in government and industry.* In some institutions this encouragement may arise from a thriving pregraduate program; in others it may evolve from an emphasis on liberal education. In all cases, departmental objectives must be realistically matched to student aspirations and to institutional goals. Wherever faculty and students share common objectives, mathematics can thrive.

Multiple Tracks

Most mathematics departments resolve the dilemma of diverse goals for the major with some sort of track system. In some institutions there are separate departments such as Applied Mathematics, Operations Research, Statistics, or Computer Science, whereas in others these options are accommodated by means of explicit or implicit choices within the offerings of a single Department of Mathematics or Department of Mathematical Sciences. Tracks within the major are a sensible strategy to respond to competing interests of students, of faculty, and of institutions.

Although tracks do accommodate student interests—and thereby help sustain enrollments—they can produce a fragmented curriculum. Whereas in the late 1960s most mathematics majors took six or seven standard courses before branching into electives, by 1981 CUPM found that there was no longer any national agreement on such an extensive core of the undergraduate mathematics major. At that time, only elementary calculus and linear algebra were universally recognized as required courses within the mathematics major. Branching occurred after the third or fourth course, rather than after the sixth or seventh.

Today however, despite institutional diversity, there is striking uniformity in the elementary and intermediate courses pursued by mathematics majors: all begin with calculus for two, three, or four semesters; most introduce linear algebra in the sophomore year and require one or two semesters of abstract algebra; virtually all require some upper division work in analysis—the "theory of calculus." Nowadays, most require some computer work (programming, computer use, or computer science) as well as some applied work (statistics, differential equations, etc.) among the electives. This restores a *de facto* six-course core to the major, typically half the total.

The rise, fall, and restoration of a core curriculum in the mathematics major paralleled similar patterns in other arts and sciences. Whereas the CUPM recommendations of the late 1960s may have had too narrow a focus, the subsequent curricular chaos of the mid-1970s may have been too unstructured. Mathematicians began worrying then—as AAC is now—about whether the typical student's experience with the mathematics major may lack appropriate coherence and depth. Too often, it seemed to many, the mathematics major had become just an accumulation of courses without sufficient structure to ensure a common core of learning.

In most colleges courses in the mathematical sciences are taught in many different departments. Even upper division mathematics courses are commonly taught in departments of application: a recent survey [28] shows that as many students study post-calculus mathematics outside departments of mathematics as do so within traditional mathematics course offerings. Examples include discrete mathematics taught in departments of computer science; methods of mathematical physics taught in departments of physics; logic and model theory taught in departments of philosophy; optimization and operations research taught in departments of economics; and mathematical foundations of linguistics taught in departments of linguistics. The practice of cross-listing such courses or counting them as electives in the major varies enormously (and arbitrarily) from campus to campus.

Whether for good or ill, the diffusion of mathematics courses both within departments of mathematical sciences and into other departments has moved the mathematics major away from a strict linear vertical pattern towards a more horizontal structure typical of the humanities or social science major. Today's major, however, retains a distinctive strength of mathematics: sequenced learning. By its very nature, mathematics builds on itself and reinforces links among related fields. All mathematics courses build on appropriate prerequisites. A student who progresses from calculus to probability to operations research sees just as many connections as does one who moves through the more traditional sequence of advanced calculus and real analysis. Although the focus of each student's work is different, the contributions made by each track to the general objectives of study in depth are comparable, and equally valuable.

Moreover, it is common for advanced courses to be offered in sequences (e.g., Abstract Algebra I, II; Real Analysis I, II; Probability and Statistics I, II) that begin with a three or four course chain of prerequisites. Many departments, following the 1981 CUPM recommendations, require mathematics majors to take some advanced sequence without specifying which particular sequence it should be. Thus most mathematics majors today take a substantial sequence of courses, but they no longer all take the *same* sequence of core courses. This is a wise policy for undergraduate mathematics in today's diverse climate: *Each student who majors in mathematics should experience the power of deep mathematics by taking some upper-division course sequence that builds on lower-division prerequisites. It is neither necessary nor wise, however, to require that all mathematics majors take precisely the same sequence.*

Flexibility with rigor can be administered in a variety of ways. In one institution with a flourishing mathematics major, the mechanism is a personal "contract" developed to suit each student's own objectives:

> Students arrange their major sequence according to a contract system. Potential majors meet with a member of the department and prepare a list of courses and activities that will constitute the major. This allows the student to arrange his or her program to suit special needs. The faculty member judges the appropriateness of the student proposal in terms of post-graduate plans, other studies, and general departmental guidelines. This contract system has two distinct advantages: it serves the personal needs of students, and the process itself enhances students' education. The process of developing the contract provides an opportunity for the student to work closely with a faculty member, to understand the variety of mathematical options in a personal framework, and to see how a program ensuring depth and breadth of study can be achieved.

Emphasizing Breadth

At the same time as we stress the value of sequential courses to study in depth, we must also emphasize the essential contribution of breadth to building mathematical insight and maturity. Whereas course sequences demonstrate depth by building in expected fashion on prior experience, the links that emerge among very different courses (tying geometry to calculus, group theory to computer science, number theory to analysis) reveal depth by indirection: such links point to deeper common principles that lie beyond the student's present understanding but are within grasp with further study. They show the mountain yet to be climbed—to shift metaphors from depth to height—and offer hints of the explanatory panorama to be revealed by some future and more profound principles.

There are other good reasons to recommend breadth as an important objective of an undergraduate mathematics major. Students who are introduced to a variety of areas will more readily discern the power of connected ideas in mathematics: unexpected links discovered in different areas provide more convincing examples of a deep logical unity than do the expected relationships in tightly sequenced courses.

For the many majors who will teach (either in high school or college), it is vitally important that their undergraduate experience provide a broad view of the discipline—since further study generally is more narrow and specialized. For those seeking their niche in the world of mathematics, a broad introduction to many different yet interconnected subjects, styles, and techniques helps pique interest and attract majors. And for the many students who may never make professional use of mathematics, depth through breadth offers a strong base for appreciating the true power and scope of the mathematical sciences. Graduates of programs that emphasize breadth will become effective ambassadors for mathematics.

Every student who majors in mathematics should study a broad variety of advanced courses in order to comprehend both the breadth of the mathematical sciences and the powerful explanatory value of deep principles. Such breadth can sometimes be achieved with courses offered by the department of mathematics, but more often than not it would be educationally advantageous for students to also select a few mathematically-based courses offered by other departments.

Effective Programs

Departments of mathematics in colleges and universities exhibit enormous variety in goals and effectiveness. In various universities, the percentage of bachelor's degrees awarded to students with majors in mathematics ranges from well under one-half of 1 percent to over 20 percent. In some departments the major is designed primarily to prepare students for graduate school. Other departments focus much of their major on preparing students to teach high school mathematics, or on preparing students for employment in business and industry. Most departments fail to attract or retain many Afro-American, Hispanic, or Native American students, whereas a few succeed in this very difficult arena.

Many measures can be used to monitor effectiveness of a mathematics major. Indicators of numbers of majors, of employability, of graduate school admissions, of eventual Ph.D.s, or of placement in teaching jobs are used by different departments according to their self-determined missions. Many mathematics departments work hard to improve their effectiveness in one or another of these different dimensions. Exploration, experimentation, and innovation—along with occasional failures—are the hallmarks of a department that is committed to effective education.

Mathematics programs that work can be found in all strata of higher education, from small private colleges to large state universities, from average to highly selective campuses. The variety of mathematics programs that work reveal what can be achieved when circumstance and commitment permit it. When faculty resolve is backed by strong administrative support, most mathematics departments can easily adopt strategies to build vigorous majors even while meeting other service obligations.

One department that has had great success in attracting students to major in mathematics bases its work on two "articles of faith:"

- We believe that faculty should relate to their students in such a way that each student in the department will know that someone is personally interested in him and his work.
- We believe that careful and sensitive teaching that helps students develop confidence and self-esteem is far more important than curriculum or teaching technique.

Another department builds strength on a foundation of excellent introductory instruction:

> We put our best teachers in the introductory courses. We put the most interesting material in the introductory courses. We try to make the statements of problems fun, not dry. We work very hard to motivate all topics, drawing on applications in other disciplines and in the working world. We are less interested in providing answers than in motivating students to ask the right questions.

Effective mathematics programs reflect sound principles of psychology as much as important topics in mathematics:

> We try to make students proud of their efforts in mathematical problem-solving, and especially proud of their partial solutions—what some might call mistakes. We look at how much is right in an answer and teach how to detect and correct the parts that are wrong.

Regular, formal recognition of student achievement at different stages of the major serves to build students' confidence and helps attract students to major in mathematics. Students know mathematics' reputation for being challenging, so recognition of honest accomplishment can provide a tremendous boost to a student's fragile self-esteem. *Effective programs teach students, not just mathematics.*

Challenges for the 1990s

Changes in the practice of mathematics and in the context of learning pose immense challenges for college mathematics. Many of those issues that pertain directly to course content, curricular requirements, and styles of instruction are under review by committees of the mathematical community. We focus here on challenges that transcend particular details of courses and curriculum:

- The *learning* problem: To help students learn to learn mathematics.
- The *teaching* problem: To adopt more effective styles of instruction.
- The *technology* problem: To enhance mathematics courses with modern computer methods.
- The *foundation* problem: To provide intellectually stimulating introductory courses.
- The *connections* problem: To help students connect areas of mathematics and areas of application.
- The *variety* problem: To offer students a sufficient variety of approaches to a mathematics major to match the enormous variety of student career goals.
- The *self-esteem* problem: To help build students' confidence in their mathematical abilities.
- The *access* problem: To encourage women and minorities to pursue advanced mathematical study.
- The *communication* problem: To help students learn to read, write, listen, and speak mathematically.
- The *transition* problem: To aid students in making smooth transitions between major stages in mathematics education.
- The *research* problem: To define and encourage appropriate opportunities for undergraduate research and independent projects.
- The *context* problem: To ensure student attention to historical and contemporary contexts in which mathematics is practiced.
- The *social support* problem: To enhance students' personal motivation and enthusiasm for studying mathematics.

These challenges have more to do with the success of a mathematics program than any curricular structure. In the diverse landscape of American higher education, successful programs differ enormously in curricular detail, but they all have in common effective responses to many of these broader challenges. The agenda for undergraduate mathematics in the 1990s must focus at least as much on these issues of context, attitude, and methodology as on traditional themes such as curricula, syllabi, and content.

Learning

One principal goal of the undergraduate mathematical experience is to prepare students for lifelong learning in a sequence of jobs that will require new mathematical skills. Departments of mathematics often interpret that goal as calling for breadth of study. But another interpretation is just as important: because mathematics changes so rapidly, undergraduates must become independent learners of mathematics, able to continue their own mathematical education once they graduate.

Most college students don't know how to learn mathematics, and most college faculty don't know how students do learn mathematics. It is a tribute to the efforts of individual students and teachers that any learning takes place at all. Effective programs pay as much attention to learning as they do to teaching.

First-year students need special attention to launch their college career on a suitable course. Typically, they carry with them a high school tradition of passive learning which emphasizes bite-sized problems to be solved by techniques provided by the textbook section in which the problem appears. Unfortunately, by maintaining this traditional teaching format which perpetuates the myth of passive mathematics learners, college calculus teachers typically contribute more to the problem than to its solution.

For example, calculus, the common entry point for potential mathematics and science majors, often fails to come alive intellectually as it should or as it is now at many institutions where calculus reform efforts are underway. One school has found that new goals for calculus can significantly enhance the entrée of students into the study of college mathematics:

> The larger goals of the major are reflected in the calculus sequence, which is founded on three principles: context, collaboration, and communication. "Context" means that we focus on the meaning and significance of calculus in the world. "Collaboration" means that students work in groups and support each other. "Communication" means the recognition that calculus is first of all a language, not only for scientists, but for economists and social scientists. Our goal is fluency.

Another institution uses calculus as a vehicle to broaden radically the view of mathematics that students bring with them from high school:

> Calculus should give students a solid base for advanced study. It is our opinion that our calculus courses were the weakest part of our program. We had, in effect, allowed the high schools to set the tone for our entire program. Our new course is so radically different from traditional calculus that our students are forced to confront the transition from school to college mathematics. It carries several important messages, e.g., mathematics is crucial for understanding science; mathematics has a strongly experimental side; mathematics is something we all are capable of understanding deeply; and mathematics is the most powerful of all the sciences.

Some institutions offer special freshman seminars as a way to encapsulate the ideal of liberal education in an intimate setting that permits students to identify with faculty mentors. However, in mathematics the massive tradition of calculus often stands in the way, so very few mathematics majors can trace the origin of their college major to a freshman seminar. Ideally calculus itself would be seen by colleges as the intellectual equivalent of a freshman seminar in which students learn to speak a new language. If that analogy were to be accepted, mathematics departments would teach calculus only in a context that placed a great deal of emphasis on one-to-one communication between student and teacher. Unfortunately, in too many institutions calculus is taught in large impersonal settings that make meaningful person-to-person dialogue unrealistic. Many efforts are now underway to reform the teaching of calculus [64]; most of these experiments emphasize student motivation and styles of learning as a primary factor in reshaping the course.

One way or another, students should learn early in their college years how to study and learn mathematics. They should learn psychological as well as mathematical strategies for solving problems. They should come to recognize that it is common even for mathematicians to hear lectures or read material that they cannot grasp, and they must learn how to pick up clues from such experiences that will fit into their personal mathematical puzzles only some time later. They should learn the value of persistence and the strategic value of going away and coming back. These "metacognitive" skills to control one's own learning are virtually never learned in high school mathematics, so they must be planned into the early stages of the college curriculum.

As students progress through their mathematical study, they need to learn the value of library and electronic resources as tools for learning. Mathematics students rarely use the library or other sources of information, concentrating instead on mastering material in course texts. They need specific assignments that focus on the big map of mathematics in order to gain perspective on their brief undergraduate tour. *Undergraduate students should not only learn the subject of mathematics, but also learn how to learn mathematics.* The major in mathematics should become more than the sum of its courses. By conscious effort to help students negotiate in unfamiliar terrain, instructors can provide them with the tools of inquiry necessary to approach the literature and learn whatever they need to know.

Teaching

The purpose of teaching, and its ultimate measure, is student learning. So in some sense one cannot discuss one without the other. However, as students must learn to learn, so teachers must learn to teach. In mathematics more than in most other subjects, the role of teaching assistants and part-time instructors is particularly important, especially in the first year [12]. Although there is no formula for successful teaching, there is considerable evidence that separates certain practices that have proven successful from those that are generally ineffective [55]. Teachers who study this evidence can learn much from the experiences of others.

Despite the general reputation of mathematics as one of the most desirable environments for developing rigorous habits of mind, criticism of undergraduate mathematics has been mounting in recent years for failure in this, mathematics' distinctive area of strength. Those who study cognitive development criticize standard teaching practices for failing to develop fully students' power to apply their mathematical knowledge in unfamiliar terrain. These critics conclude that present teaching practice in undergraduate mathematics does not do as much as it should to develop students' intellectual power.

The evidence of failure is persuasive, both locally and globally [57]. Data on the inability of the profession to attract and retain the best and brightest college graduates is confirmed by case studies of students who cannot make effective use of what they have learned. Although some very good students use a mathematics major as a platform for substantial accomplishment, the majority of those who major in mathematics never move much beyond technical skills with standard textbook problems. Passive teaching and passive learning results from an unconscious conspiracy of minimal expectations among students and faculty, both of whom find advantages in a system that avoids the challenges of active learning that fully engages both students and teacher. Both the curriculum and teaching practices must respond to this challenge of intellectual malnutrition that is all too common in today's major.

Much of the research that bears on how students learn college mathematics has been conducted in the setting either of high school mathematics or college physics. The results from these efforts are often surprising, yet not well known among university mathematicians. They show, among other things, that formal learning by itself rarely influences behavior outside the artificial classroom context in which the concept was learned [53, 54]. Students who know how to solve differential equations of motion often have no better insight into the behavior of physical phenomena described by these equations than do others who never studied the equations; students who have learned course-based tests of statistical significance frequently do not recognize statistical explanations for events in the world around them [47].

Additional evidence of how young adults learn mathematics—or more often, why they fail to learn—has accumulated in recent years as a result of many innovations in teaching tried on different campuses. For example, intervention programs designed to improve the mathematical performance of minority students show the importance of a supportive environment: constructive teamwork in a context of challenging problems in which instructors and students know each other personally builds mathematical self-esteem and, as a consequence, leads to greatly improved learning [5, 30, 63]. Very different but equally striking lessons emerge from experiences of students who study calculus in a technology-intensive environment: by forcing students (and instructors) to focus on the behavior of mathematical objects (functions, algorithms, operators) rather than on their formalism, and by integrating visual, numerical, and symbolic clues into the mathematical environment, computers reveal to students and faculty both avenues for insight and common sources of misconception [36].

A third example, but by no means the last that could be cited, can be found in evidence of improved student motivation and self-reliance that occurs in those contexts where research-like experiences are used to enrich traditional classroom and textbook experiences: students whose minds and eyes become engaged in the challenge of true discovery are frequently transformed by the experience [56].

The evidence from such diverse but non-traditional instructional environments shows clearly the effectiveness of instruction that builds self-confidence on the foundation of significant accomplishment in a context that is meaningful to the student. Here is an especially dramatic example:

> In 1986 we began a critical evaluation of our program, course offerings, and teaching methods. This examination led to profound changes in our understanding of the teacher-student relationship, and of our role in the educational process. We found, for example, that we had not engaged our students sufficiently to assume an active role in their learning of mathematics. So we deliberately modified our courses and attitude to experiment with active student participation in doing mathematics problems and theory both in class and outside class.
>
> Results were strikingly positive, and we largely discontinued the typical mathematics lecture format, since lecturing kept students in a passive role. With an active participation method, students studied the text and worked problems before class; faculty and students discussed difficult points in class. Students presented problems and results on the board in class with encouragement and guidance from the instructor. We found that students became actively and enthusiastically involved in their learning of mathematics, with the instructor acting as a coach.
>
> As a consequence of these changes, our faculty

> and students have become a community of learners and scholars. Students now do mathematics in groups outside class, and more graduating seniors are seeking advanced degrees in mathematics. The number of mathematics majors rose from 69 in 1986 to 104 in 1988; the Mathematics Department is now the largest unit in the School of Natural Sciences. Finally, and perhaps most important, faculty affirm the belief that many more students can realize their mathematical abilities.

Several barriers separate educational studies and experiments from the larger community of college and university mathematicians. First, there are very few individuals who conduct formal research dealing directly with college mathematics. Second, mathematicians tend to distrust educational research. Third, and perhaps more important, mathematicians follow habit more than evidence in their teaching styles: even well-documented reports of better methods are insufficient to influence mathematicians to change their teaching habits. (This is not really too surprising, since neither do convincing classroom explanations of effective mathematical methods suffice to eradicate deep-seated misconceptions among students.)

Too often mathematicians assume with little reflection that what was good for their education is good enough for their students, not realizing that most of their students, not being inclined to become mathematicians, have very different styles of learning. College faculty must begin to recognize the proven value of various styles of instruction that engage students more directly in their own learning. *Those who teach college mathematics must seek ways to incorporate into their own teaching styles the findings of research on teaching and learning.*

Studies of metacognition and problem solving have yielded some insights that could be useful in pedagogy, but they have also been frustrated by barriers that confront all teachers of mathematics (for example, the difficulty of assessing just what has been learned, and the great length of time required to develop effective problem-solving heuristics). Such studies may yield insights that will change for the better the way teachers teach and students learn. But so far, college-level evidence is sufficiently slim to make the case unconvincing to those who most need to be persuaded. We really don't know how to induce most students to rise to the challenge of mathematical thinking; we have much to learn about what works and what does not. *To improve our understanding of the intellectual development of college mathematics students, mathematicians should increase their efforts to conduct research on how college students learn mathematics.*

We need to experiment with new ways to evaluate teaching. One key factor in good teaching is how much students learn; other factors include such issues as how many students decide to major in mathematics, to go on to graduate school, or to work in mathematical careers. These are measures of the quality of teaching done both by an individual and by a department. They look not only to indicators such as demonstrable knowledge, but also to motivation, attitude, and enthusiasm for the discipline. *Evaluation of teaching must involve robust indicators that reflect the broad purposes of mathematics education.*

Technology

Computing has changed profoundly—and permanently—the practice of mathematics at every level of use. College mathematics departments, however, often lag behind other sciences in adapting their curricula to computing, although considerable momentum is now building within the community for greater use of computing. The delay in response may have been due in part to conservatism of mathematicians, but at least as important is the simple fact of computer power: only in the last few years have desk-top machines achieved sufficient power to provide a legitimate aid to undergraduate (and research) mathematics. As a consequence, scientific computation is becoming a third paradigm of scientific investigation—alongside experimental and theoretical science—and the role of experiment in the practice of mathematics itself is increasing [52].

Computing can enhance undergraduate study in many ways. It provides natural motivation for many students, and helps link the study of mathematics to study in other fields. It offers a tool with which mathematics influences the modern world and a means of putting mathematical ideas into action. It alters the priorities of courses, rendering certain favorite topics obsolete and making others, formerly inaccessible, now feasible and necessary [34, 68]. Computers facilitate earlier introduction of more sophisticated models, thus making instruction both more interesting and more realistic. The penetration of computing into undergraduate mathematics is probably the only force with sufficient power to overcome the rigidity of standardized textbooks [59, 66, 22].

The power of technology serves also an epistemological function by forcing mathematicians to ask anew what it means to know mathematics. Those who explore the impact of technology on education indict introductory mathematics courses for imparting to students mostly skills that machines can now do more accurately and more efficiently. It is certainly true that typical indicators of student performance document primarily that mathematics students can carry out prescribed algorithms—just what computers (or calculators) can do. College faculty can no longer avoid the deep challenge posed by computers for undergraduate mathematics: once calculations are automated, what is left that can be taught effectively to average students?

Responses to this challenge are taking shape in experimental programs in many departments of mathematics. It is, therefore, too early to describe the impact computing will have on the mathematics major. Certainly in those courses and tracks devoted to applied mathematics, computing must exert a major influence on the shape of the curriculum. In this age it would be unconscionable to offer a major in applied mathematics, statistics, or operations research without substantial and fully integrated use of computer methods. Change will come more slowly in core subjects such as topology, analysis, and algebra. In each of these subjects there are impressive computer-based applications (e.g., fractals, coding theory, dynamical systems), yet none of these applications has been central to the traditional methodology of the subject as taught in introductory courses. Despite differences in the pace of change, however, there is no turning back: computers have dramatically altered the practice of mathematics. *To ensure an effective curriculum for the twenty-first century, undergraduate mathematics should change—both in objectives and in pedagogy—to reflect the impact of computers on the practice of mathematics.*

Early experiments that make significant use of computing in undergraduate mathematics courses show that as the balance of student work shifts from computation—which machines do better than humans—to thought, the course becomes more difficult, more unsettling, and less closely attuned to student expectations [58]. As the ground rules of mathematics change from carrying out prescribed procedures to formulating problems and interpreting results, it will become more important than ever for faculty to communicate clearly to students the goals of the curriculum and how they might differ from what students have been led to believe by their prior study of school mathematics.

One institution reports that computers have changed the context of education in significant and unexpected dimensions:

> We constructed a strong computer-experimental component at all levels. Besides the obvious advantages for building experience, context, and intuition, there are less obvious payoffs. For example, laboratories are places where students spend lots of time and which become, in reality, their habitats outside of their dormitory rooms. Students form allegiances and friendships in laboratories.

Different types of surprises were revealed on another campus that has pioneered use of computers in advanced courses:

> The use of computer software made possible the introduction of topics previously reserved for graduate students. Examples include the use of MACSYMA, REDUCE, and MACAULEY in commutative algebra and algebraic geometry. For example, a 1989 honors thesis gave us strong evidence of the advances possible in learning mathematics with the help of computational aids.

Computers change not only how mathematics is practiced, but also how mathematicians think. Both changes are unsettling, yet ripe with opportunity for effective education. Indeed, in the realm of computing, students and faculty must grope together towards a new balance of power among the many components of undergraduate mathematics.

The transition of mathematics from a purely cerebral paper-and-pencil (or chalk-and-blackboard) discipline to a high technology laboratory science is not inexpensive. Space must be expanded for laboratories; classrooms and offices need to be equipped with computers and display devices; support staff must be hired to maintain both hardware and software; faculty must be given time to learn to use computers, to learn to teach with computers, and to redesign courses and entire curricula to reflect the impact of computing. Institutions must plan not only for an expensive transition, but also for continued operation at a higher plateau comparable to the traditional laboratory sciences. *Colleges must recognize in budgets, staffing, and space the fact that undergraduate mathematics is rapidly becoming a laboratory discipline.*

Foundations

Because of the highly sequenced nature of the mathematics curriculum, no student can complete an undergraduate mathematics major without having secured a proper foundation of calculus, linear algebra, and computing in the first two years of college. For many students, half of the credits required for the major are taken in the first two years. So the nature of mathematical learning in these years is of crucial importance both for individual success in completing a strong mathematics major, and for programmatic success in building a critical mass of upper class mathematics majors.

One-third of the first and second year college students in the United States are enrolled in two-year colleges, including over two-thirds of Afro-American, Hispanic, and Native American students. It is clear from these figures that any effort to strengthen the undergraduate mathematics major, especially to recruit more majors among minority students, must be carried out in a manner that includes two-year colleges as a full partner in preparing the foundation for study in depth.

The tradition of common texts and relatively standard syllabi for standard mathematics courses in the first two years has facilitated transfer of students and credits during these years even as it has mitigated against the open intellectual environment many believe to be essential for effective learning. Now, however, as momentum builds for reform of courses in the first two years, and as departments experiment in an effort to reshape the entire mathematics major, there is some risk that students from lower socio-economic backgrounds—the predominant clientele of the two-year colleges—will find themselves pursuing a course of study that is inconsistent with the efforts of four-year colleges to improve the undergraduate mathematics major.

Some four-year institutions that are engaged in curricular reform are extending the scope of their mathematics program to include informal consortia with other nearby institutions. One private Eastern liberal arts college is building just such arrangements into its mathematics program:

> Plans are underway to create a partnership with a local community college and a public school system to interest students, especially minority students, in mathematics.

Many institutions maintain regular ties with local high schools or community colleges, but it is rare to find such arrangements related specifically to mathematics departments. What is now rare should become common: *To ensure equal opportunity for access to undergraduate mathematics majors, mathematics departments should work with nearby two-year colleges to maintain close articulation of programs.*

Connections

Recent studies of the mathematical sciences [7, 8] point to two special features that have characterized twentieth-century research: the extensive growth in areas of application (no longer just limited to physics and engineering) and the impressive unity of mathematical theories (revealed by the frequent use of methods from one specialty to solve problems in another specialty). Connectedness, therefore, is inherent in mathematics. It is what gives mathematics its power, what establishes its truth, and what reveals its beauty.

Mathematics is widely recognized as the language of science. Its enabling role in the development of the physical sciences formed the paradigm of the scientific method. Today it is beginning to play a similar role in the biological sciences, where mathematical tools as diverse as knot theory, nonlinear dynamics, and mathematical logic are being applied to model the structure of DNA, the flow of blood, and the organization of the brain.

Similar connections have emerged in the human, social, and decision sciences. Statistical models undergird virtually every study of human behavior; axiomatic studies have helped establish a rigorous theory of social choice; and multi-dimensional mathematical analysis is employed widely to model the multitudinous attributes of economic, psychological, or social behavior. Today mathematics is truly the language of all science—physical, biological, social, behavioral, and economic.

Even as the connections multiply between abstract ideas of mathematics and concrete embodiments in the world, so too have the internal connections within the mathematical sciences proliferated. Key theorems and deep problems that link separate mathematical specialties have provided a force for vast growth of interdisciplinary research. Examples abound, including such areas as stochastic differential equations at the interface of probability theory and analysis; combinatorial geometry joining arithmetical methods of discrete mathematics to problems of space, shape, and position;

and control theory that employs tools from analysis, linear algebra, statistics, and computer science to formulate effective mechanisms of control for automated processes.

If the undergraduate major does not reveal connections, it has not revealed mathematics. Most mathematics courses and most mathematics majors do make substantial contributions to this objective. Indeed, it is not uncommon for sophomores to select mathematics as a major instead of chemistry or biology precisely because in their mathematics courses they can see more clearly the logical connections among different parts: in mathematics they can "figure things out" rather than just memorizing results. (Of course, many students make the opposite choice, but usually for other reasons.)

At its best, mathematics overflows with connections, both internal and external. But one must be honest: undergraduate courses do not always show mathematics at its best. At their worst, especially in lower-division courses through which both majors and non-majors must pass, they reveal mathematics as a bag of isolated tricks: problems in elementary courses are often solved more by recognition of which section of the text they come from than by any real understanding of fundamental principles. *Dealing with open-ended problem situations should be one of the highest priorities of undergraduate mathematics.* For example:

- Mathematics teachers could bring in outside ("real-world") examples to illustrate applications of material being studied in regular coursework.
- Student projects could emphasize connections, either to fields that use mathematics or from one part of mathematics to another.
- Greater emphasis on multi-step problems amenable to a variety of approaches would wean students away from the school tradition of bite-sized, self-contained problems.
- Problem-oriented seminars provide wonderful opportunities to explore links between various branches of mathematics.

Such problems would be pregnant with ambiguity, ripe with subtle connections, and overflowing with opportunities for multi-faceted analyses.

Variety

Mathematicians are fond of talking about an elusive concept called "mathematical maturity" that is the Holy Grail of undergraduate mathematics [60]. Maturity is one objective of study in depth, but its meaning must be derived from the context of a student's level and goals. Depth itself is a metaphor for many things. To a mathematician it signals knowledge, insight, complexity, abstraction, and proficiency; to some others it connotes such elusive concepts as ownership, empowerment, and control. Although most colleges equate study in depth with the major—a circumstance reflected also in this report—it is important to recognize that for some students the major may not achieve the objectives that many have for study in depth. For these students, curricular structures other than the traditional major may better approach their goals for study in depth.

Many college students study mathematics as an important adjunct to another field which is their primary interest (e.g., economics, education, biology). Some colleges offer joint majors that combine study of mathematics with study in a related field, usually tied together with some type of joint project. The ever-present danger in such options is that they merely combine two shallow minors without ever achieving the depth traditionally required in a major. Notwithstanding this risk, one must acknowledge that some objectives of study in depth are well within the range of an effective joint major, say, in mathematics and biology where senior students employ mathematical models based primarily on lower division mathematics to model a biological phenomenon and then test and modify the model based on laboratory data.

Teacher education poses a special case of particular significance, since mathematics is one of the few disciplines taught throughout all twelve grades of school. It is obviously important for our nation that school teachers be both competent and enthusiastic about mathematics. Special committees recommend standards for preparation of mathematics teachers [14, 18], and these recommendations provide one particular perspective on study in depth.

Prospective secondary school teachers of mathematics generally pursue an undergraduate degree that includes a major in mathematics, often constrained in special ways to ensure breadth appropriate to the responsibilities of high school mathematics teachers. However, the appropriate mathematical preparation of prospective elementary and middle school teachers—who commonly teach several subjects, and sometimes teach the whole curriculum—is subject to much debate these days. Many national studies have recommended that prospective elementary school teachers, like secondary school teachers, major in a liberal art or science rather than in the discipline of education. However, the traditional mathematics major is generally inappropriate for teachers at this level, and today there appears to be virtually no example of a viable alternative. Even more vexing is the question of achieving depth in mathematics appropriate to an elementary school teacher within a major in some other field. Some interesting ideas can be found in the "new liberal arts" initiative sponsored by the Alfred P. Sloan Foundation which has attempted to infuse quantitative methods in traditional liberal arts subjects [38].

Most of the issues, guidelines, and recommendations in this study focus on the traditional mathematics major, which is where most students who study mathematics in depth are to be found. However, study in depth can be done at any level and in many contexts. *Mathematics departments should take seriously the need to provide appropriate mathematical depth for students who wish to concentrate in mathematics without pursuing a traditional major.*

Self-Esteem

One of the greatest impediments to student achievement in mathematics is the widespread reputation of mathematics as a discipline for geniuses. Many facets of school and college practice conspire to portray mathematics in "macho" terms: only those who are bright, aggressive, and inclined towards arrogance are likely to succeed. Those who do not instantly understand—including many thoughtful, reflective, creative students—are made to feel "deeply dumb," like outsiders who don't get the point of an in-joke.

It is hard to overstate the power of intimidation to erode students' self-confidence. Many calculus teachers recognize the problem: bright freshman "show-offs"—usually white males—whose questions are designed not so much to elicit answers and build understanding as to demonstrate their superior intelligence to their classmates. The ritual is not unlike the bluffing maneuvers that male animals employ to claim dominant status in a herd. Many who are concerned about equality of opportunity believe that the widespread display of "geniusism" as a measure of worth in mathematics is in part a mask for sexism—an unconscious emphasis on behavior intended to preserve the *status quo* regarding access to leadership in teaching and research.

Fortunately there is a growing recognition in the mathematical community that old traditions must be replaced with new approaches better suited to the demographic realities of our age. We need to recognize that individuals bring very different but equally valuable strengths to the study of mathematics. A multiplicity of approaches that encourage student growth in many different dimensions is far more effective than a single-minded focus leading to a linear ranking in one narrow dimension of "brightness." Not every value in mathematical talent can be measured well by timed tests or intercollegiate competitions; the "Putnam powerhouse" is not the only standard by which undergraduate majors should be judged. (The William Lowell Putnam Examination, a national contest for undergraduates, is the Nobel competition of collegiate mathematics. It stumps even faculty with questions so hard that the median national score for undergraduates is frequently 0.)

Specific efforts to focus the mathematics curriculum on the interests and abilities of all students can bring dramatic results, as this campus report shows:

> At the time of the first registration for first-year students, fewer than 20 individuals in the entire freshman class indicate that mathematics is a possible major. A year later, the number is in the 50's, and by the junior year the number is over 100. One reason for this impressive increase in student interest in a mathematics major is the departmental position that mathematics is for everyone, not just the gifted. We attempt to demonstrate the power and applicability of mathematics by emphasizing breadth of study during the second and third years.

Self-confidence increases when students succeed, and decreases when they fail. "What students need to build self-confidence are genuine *small successes* of their own" [67]. Initial successes come from routine homework, but these are insufficient to the task. More effective are instructional strategies that engage the student in active learning: open-ended problems, team work that builds diverse problem-solving skills; undergraduate research experiences; independent study. *Building students' well-founded self-confidence should be a major pri-*

ority for all undergraduate mathematics instruction.

Access

Data from many sources [3, 48, 65] show that women and members of certain minority groups often discontinue their study of mathematics prematurely, before they are prepared appropriately for jobs or further school. Afro-American and Hispanic students drop out of mathematics at very high rates throughout high school and college; only a tiny fraction complete an undergraduate mathematics major [46]. In college, women major in mathematics almost as often as men do, but they persist in graduate studies at much lower rates. (Interestingly, mathematics comes closer to achieving an even balance of men and women among its undergraduate majors than virtually any other discipline; this record of equality disappears, however, in graduate school.)

Evidence from various intervention programs shows that the high drop-out rate among minority students can be reduced [4, 5, 30, 63]. Appropriate expectations that provide challenges without the stigma of "remediation" together with assignments and study environments that reinforce group learning have proved successful on many campuses. Mentoring programs of various types open doors of opportunity to women and minorities who have traditionally been under-represented in mathematically based fields. What becomes clear from these programs is that the tradition of isolated, competitive individual effort that dominates much mathematics instruction does not provide a supportive learning environment for all students.

Assignments that stress teamwork on problems chosen to relate to student interests can help many students succeed in mathematics. The experiences of students who work in teams to solve large computer science projects and of those who participate in science research groups show clearly the benefit of incentives for careful work that is created by the team atmosphere. Mathematicians must learn that the teaching strategies they recall as being successful in their own education—and in the education of a mostly white male professional class—do not necessarily work as well for those who grow up in vastly different cultures within the American mosaic.

Programs that work for minority students are built on the self-evident premise that students do not all learn mathematics in the same way. Classroom methods must fit both the goals of the major (e.g., to help students to learn to communicate mathematically) and the learning styles of individual students (e.g., need for peer support and positive feedback). These same principles apply to all students, not just to students of color. *To provide effective opportunities for all students to learn mathematics, colleges must offer a broader spectrum of instructional practice that is better attuned to the variety of students seeking higher education.*

Communication

College graduates with majors in mathematically-based disciplines are often perceived by society as being verbally inept: the stereotype of the computer hacker who cannot communicate except with a computer has permeated the business world, and tainted mathematics graduates with the same reputation. Recognizing the legitimate basis for this concern in the incomprehensible writing of their own upper-division students, many mathematics departments are beginning to emphasize writing in mathematics courses at all levels.

The forms of writing employed in mathematics courses include the standard genres used in other disciplines (expository essays, personal journals, laboratory reports, library papers, research reports) as well as some that are particularly relevant to mathematics (proofs, computer programs, solutions to problems). Many students and professors are uneasy about what writing means in a mathematics class, about how to grade it, and how to improve it. Few mathematicians know how to teach students to improve their writing or speaking, although there is increasing professional interest in this issue [31, 37, 61].

One department focuses on communication throughout the major, and stresses writing and speaking mathematics in a required senior colloquium:

> The conclusion of the major features the colloquium course "Mathematical Dialogues." The emphasis here, as in earlier courses, is on communication, as well as on the connections among the different branches of mathematics. Mathematical Dialogues consists of lectures from invited scholars, discussions, and independent work. Students are expected to read papers and write reviews, to listen to talks and to deliver them.

In industry, one of the most important tasks for a mathematician is to communicate to nonmathematicians in writing and orally the mathematical formulation and solution of problems. Each student's growth in mathematical maturity depends in essential ways on continual growth in the ability to communicate in the language of mathematics: to read and write, to listen and speak. Students must learn the idioms of the discipline, and the relation of mathematical symbols to English words. They need to learn how to interpret mathematical ideas arising in many different sources, and how to suit their own expression of mathematics to different audiences. *Mathematics majors should be offered extensive opportunities to read, write, listen, and speak mathematical ideas at each stage of their undergraduate study.* Indeed, writing and speaking is the preferred test of comprehension for most of the broad goals of study in depth.

Transitions

As students grow in mathematical maturity from early childhood experiences to adult employment, they face a series of difficult transitions where the nature of mathematics seems to change abruptly. These "fault lines" that cross the terrain of mathematics education appear at predictable stages:

- Between arithmetic and algebra, when letter symbols, variables, and relationships become important.
- Between algebra and geometry, when logical proof replaces calculation as the methodology of mathematics.
- Between high school and college, when the expectation for learning on one's own increases significantly.
- Between elementary and upper-division college mathematics, when the focus shifts from techniques to theory, from solving problems to writing proofs.
- Between college and graduate school, when the level of abstraction accelerates at a phenomenal rate.
- Between graduate school and college teaching, when the realities of how others learn must take precedence.
- Between graduate school and research, when the new Ph.D. must not just solve a serious problem, but learn to find good problems as well.

Students experience real trauma in crossing these transitions; many drop out of the mathematics pipeline as a consequence, often to the detriment of their future study in many disciplines. Mathematics education at all levels, from grade school through graduate school, should take as a goal to smooth out the roughness caused by these difficult transitions. College mathematics departments should, in particular, seek to streamline the transition of students to college, to upper-division mathematics, and to graduate school.

In college, students often experience a different type of transitional problem that applies in virtually all courses: to understand the relation between theory and applications. This is probably the most common complaint that students and faculty in collateral disciplines raise about undergraduate mathematics courses: they are often perceived as being too theoretical and insufficiently applied. Although in some cases this perception may be well justified, in many other instances the problem rests more with insufficient effort to demonstrate the value of theory to application than with an actual excess of theory. The problem is not that the transition from application to theory is inappropriate, but that it is often taken without sufficient effort to build appropriate motivation or connections. *Smooth curricular transitions improve student learning and help maintain momentum for the study of mathematics.*

Research

The role of so-called "capstone experiences" such as undergraduate research, theses, or senior projects is one of the more controversial ingredients in discussions of the mathematics major. Typically, such requirements are common in the humanities and the sciences, especially in more selective institutions. In the humanities they are viewed as opportunities for integration; in the sciences, as opportunities for research. In both science and humanities, capstone requirements offer apprenticeships in the investigative methods of the field.

In mathematics, however, there has been little consensus about objectives, feasibility, or benefits of this type of requirement. Very few institutions heeded the 1981 CUPM call for a required course in mathematical modelling for all majors. Many mathematicians believe in coverage as more crucial to understanding: standard theorems, paradigms of proof, and significant counterexamples in all major areas must be covered before a student is ready to advance to the next stage of mathematical maturity. In this view, learning what is already known is a prerequisite to discovering the unknown. Moreover, special capstone courses appear superfluous since each course provides its own capstone—the fundamental theorem of calculus, the central limit theorem in statistics, the fundamental theorem of algebra—which ties together a long chain of prior study. When forced to choose between a capstone experience or another advanced course, advocates of coverage will unhesitatingly choose the latter.

Because of mathematics' austere definition of "research"—a definition which, incidentally, rules out the professional work of more than half the nation's mathematics faculty—many mathematicians believe that except in very rare cases, undergraduates cannot do research in mathematics. Moreover, in most areas of mathematics, students cannot even assist in faculty research, as they do quite commonly in the laboratory sciences. The exceptions in mathematics are principally where computer investigation—the mathematician's laboratory—can aid the research effort. As a consequence, many mathematicians believe that further coursework would better serve the goals of integration (because the higher one progresses in mathematics, the more internal links one can see) and at the same time help advance the student towards better preparation for further study or application of mathematics.

Others feel that any encounter with a substantial problem that a student does not know how to solve can provide a legitimate and rewarding research experience. Indeed, many colleges have used summer experiences with undergraduate research as an effective strategy to recruit students to careers in the mathematical sciences [26, 56], and the National Science Foundation is actively supporting such programs. There are now many diverse programs offering research experiences for undergraduate mathematics majors.

In applied areas—especially in statistics, computing, and operations research—it is easier to develop projects that are sufficiently rich and varied so that students can make progress along various lines of investigation. Computers now are making inroads in theoretical areas of mathematics, permitting exploration of conjectures that heretofore were beyond the range of any undergraduate. Students preparing to teach mathematics in high school also have open an enormous range of appropriate projects to translate interesting newer mathematics into curriculum appropriate to the schools. In some cases students may want to undertake research into how people learn mathematics, to explore for themselves the effectiveness of various instructional strategies and the impact of computers on development of mathematical understanding.

Internships in industry, co-op programs that mix study with work, and summer research opportunities in industrial or government laboratories provide rich environments for breaking down the artificial barriers of courses and classrooms. They enable students to integrate mathematics learned in several different courses; to experience the role of mathematical models; to extend their mathematical repertoire beyond just what has been taught; and to establish mathematical concepts in a context of varied use, applications, and connections.

Experiences of departments with long-standing traditions of undergraduate research or senior projects confirm both the value of such work and the effort required for success:

> While these projects require a great deal of time and effort on the part of students and faculty, we generally feel that it is well worth it. Most of the students report that they had worried about the senior project for their first three years, but had ultimately found it to be a very worthwhile and stimulating part of their college experience. All recommended that this important aspect of the undergraduate experience be retained.

One department in an institution whose academic calendar permits extended blocks for full-time study in one subject requires all majors to complete a major project in the senior year:

> The final project in the major field should demonstrate application of the skills, methods, and knowledge of the discipline to the solution of a problem that would be representative of the type to be encountered in one's career. Project activities encompass research, development and application, involve analysis or synthesis, are experimental or theoretical, emphasize a particular subarea of the major or combine aspects of several subareas.

Another department uses summers to provide opportunities for research experiences: student participants range from freshmen to seniors, and engage in a wide variety of mathematical investigations:

> We are convinced that everyone working in mathematics can find problems appropriate for undergraduates. Many problems can be attacked

> without any knowledge of the complex machinery which generated them. Mathematicians know how exciting mathematical research can be. The best way to generate interest in mathematics is to provide undergraduates with the chance to experience that excitement.

Since hard work by itself is insufficient to ensure reasonable progress on a mathematical problem, there is ever-present danger that undergraduates confronted with difficult theoretical problems will flounder and become discouraged. Strong faculty intervention can prevent disaster, but excessive supervision undermines the independence that is supposed to result from the project. Effective undergraduate research experiences require careful planning and steady, unobtrusive leadership. One must carefully choose problems to be suggested to undergraduates for the research experiences: they must be tailored to the individual undergraduate.

Effective programs provide stepping stones to help students progress from routine homework to independent investigation. For example, one institution plans a progression leading to the senior project:

> Mathematics majors enroll in a Junior Seminar where they are asked to read critically two senior projects from earlier years to describe the strengths and weaknesses of these papers, and to suggest how they would improve on these papers had they written them. This Seminar also helps acquaint these students with appropriate standards of exposition in mathematics.

The range of opportunities for independent investigation is so broad and the evidence of benefit so persuasive as to make unmistakably clear that research-like experiences should be part of every mathematics student's program. *Undergraduate research and senior projects should be encouraged wherever there is sufficient faculty to provide appropriate supervision.* Effective programs must be tailored to the needs and interests of individual students; no single mode of independent investigation can lay claim to absolute priority over others. Flexibility of implementation is crucial to ensure that all experience the exhilaration of discovery which accompanies involvement with mathematical research.

Context

Mathematics courses—especially those taken by majors—have traditionally been taught as purely utilitarian courses in techniques, theory, and applications of mathematics. Most courses pay no more than superficial attention to the historical, cultural, or contemporary context in which mathematics is practiced. Today, however, as mathematical models are used increasingly for policy and operational purposes of immense consequence, it is vitally important that students of mathematics learn to think through these issues even as they learn the details of mathematics itself.

Examples abound of mathematical activity that leads directly to decisions of great human import. Software written for the Strategic Defense Initiative depends on mathematical theories of orbital dynamics for its performance, and on the ability of logicians and computer scientists to verify that complex untestable programs will perform correctly under any possible situation. Debates about the relation of carbon dioxide buildup to global warming and consequent implications for governmental and industrial policies center in large part on different interpretations of statistical and mathematical projections. Computer-controlled trading of stocks, epidemiological studies of AIDS, and implications of various voting rules offer other examples where mathematics really matters in important decisions affecting daily life.

Students of mathematics should be encouraged to see mathematics as a human subject whose theories often begin in ambiguity and controversy. It takes decades, sometimes centuries, for scholars to sculpt and polish the precise theories that are expounded in today's textbooks. Historical analogs provide useful yardsticks to students (and faculty) who seek to understand the limits of what mathematics can contribute to public policy. As society comes to rely increasingly on mathematical analyses—often well-disguised—of social, economic, or political issues, mathematics majors must confront the social and ethical implications of such activity. All such issues can be enlightened by appropriate historical case studies, and motivated by compelling debates of our age. *All mathematics students should engage in serious study of the historical context and contemporary impact of mathematics.*

One possible strategy to achieve both this objective and several others as well is to adapt a modelling project or course to problems of significant societal impact. In such a setting students could undertake original investigation, gain experience in reading, writing, listening, and speaking about mathematically rich material, explore historical antecedents and contemporary debates, and gain experience in team work to address complex, open-ended problems. For many students a capstone project on a public policy issue would be a fitting way to relate their mathematics major to liberal education.

Social Support

The abstract, austere nature of mathematics provides relatively few intrinsic rewards for the typical undergraduate who is trying to pursue a field of study and at the same time learning to establish and maintain personal friendships. In this context the social support provided by departmental activities can be decisive in tipping the balance either for or against a mathematics major. Peer group support helps build mathematical self-confidence and enhances the intrinsic rewards that come from mathematical achievement.

Virtually all successful mathematics departments instigate and support a variety of extracurricular activities. Examples include mathematics clubs, student chapters of the Mathematical Association of America, or chapters of the mathematics honorary society Pi Mu Epsilon. Another common feature of successful departments is informal faculty-led sessions to help students solve problems posed in collegiate periodicals or to prepare for national contests such as the Putnam Examination or the Mathematical Modelling Contest. Banquets, picnics, and barbecues lend a light touch that help students become acquainted with each other and with the faculty of the entire department.

Other activities can enrich students' experiences with their courses by providing links to the world beyond the campus. Undergraduate colloquia with visiting mathematicians from industry or universities is one common mechanism. Alumni involvement through career nights or other activities can help students imagine what they too could do with their major. Current students will be inspired when departments make visible the variety of accomplishments of their graduates—not only those who have become mathematicians but also the majority who have used their undergraduate mathematics for other ends. *Mathematics departments should exert active leadership in promoting extracurricular activities that enhance peer group support among mathematics majors.*

Mechanisms for Renewal

Constant vigilance is needed to maintain quality in any academic department. This is especially true in mathematics, where the subject is continually evolving, where external departments impose their own often-conflicting demands, where so much teaching effort is devoted to remedial, elementary, and lower division work, and where the very ability of the discipline to attract sufficient numbers of students to careers in the mathematical sciences is now in serious doubt. We focus here on five mechanisms of renewal:

- Dialogue: To talk with students and colleagues.
- Assessment: To measure what is happening.
- Faculty Development: To improve intellectual vitality.
- Departmental Review: To listen to colleagues and clients.
- Graduate Education: To provide leadership for improvement.

The key ingredient is listening—to one's students, to one's discipline, to one's colleagues, to one's friends, and to one's critics. Departments that listen—and learn—will thrive.

Dialogue

Departments often know very little about their students' views of the undergraduate mathematics major. That different students pursue mathematics for very different reasons is clear. Most departments must accommodate students with quite different purposes, although certain departments tend to focus their programs on one or another objective (for example, preparation for jobs, preparation for teaching, preparation for graduate school). Many departments, especially small departments, find it impossible to sustain several different programs of equal high quality.

Mathematicians also frequently know almost nothing about the expectations held by their colleagues in cognate disciplines for the mathematical preparation of students with other majors. It is not uncommon for the three interested parties—mathematics professors, science faculty advisors, and students—never to discuss goals or objectives, but only credit hour requirements. It should come as no surprise that in the absence of good communication, misunderstandings flourish.

Undergraduate mathematics shares many borders with other subjects and institutions: vertically with high schools below and graduate schools above; horizontally with science, business, and engineering. Each border is a potential impediment to the smooth flow of ideas and students. Mathematics departments must work hard to maintain effective articulation across these many boundaries:

- With high schools whose curriculum is also changing and whose students will arrive at college with new expectations.
- With departments in the physical sciences and engineering whose students use advanced mathematics.
- With graduate schools in the mathematical sciences, which attract and retain far too few U.S. students.
- With employers of bachelors degree graduates who expect employees who can function effectively in a work environment.

Regular discussion is essential to maintain effective policies that will satisfy these many boundary conditions.

To the extent that resources permit, departments should seek to determine and then accommodate different student career interests. This means that even small departments should provide mechanisms (e.g., independent study, special seminars) to allow students of diverse interests to receive a major suitable to their career objectives. Mathematics is too diverse and student purposes too different for any single set of eight to ten courses to meet all needs equally well.

Students too must recognize that the practice of mathematics is quite different from the textbook image they usually bring with them from high school. Often students expect of college mathematics merely advanced topics in the spirit of school mathematics: a succession of techniques, exercises, and test problems, each explained by the instructor with sufficient clarity that what remains for the student is only the requirement of practice and memorization. Such expectations do little to foster creativity, independence, criticism, and perspective—the more important goals of liberal education.

The different perspectives of mathematics student and mathematics professor often approach caricature. Eager students expect of college classes directed instruction in tools of the trade with which they can, upon graduation, get jobs that pay more than their professors earn. Professors, in contrast, expect students who are eager to take on challenging problems and who will learn on their own whatever they need to make progress. Students, in this exaggerated portrait, feel responsible only for what they have been taught, whereas faculty judge as truly significant only those things students can do which they have not been taught.

It is important for mathematics departments to help faculty and students recognize their own perspectives on mathematics and understand the perspectives of others. Doing this is not the same as covering a syllabus of mathematical topics; it involves instead various strategies to enable faculty and students to discuss mathematics in informal ways. Such discussions are an important part of the process by which students grow from the limited school perspective to the self-directed stance of a professional.

Announcing or publishing department goals is not sufficient to achieve this important objective. What is required is a process that engages all students in significant and repeated discussion of individual goals throughout their undergraduate study of mathematics. In particular, *careful and individualized advising is crucial to students' success.* Effective advising builds an atmosphere of mutual respect among faculty and students. Courses, career objectives, motivations, fears, celebrations are all part of advising, and of special importance in the long, slow process of building students' self-confidence.

Assessment

Many would argue that goals for study in depth can be effective only if supported by a plan for assessment that persuasively relates the work on which students are graded to the objectives of their education. Assessment in courses and of the major as a whole should be aligned with appropriate objectives, not just with the technical details of solving equations or doing proofs. Many specific objectives can flow from the broad goals of study in depth, including solving open-ended problems; communicating mathematics effectively; close reading of technically-based material; productive techniques for contributing to group efforts; recognizing and expressing mathematical ideas embedded in other contexts. Open-ended goals require open-ended assessment mechanisms;

although difficult to use and interpret, such devices yield valuable insight into how students think.

Relatively few mathematics departments now require a formal summative evaluation of each student's major. The few that do often use the Graduate Record Examination (or an undergraduate counterpart) as an objective test, together with a local requirement for a paper, project, or presentation on some special topic. Many institutions, frequently pressured by mandates from on high, are developing comprehensive plans for assessing student outcomes; a few are exploring innovative means of assessment based on portfolios, outside examiners, or undergraduate research projects. Here's one example that blends a capstone course with a senior evaluation:

> The Senior Evaluation has two major components to be completed during the fall and spring semesters of the senior year. During the fall semester the students are required to read twelve carefully selected articles and to write summaries of ten of them. (Faculty-written summaries of two articles are provided as examples.) This work comprises half the grade on the senior evaluation. During the fall semester each student chooses one article as a topic for presentation at a seminar. During the spring semester the department arranges a seminar whose initial talks are presented by members of the department as samples for the students. At subsequent meetings, the students present their talks. Participation in the seminar comprises the other half of the grade for the Senior Evaluation.

Because of the considerable variety of goals of an undergraduate mathematics major, it is widely acknowledged that ordinary paper-and-pencil tests cannot by themselves constitute a valid assessment of the major. Although some important skills and knowledge can be measured by such tests, other objectives (e.g., oral and written communication; contributions to team work) require other methods. Some departments are beginning to explore portfolio systems in which a student submits samples of a variety of work to represent just what he or she is capable of. A portfolio system allows students the chance to put forth their best work, rather than judging them primarily on areas of weakness.

The recommendations [41] from the National Council of Teachers of Mathematics for evaluation and assessment of school mathematics convey much wisdom that is applicable to college mathematics. Assessment must be aligned with goals of instruction. If one wants to promote higher order thinking and habits of mind suitable for effective problem solving, then these are the things that should be tested. Moreover, assessment should be an integral part of the process of instruction: it should arise in large measure out of learning environments in which the instructor can observe how students think as well as whether they can find right answers. *Assessment of undergraduate majors should be aligned with broad goals of the major: tests should stress what is most important, not just what is easiest to test.*

Faculty Development

The relation of research and scholarship to faculty vitality is one of the most difficult issues facing many departments of mathematics, especially in smaller institutions. Professional activity is crucial to inspired teaching and essential to avoid faculty burn-out. Mathematical research in its traditional sense plays only a small role in the mechanisms required to maintain intellectual vitality of a mathematics department: only about one in five full-time faculty in departments of mathematics publish regularly in research journals, and fewer than half of those have any financial support for their research. Clearly the community needs to encourage and support a broader standard as a basis for maintaining faculty leadership both in curriculum and in scholarship.

The first step is to expand the definition of professional activity from "research" to "scholarship," more in a manner akin to that currently recognized in some other academic disciplines. Applied consulting work, software development, problem solving, software and book reviews, expository writing, and curriculum development are examples of activities that serve many of the same purposes as research: they advance the field in particular directions, they engage faculty in active original work, they serve as models for students of how mathematics is actually practiced, and they provide opportunities for student projects.

Teaching in new areas is also a form of scholarship in mathematics. Unlike many other disciplines where faculty rarely teach outside their own areas of specialty, mathematicians are generally expected to teach a wide variety of courses. Learning and then teaching a course far outside one's zone of comfort is an effective way to build internal connections which then spill over in all courses one teaches. A teacher who is still an active learner sets a fine example for students concerning the true meaning of scholarship.

The second step is to insist on greater communication about professional activity in mathematics so that it becomes public. Only the bright light of public scrutiny by colleagues in various institutions—not only on one's own campus—can affirm the quality and value of professional work. "Public" need not mean merely publication; lectures, workshops, demonstrations, reports of various sorts can serve the same objective. What matters is that the result become part of the profession, and be evaluated by the profession. *To ensure continued vitality of undergraduate mathematics programs, all mathematics faculty should engage in public professional activity, broadly defined.*

Department Review

More than any other academic discipline, mathematics is constrained to serve many masters: the many sciences that depend on mathematical methods; the demand of quantitative literacy that undergirds general education; the need to educate teachers for our nation's schools; the need of business and industry for mathematically literate employees; the expectation of mathematical proficiency by faculty and students in natural science, business, engineering, and social science; the professional standards of employers for entry-level technical personnel; and the requirements of the mathematical sciences themselves for well-prepared graduate students. It is an enormous challenge for a department of mathematics, one that very few are able to fulfill with distinction in every dimension.

Because of these diverse demands, it is especially important that departments of mathematics undergo regular review, with both external and internal mechanisms to provide evaluation and advice. External requirements mandate periodic review of all departments in many colleges and universities, especially in public institutions. But in other institutions, department goals are defined implicitly without self-reflection or benefit of external perspectives. At worst, the goals of such departments are defined by coverage of standard textbooks. Often it takes a crisis—such as when the engineering or business school complains about certain courses—for departments to step back and examine their objectives. Reviews should take place regularly, not just when some crisis threatens the *status quo.*

Client disciplines expect from mathematics departments an amazing repertoire of support services for students who will major in other fields [44]. Some demand a magic bullet—a perfect infusion of just those mathematical methods (and no more) needed in the other field; others expect a rigorous filter that will pass on only those students who are sufficiently bright to function ably in upper-division work in other fields. Occasionally, but all too rarely, an external discipline will require mathematics primarily to enable students to benefit from the intrinsic values of mathematics: logic, rigor, analysis, symbol-sense, etc. Since the expectations of other departments are often not clearly conveyed by the list of mathematics courses that they recommend or require, regular reviews provide a good mechanism—but not the only one—to ensure that different departments at least understand their differing perspectives and objectives.

Virtually all departments receive informal feedback from graduates, employers, and graduate schools. Speaker programs that bring students and faculty into contact with users of mathematics serve both to inform students about the broad world of mathematics beyond their classroom walls and to provide informal feedback to help regulate the curriculum and keep it properly tuned to the needs of graduates. All such informal means of feedback are valuable and must be encouraged. However, they are no substitute for formal, regular, external review. *Both external reviews and informal feedback are needed to assure quality in departments of mathematics.*

There are many advantages to a regular program of external reviews that should form the basis of all reviews:

- A broad-based review provides a strategic opportunity to document the accomplishments of a department. Well-structured reviews can effectively counter external (political) demands for narrow or inappropriate instruments of assessment such as a multiple-choice examination of all graduates.
- Reviews provide a structured and neutral forum for mathematicians to discuss with those who use mathematics both the mathematical needs of client disciplines and the common issues that both mathematics and the client discipline face

in accommodating changes that are underway in the mathematics curriculum.

- By involving members of the faculty outside the department in the review—especially those from fields that are served by mathematics and those involved in faculty curriculum committees—a department of mathematics can help educate colleagues across the campus about the special opportunities and challenges of teaching mathematics. Ignorance can usually be turned to understanding through discussions prompted by the occasion of a regular review.
- By including non-academic reviewers such as industrial executives, scientists, professionals, and community leaders, the department can gain valuable insight into the qualities that will be expected of graduates who enter the work force.
- Regular reviews encourage faculty members to think about the department's program as a whole, rather than only about the courses they teach. Such discussions make it more likely that the curriculum will remain responsive to student needs, and to the changing demands of the mathematical sciences. Reviews provide an ideal mechanism for the department to assert control over its own program.

The Mathematical Association of America can provide advice to departments both about the structure of effective reviews and about appropriate consultants or reviewers.

Graduate Education

Even though relatively few mathematics majors go on to receive a graduate degree in the mathematical sciences, the health of college mathematics is inextricably linked with the status of graduate education. As the sole agent for advanced degrees, graduate schools bear alone the responsibility for preparing college mathematics teachers; as the primary locus of mathematical research, graduate schools shape the nature of the discipline, and hence of the curriculum. Much of the responsibility for renewing undergraduate mathematics rests with the graduate schools, since it is they who provide the primary professional education of those who are responsible for undergraduate mathematics: college faculty.

Indicators from many sources [12, 35, 44] suggest that the match between undergraduate and graduate education in mathematics is not now serving U.S. interests especially well:

- Too few U.S. mathematics majors choose to enter graduate school in a mathematical science.
- U.S. mathematics students do less well in graduate school—and drop out more often—than foreign nationals.
- Many students finish graduate school ill-equipped for the breadth of teaching duties typically expected of undergraduate mathematics teachers.
- Relatively few who finish doctoral degrees in mathematics actually go on to effective research careers in mathematics.

In the 1970s, as the number of U.S. students applying to graduate school in mathematics began to decline, the graduate schools responded by increasing the number of international students, most of whom had completed a more intense and specialized education in mathematics than is typical of American undergraduates. Hence the level of mathematics expected of beginning graduate students gradually shifted upward to an international standard that is well above current U.S. undergraduate curricula. Consequently, the failure or drop-out rate of U.S. students increased, creating pressure for more international students and even higher entrance expectations.

It is time to break this negative feedback loop by encouraging better articulation of programs and standards between U.S. undergraduate colleges and U.S. graduate schools. Such cooperation is needed both to enhance the success of U.S. students and to enable the graduate schools to better match their programs with the needs of the colleges and universities who employ a majority of those who receive advanced degrees. *Renewal of undergraduate mathematics will require commitment, leadership, and support of graduate schools.*

One good mechanism for such cooperation would be an exchange of visitors between undergraduate and graduate institutions so that each can learn about the needs of the other. Especially as change occurs in the content and nature of the undergraduate major, it is very important that graduate schools maintain programs of study and research that are appropriately linked to the undergraduate program in mathematics.

Summary

Without becoming entangled in specific curriculum and course recommendations—which are the proper province of other committees of mathematics professional organizations—we can nevertheless enumerate several broad principles implied by our study of the undergraduate mathematics major:

GOALS AND OBJECTIVES

- The primary goal of a mathematical sciences major should be to develop a student's capacity to undertake intellectually demanding mathematical reasoning.
- The undergraduate mathematics curriculum should be designed for all students with an interest in mathematics.
- Applications should motivate theory so that theory is seen by students as useful and enlightening.
- Mathematics majors should be offered extensive opportunities to read, write, listen, and speak mathematical ideas at each stage of their undergraduate study.

BREADTH AND DEPTH

- All students who major in mathematics should study some sequence of upper division courses that shows the power of study in depth.
- Every student who majors in mathematics should study a broad variety of advanced courses.
- Mathematics departments should take seriously the need to provide appropriate mathematical depth to students who wish to concentrate in mathematics without pursuing a traditional major.
- Mathematics majors should complete a minor in a discipline that makes significant use of mathematics.

LEARNING AND TEACHING

- Instruction should encourage students to explore mathematical ideas on their own.
- Undergraduate students should not only learn the subject of mathematics, but also learn how to learn mathematics.
- Those who teach college mathematics should seek ways to incorporate into their own teaching styles the findings of research on teaching and learning.
- Mathematicians should increase their efforts to understand better how college students learn mathematics.
- Evaluation of teaching must involve robust indicators that reflect the broad purposes of mathematics education.

ACCESS AND ENCOURAGEMENT

- Effective programs teach students, not just mathematics.
- National need requires greater encouragement for students to continue their study of mathematics beyond the bachelor's degree.
- To provide effective opportunities for all students to learn mathematics, colleges should offer a broader spectrum of instructional practice that is better attuned to the variety of students seeking higher education.
- To ensure for all students equal access to higher mathematics education, mathematics departments should work with nearby two-year colleges to maintain close articulation of programs.
- Smooth curricular transitions improve student learning and help maintain momentum for the study of mathematics.

USING COMPUTERS

- The mathematics curriculum should change to reflect in appropriate ways the impact of computers on the practice of mathematics.
- Colleges must recognize in budgets, staffing, and space the fact that undergraduate mathematics is rapidly becoming a laboratory discipline.

DOING MATHEMATICS

- Dealing with open-ended problem situations should be one of the highest priorities of undergraduate mathematics.
- All undergraduate mathematics students should undertake open-ended projects whose scope extends well beyond typical textbook problems.
- Undergraduate research and senior projects should be encouraged wherever there is sufficient faculty to provide appropriate supervision.
- Students majoring in mathematics should undertake some real-world mathematical modelling project.

STUDENTS

- Building students' well-founded self-confidence should be a major priority for all undergraduate mathematics instruction.

- Careful and individualized advising is crucial to students' success.
- All mathematics students should engage in serious study of the historical context and contemporary impact of mathematics.
- Mathematics departments should actively encourage extracurricular programs that enhance peer group support among mathematics majors.

RENEWAL

- It is important for mathematics departments to help faculty and students recognize their own perspectives on mathematics and understand the perspectives of others.
- Assessment of undergraduate majors should be aligned with broad goals of the major; tests should stress what is most important, not just what is easiest to test.
- To ensure continued vitality of undergraduate mathematics programs, all mathematics faculty should engage in public professional activity, broadly defined.
- Regular external reviews and informal feedback are needed to assure quality in departments of mathematics.
- Renewal of undergraduate mathematics will require commitment, leadership, and support of graduate schools.

In most respects both prevailing professional wisdom and current practice for the mathematics major reflect well the major goals of AAC's *Integrity.* Discussion continues on many campuses about whether the major should focus inward towards advanced study in the mathematical sciences or outward towards preparation for diverse careers in science and management. These discussions are more about strategies than long-term goals, however, since either emphasis can advance the broad AAC goals of coherence, connections, and intellectual development.

Liberal education provides a versatile background for a life of ever-changing challenges. Among the many majors from which students can choose, mathematics can help ensure versatility for the future. Habits of mind nurtured in an undergraduate mathematics major are profoundly useful in an enormous variety of professions. The challenge for college mathematicians is to ensure that the major provides—and is seen by students as providing—not just technical facility, but broad empowerment in the language of our age.

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Mathematics Magazine

Martha J. Siegel, Editor

The aim of *Mathematics Magazine* is to provide lively mathematical exposition in both long articles and short notes. This journal, published five times a year, strives to provide material of interest to collegiate mathematicians—both faculty and students. Of particular appeal to the Editorial Board are manuscripts that stress the relationships among several mathematical areas as well as the use of mathematics in other disciplines.

We welcome papers on historical topics and look for the kind of writing that students in advanced undergraduate and beginning graduate courses would read and enjoy. Although all papers are refereed, the *Magazine* is not a research journal. There is a hope, however, that the articles and other features will stimulate research. Every issue includes a lively "Reviews" section (featuring articles and books about mathematics but outside the mainstream of mathematics literature), "Problems," "Letters," and "Notes." In three issues each year, we present problems and solutions for major national and international mathematical contests. The new Editorial Board has agreed to publish short classroom gems: clever and innovative mathematics suitable for presentation in the classroom. Student members of the MAA and their advisors should find the *Magazine* particularly suited to their interests.

1990 MAA Journal Awards

Each year the MAA's three journal awards committees honor the most accomplished articles from each of the Association's periodicals—the Carl B. Allendoerfer Award for papers in *Mathematics Magazine;* the Lester R. Ford Award for papers in *The American Mathematical Monthly;* and the George Pólya Award for papers in *The College Mathematics Journal.* Winners received a check, a certificate, and, most importantly, the respect and admiration of their colleagues. The mathematical community recognizes these authors for their exceptionally skillful mathematics exposition and for generously sharing their insights with us.

Carl B. Allendoerfer Award Fan R. K. Chung, Martin Gardner, and Ronald L. Graham received this award for "Steiner Trees on a Checkerboard," in *Mathematics Magazine,* **62** (1989): 83–96. Gardner has written several books published by the MAA and, for twenty-five years, authored *Scientific American's* "Mathematical Games" column. Chung and Graham are associated with Bell Communications Research and AT&T Bell Laboratories, respectively. Their "presentation is accessible and engaging and includes problems and examples to involve the reader at various levels."

The Allendoerfer Committee also honored Thomas W. Archibald of Arcadia University for "Green's Second Identity in Its First Fifty Years," in *Mathematics Magazine,* **62** (1989): 219–232. In his historical account of Green's second identity, Archibald illustrates "the vital role of interplay between mathematics and physics in the evolution of some fundamental ideas of both disciplines. The author developed his theme with selections from the lives and writings of familiar figures in the scientific community of the nineteenth century." When asked what motivated him to explore this topic, Archibald responded that "The relationship between connectivity and smoke-rings which I described in my paper seemed to me to illustrate . . . rather well . . . sources of abstract mathematical ideas in the concrete world."

Lester R. Ford Award Jacob Eli Goodman of City College, City University of New York, and János Pach and Chee K. Yap, both of the Courant Institute of Mathematical Sciences, New York University accepted this award for "Mountain Climbing, Ladder Moving, Ring Width of a Polygon," in *The American Mathematical Monthly,* **96** (1989): 494–510. Their paper realizes "a unified approach to three elegant results that generally confirm one's intuition about movement problems along polygonal paths in the plane. For example, the authors show that under modest hypotheses on the path, a ladder of fixed length can be carried along a polygonal path by two people holding the ladder at its ends. Intuition in this area can be deceiving." Pack explained that the challege of "an open problem posed by Chee Yap on a computational geometry day at the Courant Institute" piqued his interest. Moreover, he expressed delight with the "beauty and simplicity of the mathematical tools required for the solution." Yap described the essay as an attempt to solve two conjectures which he had posed in his earlier paper, "How to Move a Chair through a Door." The award winning essay "partially solves" one of these two conjectures.

The Ford Committee also recognized Doron Zeilberger of Temple University for "Kathy O'Hara's Constructive Proof of the Unimodality of the Gaussian Polynomials" in *The American Mathematical Monthly,* **96** (1989): 590–602. "Often in mathematics, one has to use techniques quite distant from the field in which a problem is found in order to give a proof. Thus, combinatorialists have for many years regretted that all proofs of the unimodality of Gaussian polynomials, which occupy a central position in combinatorics, use analysis or algebra. In 1987, Kathy O'Hara offered a brilliant, purely combinatorial, but very difficult proof. In presenting this difficult proof to *Monthly* readers, Zeilberger offers several simplifications but modestly downplays his own contributions so that the focus remains on O'Hara's resplendent triumph."

George Pólya Award In 1990, two authors accepted this award for their outstanding mathematical exposition: Richard Dean Neidinger of Davidson College for "Automatic Differentiation and APL," in the *College Mathematics Journal,* **20** (1989): 238–251; and Israel Kleiner of York University for "Evolution of the Function Concept: A Brief Survey," in the *College Mathematics Journal,* **20** (1989): 282–300. Neidinger's essay is a "beautifully written article that fulfills its stated goal while at the same time teaching the reader a programming language (APL). The recursive techniques are interesting since they can be implemented so easily in APL and, conversely, the reader is prompted to learn APL because it is perfect for the job at hand. The level is appropriate for a calculus student who could be drawn into looking in a new way at the familiar, and sometimes boring, rules for differentiation." Neidinger reports that an article in *Mathematics Magazine* inspired him to teach students about the computer language APL—he regarded this article as a "natural means to explain the idea and practical significance of automatic differentiation."

In his paper, Israel Kleiner, the second Pólya Award recipient, "makes a point that is particularly important for students and teachers to understand these days when graphing calculators have further reinforced the naïve idea of functions as graphs. At a time when discoveries about dynamical systems are making clear the ubiquity of everywhere-discontinuous functions, continuous but nowhere differentiable functions, and other functions formerly thought to be pathological, it is a mistake to limit students to a seventeenth century viewpoint by identifying the function concept with the geometric notion of a graph. Kleiner's article is well-written, with apt quotations, and an excellent bibliography."

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DEPARTMENT OF MATHEMATICS
University of Alberta

Applications are invited for a tenure-track position (File GP-1) at the assistant professor level, beginning July 1, 1991. The possibility of an appointment at the associate professor level is not precluded. Requirements are a PhD and proven ability or demonstrated potential for research and teaching. Salary for assistant professor is currently $36,910–$53,374; for associate professor currently $45,694–$67,658. Send vita and arrange for three letters of reference to be sent to: Professor R. Bercov, Chairman, Department of Mathematics, University of Alberta, Edmonton, Alberta, Canada T6G 2G1. In accordance with Canadian Immigration requirements, priority will be given to Canadian citizens and permanent residents of Canada. Closing date for applications is January 15, 1992. Please quote file number when responding to this advertisement. The University of Alberta is committed to the principle of equity in employment. The University encourages applications from aboriginal persons, disabled persons, members of visible minorities, and women.

DEPARTMENT OF MATHEMATICS
University of Alberta

Applications are invited for a tenure-track position in Approximation Theory (File AP-1) at the assistant professor level, beginning July 1, 1991. The possibility of an appointment at the associate professor level is not precluded. Requirements are a PhD and proven ability or demonstrated potential for research and teaching. Salary for assistant professor is currently $36,910–$53,374; for associate professor currently $45,694–$67,658. Send vita and arrange for three letters of reference to be sent to: Professor R. Bercov, Chairman, Department of Mathematics, University of Alberta, Edmonton, Alberta, Canada T6G 2G1. In accordance with Canadian Immigration requirements, priority will be given to Canadian citizens and permanent residents of Canada. Closing date for applications is January 15, 1992. Please quote file number when responding to this advertisement. The University of Alberta is committed to the principle of equity in employment. The University encourages applications from aboriginal persons, disabled persons, members of visible minorities, and women.

SOUTHWESTERN UNIVERSITY
Georgetown, Texas

Southwestern University invites applications for a tenure-track position in computer science, beginning August 1991. Responsibilities include teaching nine credit hours per semester, continuing scholarly activity, and assisting in the design of a new curriculum emphasizing formal mathematical methods such as derivation and verification of programs. A PhD in computer science or in mathematics with strong supporting work in computing is required. Rank and salary are commensurate with experience and expertise.

Southwestern University is a selective, undergraduate institution committed to a broad-based liberal arts and sciences education. Affiliated with the United Methodist Church, it has over 1,200 students and a history of stable enrollment. Southwestern's endowment of more than $126 million ranks among the highest per student of undergraduate institutions in the country. The University has a strong commitment to faculty support, with faculty salaries at the 92nd percentile for IIB institutions, and an endowment in excess of $1 million for faculty development. The University is located in Georgetown, Texas, 28 miles north of Austin, the state capital, and site of the University of Texas.

Applicants should send a letter of application, vita, and three current letters of recommendation to: Search Committee, Department of Mathematics and Computer Science, Box 6399 SU Station, Georgetown, TX 78626. Review of applications will begin on January 8, 1991. Women and minority candidates are encouraged to apply. Southwestern University is an Affirmative Action, Equal Employment Opportunity Employer.

ASSISTANT PROFESSOR OF MATHEMATICS
Roanoke College

Tenure-track appointment at the assistant professor level beginning August 1991. PhD in mathematics required. Salary commensurate with qualifications. Excellent teaching emphasized, active scholarship encouraged. Commitment to liberal learning expected. Roanoke College is a private, liberal arts college affiliated with the Lutheran Church and located in the Roanoke Valley of Virginia. Position is open until filled. Send vita, graduate transcript, and three letters of recommendation to: Dr. W. D. Ergle, Department of Math, Computer Science, and Physics, Roanoke College, Salem, VA 24153; (703) 375-2449. AA/EOE.

CHAIR
Department of Mathematics & Statistics
Saint Mary's College of Minnesota

Applications are invited for the position of department chair. Nine-month, tenure-track position to lead a seven-member department, starting 8/91. PhD in mathematics or statistics required, as well as demonstrated excellence in teaching and communication at the undergraduate level, and experience or potential for departmental administration.

Review begins December 1 and continues until the position is filled. Send vita and names & addresses of three references to: Louis A. Guillow, Campus Box 4, Saint Mary's College of Minnesota, Winona, MN 55987.

Saint Mary's College of Minnesota is committed to affirmative action and encourages applications from minorities and women.

DEPARTMENT OF MATHEMATICS
University of Alberta

Applications are invited for a tenure-track position in Numerical Analysis (File NA-1) at the assistant professor level, beginning July 1, 1991. The possibility of an appointment at the associate professor level is not precluded. Requirements are a PhD and proven ability or demonstrated potential for research and teaching. Salary for assistant professor is currently $36,910–$53,374; for associate professor currently $45,694–$67,658. Send vita and arrange for three letters of reference to be sent to: Professor R. Bercov, Chairman, Department of Mathematics, University of Alberta, Edmonton, Alberta, Canada T6G 2G1. In accordance with Canadian Immigration requirements, priority will be given to Canadian citizens and permanent residents of Canada. Closing date for applications is January 15, 1992. Please quote file number when responding to this advertisement. The University of Alberta is committed to the principle of equity in employment. The University encourages applications from aboriginal persons, disabled persons, members of visible minorities, and women.

HOBART AND WILLIAM SMITH COLLEGES

Department of Mathematics and Computer Science

Two assistant professor, tenure-track positions starting in September 1991. Salary is competitive.

For the first position applicants should have a PhD in computer science or a PhD in mathematics and experience in computer science. Duties include teaching undergraduate computer science, participating in the Colleges' Interdisciplinary General Curriculum, and the possibility of teaching some mathematics (depending on interests and qualifications).

For the second position applicants should have a PhD in mathematics; specialty open, but preference may be given to algebraists, applied mathematicians, or those with demonstrated computer science expertise. Duties include teaching undergraduate mathematics, participating in the Colleges' Interdisciplinary General Curriculum, and the possibility of teaching some computer science (depending on interests and qualifications).

For both positions a strong commitment to teaching and promise of continued scholarly activity is required. Teaching load: two courses per trimester. Hobart College for men and William Smith College for women are coordinate, four-year, liberal arts colleges committed to teaching and interdisciplinary study with a combined enrollment of 1,900 students. Within an hour's drive are three major universities: Cornell, Rochester, and Syracuse.

Send detailed resumé, three letters of recommendation (at least one including comments on teaching), and undergraduate and graduate transcripts (photocopies acceptable) to: Prof. Kevin Mitchell, Faculty Box 75, Department of Mathematics and Computer Science, Hobart and William Smith Colleges, Geneva, NY 14456. Evaluation of applications will begin December 15, 1990 and will continue until the position is filled. Women and minorities are strongly encouraged to apply. An Equal Opportunity/Affirmative Action Employer.

DARTMOUTH COLLEGE

John Wesley Young Research Instructorship

The John Wesley Young Research Instructorship is a two-year, postdoctoral appointment for promising new or recent PhDs whose research interests overlap a department member's. Current departmental interests include areas in algebra, analysis, algebraic geometry, combinatorics, computer science, differential geometry, logic and set theory, number theory, probability, and topology. Teaching duties of four ten-week courses spread over two or three quarters typically include at least one course in the instructor's speciality and include elementary, advanced, and (at instructor's option) graduate courses. Nine-month salary of $32,500, supplemented by summer (resident) research stipend of $7,150 (two-ninths). Send letter of application, resumé, graduate transcript, thesis abstract, description of other research activities and interests if appropriate, and 3 or preferably 4 letters of recommendation (at least one should discuss teaching) to: Phyllis A. Bellmore, Department of Math and CS, Bradley Hall, Hanover, NH 03755. Applications received by Jan. 15 receive first consideration; applications will be accepted until position is filled. Dartmouth College is committed to affirmative action and strongly encourages applications from minorities and women.

NORTHERN KENTUCKY UNIVERSITY

Department of Mathematics and Computer Science

The Department of Mathematics and Computer Science anticipates having two tenure-track positions. The positions are pending funding and would begin in August 1991. A doctorate in mathematics or a mathematical science is required. Statisticians are encouraged to apply. Responsibilities include undergraduate teaching (12 hours/semester), scholarly activity, and service. The department's emphasis is on quality teaching. Good oral and written communication skills in English are mandatory.

Review of applicants will begin February 1, 1991. Apply to: Professor Frank Dietrich, Chair of Search Committee, Department of Mathematics and Computer Science, Northern Kentucky University, Highland Heights, KY 41706.

NKU is an Affirmative Action/Equal Opportunity Employer and actively seeks the candidacy of minorities and women.

WESTERN NEW ENGLAND COLLEGE

Department of Mathematics/Computer Science

Assistant/associate professor of mathematics, tenure-track position(s) commencing September 1991. Candidate will instruct 12 credit hours per semester. Applicants must have a PhD in mathematics or mathematical sciences with excellence in and a strong commitment to teaching. Scholarly activity is expected and supported through released time, summer research grants, and professional travel funds. There are no restrictions as to area of specialization. Salary is competitive and commensurate with credentials and experience. Western New England College is an independent, nondenominational, coeducational institution with Schools of Arts and Sciences, Business, Engineering, and Law, with a total day-evening enrollment of over 5,000 students. The department has 11 full-time faculty members and over 70 undergraduate majors. Send cover letter (indicate if you plan to attend AMS/MAA annual meeting in San Francisco), curriculum vitae, and three letters of recommendation to: Dennis M. Luciano, Chairman, Department of Mathematics and Computer Science, Western New England College, Springfield, Massachusetts 01119. Applications will be screened on a rolling basis until the positions are filled. Western New England College is an Equal Opportunity/Affirmative Action Employer and encourages applications from women and minority candidates.

MIAMI UNIVERSITY
Oxford, Ohio

Mathematics Education Position
Department of Mathematics and Statistics

Tenure-track assistant professorship beginning August 1991 in the area of mathematics education. Duties include teaching 8–9 hours per semester, continuing scholarship, and service. Applicants should have (by 8/91) a doctorate in mathematics education or a doctorate in mathematics with expertise in mathematics education. Please send vita, transcripts, and three reference letters to: John Skillings, Math Education Search, Department of Mathematics and Statistics, Miami University, Oxford, Ohio 45056. Review of applications will commence on January 15, 1991. Women and minorities are encouraged to apply. Miami provides equal opportunity in employment and education.

EMORY & HENRY COLLEGE

Tenure-track position beginning August 1991. PhD or near completion. Rank and salary open. Able to teach wide range of undergraduate mathematics and beginning computer science courses. Commitment to excellent teaching essential. Apply with letter, cv, names of three references to: Richard Pfau, Dean, Emory & Henry College, Emory, VA 24327. AA/EOE.

COLBY

Department of Mathematics

Colby anticipates having two regular (i.e., tenure track) faculty openings (at any level) in mathematics/computer science, commencing September 1991. Our aim is to fill one vacancy in algebra and one in computer science (exact interests not important). A strong research program plus a firm commitment to undergraduate teaching are essential for these positions. The PhD degree is required.

We particularly welcome applications from women, and well-qualified female applicants at the level of Assistant or Associate Professor will in addition be considered for one of the college's Clare Boothe Luce Chairs for women scientists.

Colby is a highly selective, private, liberal arts college with an enrollment of 1700 students. The Mathematics Department has 9 full-time and one part-time faculty, and teaches bachelor's courses in mathematics and computer science. The college provides generous support for faculty research.

The college offers a rich computing environment based on VAX mainframes and Apple Macintosh workstations (IIfx, IIcx and SE30). The Mathematics Department in-house teaching/research laboratory comprises 18 Macintosh II/IIcx computers, a central fileserver, and associated visual-display apparatus. An Internet connection provides access to supercomputers and other networked resources. Colby is an acknowledged national leader in the development of computer-aided instruction in mathematics.

Please send a letter of application and a resume, and the names of three referees, to: **Professor Keith Devlin, Chair, Department of Mathematics, Colby College, Waterville, ME 04901.** Review of applications will begin December 5, 1990, and continue until the positions are filled. Colby is an AA/EO Employer and strongly encourages applications from women and minorities.

JOHNS HOPKINS UNIVERSITY

Applications are invited for a junior position in statistics, to begin in fall 1991. Selection is based on demonstration and promise of excellence in research, teaching, innovative application. AA/EOE. Applicants are asked to furnish a vita, transcripts, a letter describing professional interests and aspirations, and arrange for three letters of recommendation to be sent to: Prof. John D. Wierman, Chairman, Mathematical Sciences Department, The Johns Hopkins University, Baltimore, MD 21218.

JOHNS HOPKINS UNIVERSITY

The Mathematical Sciences Department invites applications for the 1991-92

ELIEZER NADDOR
POSTDOCTORAL FELLOWSHIP.

The Fellow is to be an outstanding graduating doctoral student in mathematics, statistics, or operations research, who plans an academic research career. The fellowship provides full support for 12 months of postdoctoral study at the department in an area of interest to some department faculty member, free from teaching and administrative duties. Selection is made without discrimination on the basis of race, color, religion, sex, or national origin. Applicants should provide a current vita, a letter describing career aspirations and a research plan for the fellowship year, and transcripts, and should arrange for three letters of recommendation to be sent, by January 15, 1991, to:

Professor John C. Wierman, Chairman
Mathematical Sciences Department
220 Maryland Hall
The Johns Hopkins University
Baltimore, Maryland 21218

Applicants for positions in algebra, analysis, differential equations, geometry, number theory, and topology should contact the Mathematics Department instead of the Mathematical Sciences Department.

UNIVERSITY OF FLORIDA

Anticipated, two LECTURER openings, non-tenure-accruing but continuing full-time positions, to teach undergraduate statistics courses starting August 1991. Requires master's level training in statistics and strong commitment to quality teaching. Experience desirable, but not necessary. Apply by January 15, 1991. Women and minorities are encouraged to apply. Send curriculum vitae, graduate transcripts, three letters of recommendation to: Geoff Vining, Dept. of Statistics, 483 Little Hall, Univ. of FL, Gainesville, FL 32611-2049. AA/EOE.

WAKE FOREST UNIVERSITY

Department of Mathematics
and Computer Science

Applications are invited for two tenure-track positions in mathematics at the assistant professor level beginning August 1991. Duties include teaching mathematics at the undergraduate and graduate levels and continuing research. A PhD is required. The department has 22 members and offers a BS and MA in mathematics and a BS in computer science. Send letter of application and resumé to: Richard D. Carmichael, Chairman, Department of Mathematics and Computer Science, Wake Forest University, Box 7311, Winston-Salem, NC 27109. AA/EO Employer.

TRINITY COLLEGE

The Department of Mathematics at Trinity College invites applications for a tenure-track position at the rank of assistant professor, beginning in the academic year '91–'92. The normal teaching load is five semester-courses per year ("3/2"). While we will be happy to receive applications from those with any specialty, we will be particularly interested in algebraists, logicians, and persons whose research interests might intersect with current department members' areas: complex analysis, functional analysis, geometry, graph theory, combinatorics, and mathematical statistics. Requirements for the position: PhD in mathematics, evidence of teaching excellence at the undergraduate level, indications of promise in research, and interest in curriculum development.

Applicants should send a cv, three letters of reference (at least one of which addresses teaching) and a statement of teaching and research interests to:

Search Committee Chair
Dept. of Mathematics
Trinity College
Hartford, CT 06106

No decision will be made prior to January 21, after which the position may be filled at any time.

Representatives of the Department will attend the employment register at the Joint Annual Meetings in San Francisco in January 1991.

Trinity College is an Equal Opportunity/Affirmative Action employer. Women and members of minority groups are especially encouraged to apply.

FURMAN UNIVERSITY

Greenville, South Carolina 29613

The Department of Mathematics at Furman University, an undergraduate, liberal arts college, invites applications for a tenure-track assistant professorship beginning September 1, 1991. A PhD in a mathematical science is required. All areas of specialization are acceptable. Excellence in teaching and continued scholarly activity are expected of all faculty. A vita, graduate and undergraduate transcripts, and three letters of recommendation should be send to: Robert Fray, Department of Mathematics. Application deadline: February 1, 1991. EOE/AAE.

HUMBOLDT STATE UNIVERSITY (HSU)

Arcata, CA 95521

Applications are invited for a tenure-track assistant or associate professor position for fall 1991. HSU, located on the California north coast, has an active mathematics faculty and a strong undergraduate major with options in applied mathematics, computer science, and teacher preparation, as well as a master's program in mathematics modeling of environmental systems. Candidates must have a doctorate in a mathematical science or mathematics education. All qualified applicants with a commitment to teaching excellence and professional activities will be considered. Preference will be given to applicants with a teaching credential and/or teaching experience in elementary or secondary schools. Women and minorities are especially encouraged to apply. Applicants should send vita, transcripts, and three letters of reference to: Search Committee, Department of Mathematics, Humboldt State University, Arcata, CA 95521 by February 1 for full consideration. HSU is an Equal Opportunity/Affirmative Action Employer.

EASTERN OREGON STATE COLLEGE

Mathematics

Applications are invited for two tenure-track positions in mathematics at the assistant or associate professor level beginning in the fall of 1991. The successful candidates will be committed to teaching a wide range of undergraduate mathematics courses. An interest and background in geometry is preferred for one of the positions, but applications from all areas of mathematics are welcome. A PhD in mathematics or a related discipline is required. Applications, including a vita and three current (1987--90) letters of recommendation, should be sent to: Mathematics Search Committee Chair, Badgley Hall of Science, Eastern Oregon State College, La Grande, OR 97850-2899. Application deadline: February 1, 1991, or until positions are filled. Telephone inquiries: Professor Robert Brandon, (503) 962-3631. AA/EOE.

HAVERFORD COLLEGE

Mathematics
Haverford, PA 10941

Haverford College announces a tenure-track opening for 1991–92 in the Department of Mathematics, at the assistant (or possibly associate) professor level. Applications are invited from candidates with research interests in any field of mathematics. Candidates should demonstrate a strong commitment to teaching a broad spectrum of undergraduate courses, and to research. Send curriculum vitae, statement of research and teaching interests, and three letters of recommendation to: Curtis Green, Chair, Department of Mathematics, Haverford College, Haverford, PA 19041. Haverford College is an EO/AA Employer. Women and minority candidates are encouraged to apply.

Deadline for applications: December 7, 1990

(Late applications may be considered until the position is filled, but this cannot be guaranteed.)

FERRIS STATE UNIVERSITY

Head, Department of Mathematics

Ferris State University invites nominations and applications for the position of Head of the Department of Mathematics. The Department of Mathematics currently has 22 faculty and is responsible for undergraduate education in math and computer science and for baccalaureate programs in applied mathematics and actuarial science. QUALIFICATIONS: An earned doctorate in mathematics or applied mathematics, or an earned doctorate in mathematics education with a master's in mathematics; professional development and teaching experience appropriate to senior rank; familiarity with a broad spectrum of math instruction; ability to work with others in a broad array of disciplines; and personal qualities of integrity, industriousness, organization, leadership, and interpersonal skills.

Ferris State University is a career-oriented, open-admission, state-funded institution in western Michigan with 12,000 students and over 120 degree programs. Its schools include Allied Health, Arts and Sciences, Business, Education, Pharmacy, Technology, and the College of Optometry.

Review of applications is in progress and will continue until the position is filled. Send letter of interest, curriculum vitae, 3 letters of reference, and official transcripts to: George Wales, Search Committee Chair, Starr 120, Ferris State University, Big Rapids, MI 49307. EO/AAE.

GANNON UNIVERSITY

Mathematics Department
University Square, Erie, PA 16541

The Department of Mathematics invites applications for a tenure-track position beginning August 1991. Applicant must possess a PhD with strength in probability and statistics, and demonstrate evidence of good undergraduate teaching with research potential. Send resumé and three letters of recommendation to: Dr. Rafal Ablamowicz, Chairman. For full consideration, please apply by March 1, 1991. Gannon University is an Equal Opportunity/Affirmative Action Employer.

TRANSYLVANIA UNIVERSITY

Lexington, KY 40508

Computer Science: assistant/associate professor, Transylvania University. Full-time, tenure-track. PhD in computer science, or master's in computer science with PhD in a closely related field. Rank and salary dependent upon background. Exceptionally well-qualified candidates may be considered for a Bingham Award for Excellence in Teaching; smaller "start-up" grants are available for less experienced faculty. This recognition provides a supplement of up to 50% of base salary for the position. Transylvania is a private, liberal arts college with a strong commitment to excellence in undergraduate education. The program in computer science is of longstanding and is recognized for its outstanding quality. Please send letter of application, curriculum vitae, and names of three references to: Dr. Dwight W. Carpenter, Computer Science Program, Transylvania University, Lexington, KY 40508. An Equal Opportunity Employer.

TRANSYLVANIA UNIVERSITY

Lexington, KY 40508

The Mathematics Program invites applications for a tenure-track position commencing in the fall of 1991 and a possible one-year sabbatical replacement for 1991–92. Transylvania University is a private, liberal arts college with a strong commitment to academic excellence. Applicants must have a PhD in mathematics and a commitment to undergraduate teaching. Salary and rank will depend on qualifications and experience. Exceptionally well-qualified candidates may be considered for a Bingham Excellence in Teaching Award which supplements faculty member's salary by up to 50%; smaller, "start-up"grants are available for less experienced faculty. Send letter of application, resumé, undergraduate and graduate transcripts, and three letters of references to: David L. Shannon, Mathematics Program Director, Transylvania University, Lexington, KY 40508. The search will remain open until the positions are filled. Transylvania University is an Equal Opportunity Employer.

ITHACA COLLEGE

The Department of Mathematics and Computer Science at Ithaca College has two tenure-eligible positions in mathematics available for the 1991–92 academic year. Qualifications: PhD preferred, active ABDs considered. Rank: assistant professor or above. All successful candidates will be expected to teach a wide variety of mathematics courses at the undergraduate level. Screening begins December 17, 1990. Send vita to: Dr. Eric Robinson, Chair, Department of Mathematics and Computer Science, Ithaca College, Ithaca, NY 14850. An Affirmative Action/Equal Opportunity Employer.

NATURAL SCIENCES AND MATHEMATICS DEPARTMENT

Trinity College of Vermont
Burlington, VT 05401

Trinity College invites applications for a ***faculty position in mathematics,*** August 1991. Trinity is located in Burlington, VT, a progressive and intellectually stimulating community. Trinity's mission stresses academic excellence, the education of women, combining the liberal arts with career preparation, and supporting life-long learning. PhD in math and demonstrated independent teaching ability required. Duties: 24 credit hours of instruction per year, student advising, committee work. Continued faculty development expected and supported. Rank and salary depend on qualifications and experience. Application: letter, vita, graduate transcript, one letter of reference addressing teaching, names/addresses/phone numbers of 2–3 additional references. Submit to: Dr. John F. Heinbokel, Chair, NSM Department, Trinity College, Burlington, VT 05401. Review of applications will continue until position filled. Applications from women and minorities are encouraged. AA/EOE.

CLARION UNIVERSITY

Clarion, Pennsylvania

Applications are invited for a tenure-track position in mathematics at the assistant/associate professor level, beginning fall 1991. PhD (by August 1991) in the mathematical sciences and evidence of successful mathematics teaching are required, Faculty load is 12 hours per semester. Salary and benefits are highly competitive. Located in northwestern Pennsylvania, adjacent to Interstate 80, Clarion University is one of fourteen institutions in the State System of Higher Education. With 6,100 students, it offers the bachelor's degree in mathematics, with options in applied mathematics, actuarial science, computer science, and in mathematics education. Send letter of application, personal vita, transcripts, and three current letters of reference to: Main Campus Search Committee, Department of Mathematics, Clarion University, Clarion, Pennsylvania 16214. Screening will begin December 1, 1990, but applications will be accepted until the position is filled. Clarion University actively seeks minorities and women applicants, and is an affirmative action/equal opportunity employer.

THE UNIVERSITY OF SOUTH CAROLINA

Department of Mathematics

Applications are invited for anticipated tenure-track faculty positions at all ranks. Applicants in all areas of mathematics will be considered. The department is building on existing research strengths and is increasing the scope of its program in applied mathematics. The PhD degree or its equivalent is required, and all appointments will be consistent with the department's commitment to excellence in research and teaching at the graduate and undergraduate levels. A resumé, containing a summary of research accomplishments and goals, and four letters of recommendation should be sent to:

Dr. Colin Bennett, Chairman
Department of Mathematics
University of South Carolina
Columbia, South Carolina 29208

The closing date for applications is January 31, 1991. The University of South Carolina is an Affirmative Action/Equal Opportunity employer.

CALIFORNIA STATE UNIVERSITY

Long Beach

Tenure-track position Statistics, begin fall 1991: Requires PhD math, applied math, or statistics, strong coursework math, research interests stat; evidence effective teaching and strong research potential; knowledge actuarial math desirable. Teach 3–4 classes per semester undergrad math and stat, MA-level stat; research in speciality. Asst. or assoc. prof. preferred. Send resumé, transcripts, 3 references letters to: Samuel G. Councilman, Chair, Math Dept., CSU Long Beach, CA 90840-4502. Position open until filled; selection begins 12/1/90. CSULB is an Equal Opportunity/Affirmative Action/Title IX Employer.

MATHEMATICS DEPARTMENT UNIVERSITY OF NORTH DAKOTA

Box 8162 University Station
Grand Forks, ND 58202-8162

Applications are invited for 2 or more tenure-track positions at the assistant professor level starting August 16, 1991. Consideration will be given to all areas of mathematics, as well as statistics and math education. Applicant must possess a strong commitment to teaching and research and have completed PhD requirements by starting date. Teaching loads are three courses/semester. Must be eligible to work in US. Salary and fringes competitive. Application deadline: March 1, 1991. Send resumé, copy of transcripts, and three letters of reference to: Selection Committee. UND is an AA/EOE.

WESTERN KENTUCKY UNIVERSITY

DEPARTMENT HEAD

DEPARTMENT OF MATHEMATICS

The Department of Mathematics is seeking an energetic and enthusiastic individual to serve as Head of the Department. The Department consists of 28 full-time faculty and offers baccalaureate and masters degree programs with approximately 270 undergraduate majors and minors.

Western has an enrollment of 15,000 students and is located in Bowling Green, KY (population approximately 50,000), two hours south of Louisville and one hour north of Nashville, TN along interstate I-65.

The successful candidate will hold a Doctorate in Mathematics and have at least five years of college teaching experience. Applicants should demonstrate administrative leadership skills and provide evidence of effective teaching, public service, and research/scholarly activities.

Review of applications will begin February 1, 1991, and will continue until the position is filled, with expected date of appointment July 1, 1991. Send letter of application, vita, and names, addresses and phone numbers of at least three references to: Office of Academic Affairs, Mathematics Department Head Search, Western Kentucky University, Bowling Green, KY 42101.

An Affirmative Action/Equal Opportunity Employer.
Women and minorities are encouraged to apply.

WEST CHESTER UNIVERSITY

Mathematics Education and Computer Science

West Chester University seeks applicants for a tenure-track assistant or associate professorship in mathematics education beginning September 1991 to teach pre- and inservice mathematics content and methods courses for elementary majors, as well as other mathematics service courses. Also, will be responsible for the recruitment and advisement of elementary education majors who concentrate in mathematics. A doctorate in mathematics education is preferred but a candidate at dissertation level may be considered. Applicants must have experience teaching at the elementary school level and/or teaching preservice mathematics for elementary majors. Preference will be given to applicants with a demonstrated interest in research and grant proposal writing in K–8 mathematics education. Please send transcripts, vita, and three letters of reference, postmarked by January 31, 1991, to: Prof. John Kerrigan, Mathematics Dept., WEST CHESTER UNIVERSITY, West Chester, PA 19383. AA/EOE. Women and Minorities Are Encouraged to Apply.

DEPARTMENT HEAD

Mathematics and Computer Science at the University of Northern Iowa, a department with a strong tradition of excellence in teaching and teacher preparation and a growing program of scholarship and research in mathematics, computer science, and mathematics education, is seeking a new department head. Responsibilities include budgeting; faculty assignment, evaluation & development; external relations; and some teaching and scholarly activity. Required characteristics include being appointable as a full professor in the department, leadership & academic administrative skills, and good communication skills.

Appointment will cover the academic year plus summer session and begin June or August 1991. Year salary will be near $70,000 plus excellent fringe benefits. Application screening begins January 14, 1991. For further information, contact: Philip East, Mathematics and Computer Science, University of Northern Iowa, Cedar Falls, IA 50614-0506.

UNI is an affirmative action/equal opportunity educator and employer.

FROSTBURG STATE UNIVERSITY

Mathematics

Full-time, tenure-track, instructor/assistant professor beginning fall 1991, to teach 12 credits of introductory level mathematics per semester and share departmental responsibilities. Salary range is $25,000–$35,000. Master's degree in mathematics required, doctorate preferred in mathematics or mathematics education. Candidate must have a strong commitment to undergraduate teaching and a continuing interest in mathematical development. Preferable attributes include teaching experiences, experience with applications of mathematics, and interest in applications of technology to classroom teaching. Send letter of interest, resumé, transcripts and three letters of recommendation no later than January 15, 1991 to: Mr. C. Douglas Schmidt, Director of Personnel Services, Frostburg State University, Frostburg, MD 21532. Questions may be directed to: Dr. Richard Weimer, Department Chair; (301) 689-4377. Women and minorities are encouraged to apply. AA/EEO.

EAST STROUDSBURG UNIVERSITY

East Stroudsburg, PA 18301

Full-time, tenure-track position. Assistant/associate professor depending on qualifications. Master's plus 10 credits in mathematics required; doctorate degree and discrete mathematics background preferred. Primary duties in undergraduate instruction. 24 hour per academic year load. Competitive salary and excellent benefits. Send current resumé, transcripts, and 3 letters of recommendation to: Prof. Edward Hogan, Search Committee Chairperson, Mathematics Department, East Stroudsburg University, East Stroudsburg, PA 18301. Deadline for applications is February 1, 1991. A State System of Higher Education University. An Affirmative Action/Equal Opportunity Employer.

STATE UNIVERSITY OF NEW YORK AT PURCHASE

Four-year liberal arts college near New York City has available a tenure-track assistant professorship in mathematics starting September 1991. Responsibilities include teaching five courses per year, supervising senior theses, and conducting research. Must be able to teach elementary computer science courses. Interest in teaching general education courses a plus. Send three letters of reference and resume to: Dr. Martin Lewinter, Chair, Mathematics Search Committee, State University of New York, 735 Anderson Hill Road, Purchase, NY 10577-1400. Letters of reference should address teaching and research potential. Application deadline is January 4, 1991 or until position is filled. Equal Opportunity/Affirmative Action Employer.

BOWDOIN COLLEGE

Brunswick, Maine 04011

Mathematics Department: Two tenure-track assistant professorships starting fall 1991. Initial appointment for three years with renewal possible. PhD required and strong research record or potential expected. Field open, but a preference will be given to candidates in applied mathematics for one position. Normal teaching load is two courses per semester. Candidates with record of effective undergraduate teaching preferred. Review of candidates begins 15 January, but applications will be considered until both positions are filled. Women and minorities are encouraged to apply. Send resumé and 3 letters of recommendation to: Wells Johnson, Chair, Department of Mathematics, Bowdoin College, Brunswick, ME 04011. Bowdoin College is committed to Equal Opportunity through Affirmative Action.

GRAND VALLEY STATE UNIVERSITY

Allendale, Michigan

Tenure-track positions are open for fall 1991 in mathematics, mathematics education, statistics, computer science, and information systems. Duties include teaching undergraduate and/or graduate courses, student advising, and professional development. Earned doctorate and strong teaching recommendations required.

GVSU is located in greater Grand Rapids, the second largest metropolitan area in Michigan, and offers numerous cultural and recreational opportunities. Cost of living is moderate and quality of life is high. Applications accepted until positions are filled. Send resumé and names of three references to: Faculty Search Committee, Math & CS Dept., GVSU, Allendale, MI 49401. An EO/AA Institution.

MILLERSVILLE UNIVERSITY

Millersville University of PA, Mathematics Department. Tenure-track assistant professor in mathematics beginning August 1991. Duties include teaching upper and lower division undergraduate mathematics courses. Must have strong commitment to excellence in teaching and scholarship. PhD in mathematics or near completion. Candidates with specialty in any field of mathematics may apply; some preference will be given to applications received by 1/28/91, but consideration of applications will continue until position filled. Submit resumé, copies of transcripts, and three letters of recommendation (at least two attesting to teaching effectiveness) to: Dr. Dorothee Jane Blum, Search Committee/FO1190, Department of Mathematics, MILLERSVILLE UNIVERSITY, Millersville, PA 17551. An Affirmative Action/Equal Opportunity Employer.

THE UNIVERSITY OF SCRANTON

Mathematics Department

The University of Scranton is a Jesuit university with over 3,500 undergraduates. The Mathematics Department has 15 full-time faculty and about 50 majors.

One (possibly two) tenure-track position is available for fall 1991 for faculty interested in a teaching environment where research is encouraged and supported. Individuals with expertise in any area of mathematics will be considered. Preferred areas include applied mathematics, probability/statistics, actuarial mathematics, algebra, and analysis. Rank and salary are open and competitive.

Submit a vita, transcripts, and three references to: Mathematics Faculty Search Committee, University of Scranton, Scranton, PA 18510 or phone (717) 914-6113. Screening will begin at once and applications will be considered until all positions have been filled. An AA/EO Employer and Educator.

MURRAY STATE UNIVERSITY

Department of Mathematics & Statistics

Applications are invited for tenure-track positions at the assistant/associate professor level beginning August 1991. Preference will be given to applicants in statistics and mathematics education.

Responsibilities include a maximum three course teaching load consisting of a wide variety of undergraduate and graduate level courses, continuing research/scholarly activities, and university and departmental service. A PhD in statistics or a PhD in applied mathematics with a statistics background is required for the statistics position. The mathematics education position requires a PhD in mathematics education or a PhD in mathematics with a background in mathematics education. Salary will be competitive. Screening will begin January 14, 1991 and continue until the positions are filled.

Applicants who are not US citizens must provide their visa status and any other information relevant to their ability to accept employment.

Send letter of application with vita, graduate transcripts or a list of graduate courses taken, and direct three letters of recommendation to:

Dr. Robert Pervine, Search Committee Chair
Department of Mathematics & Statistics
Murray State University
Murray, KY 42071

MSU is an EO/AA employer.

CENTRAL MICHIGAN UNIVERSITY

Three tenure-track positions and one tentative tenure-track position. All are at assistant professor rank. Priorities for the three indicated positions are 1. Functional analysis/operator theory; 2. Combinatorics/design theory; and 3. Mathematics education. The tentative position is in statistics and its status will be determined by January 1991. Statistics candidates may also be considered as a fourth priority for one of the three positions mentioned above. Candidates for all positions should have a doctorate in the appropriate field of mathematics, show promise of excellence in teaching, and have demonstrated research ability. Candidates in mathematics education should have teaching experience in K–12 and the ability to teach undergraduate mathematics courses. Duties include teaching and research with a normal teaching load of 9 semester hours. Preference will be given to candidates who complement existing research interests in the department. Salaries are competitive and benefits include university-paid TIAA, medical, dental, and group life. Send resumé, transcripts, and three letters of recommendation to: R. J. Fleming, Dept. of Mathematics, Central Michigan Univ., Mt. Pleasant, MI 48859 by January 21, 1991. Late applications will be received until the positions are filled. Central Michigan University is an affirmative action/equal opportunity employer. All persons including members of minority groups, women, handicapped persons, disabled veterans, and veterans of the Vietnam Era are encouraged to apply.

KINGS'S COLLEGE

Wilkes-Barre, PA 18711

Math Position: assistant professor. PhD required. ABD will be considered. Successful applicant will teach an advanced course for majors in addition to introductory courses. Primary interest and commitment must be to quality undergraduate instruction and to formally training young mathematicians. Public scholarship and active involvement in the mathematical community are also expected. Applications, consisting of teaching and research interests, vita, transcripts, and three letters of recommendation should be submitted to: Dr. William Shergalis, Dean of the College of Arts & Sciences, King's College, Wilkes-Barre, PA 18711. Review of applicants will begin on December 1, 1990. KING'S COLLEGE IS AN EQUAL OPPORTUNITY EMPLOYER AND SPECIFICALLY INVITES AND ENCOURAGES APPLICATIONS FROM WOMEN AND MINORITIES.

PURDUE UNIVERSITY

Tenure-track position in the area of quantitative-mathematics psychology beginning fall 1990.

Strong research credentials or research potential essential. Salaries and benefits highly competitive, excellent research facilities, and support available. Curriculum vitae, three letters of recommendation, and representative preprints and/or reprints should be sent to: Dr. Jerome Busemeyer, c/o Department of Psychological Sciences, Purdue University, West Lafayette, Indiana 47907. Applications received prior to November 1 will be given fullest consideration though applications will be accepted until filled. Women and minorities are especially encouraged to apply.

Purdue University is an Equal Opportunity Affirmative Action Employer.

BENTLEY COLLEGE

Waltham, Massachusetts

The Department of Mathematical Sciences anticipates at least one opening for a tenure-track position starting in fall 1991. A PhD in mathematics, statistics, quantitative methods, operations research, or a related field is required. Located in suburban Boston, Bentley College has long been known for its leadership in the education of business professionals. In recent years, the school has experienced dramatic growth and currently enrolls about 7,500 graduate and undergraduate students in both business and liberal arts programs. While heavy emphasis continues to be placed on quality teaching, research and other scholarly activities are also encouraged and expected. Send resumé to: Prof. Charles. R. Hadlock, Chair, Department of Mathematical Sciences, Bentley College, 175 Forest Street, Waltham, MA 02154-4705. Bentley College is an Equal Opportunity/Affirmative Action employer.

HOPE COLLEGE

Holland, Michigan 49423

The Department of Mathematics at Hope College invites applications for two tenure-track positions, beginning August 1991. Candidates should have a PhD in mathematics (or one of the mathematical sciences), a commitment to excellence in teaching, and evidence of continuing professional activity. Scholarship and research are encouraged and supported. Candidates should have a strong interest in research projects involving undergraduates. A letter of application, vita, graduate and undergraduate transcripts, and at least three letters of recommendation should be sent to: Prof. John Stoughton, Mathematics Search. Departmental representatives will interview at the San Francisco meetings. Hope College is a Christian, coeducational, residential, liberal arts college affiliated with the Reformed Church in America. Hope College complies with federal and state requirements for nondiscrimination in employment. Applications are strongly encouraged from women and minority persons.

UNIVERSITY OF TENNESSEE AT CHATTANOOGA

Department Head

The University of Tennessee at Chattanooga invites applications for Head of the Department of Mathematics. A PhD in a mathematical science and at least five years of college mathematics teaching experience are required. Applicants should provide evidence of leadership in curriculum development, teaching, public service, and research/scholarly activities. In this primarily undergraduate institution, the faculty is expected to exhibit excellence in teaching while maintaining a strong commitment to research and public service. The mathematics department has 21 faculty members including a Chair of Excellence in applied mathematics. Located in a very scenic metropolitan area of 400,000, UTS has a student enrollment of 7,800. Send application with current vita to: Dr. Paul L. Gaston, Dean, College of Arts and Sciences, 119 Holt Hall, UTC, Chattanooga, TN 37403-2598. Applications received by January 31, 1991 will be assured full consideration. Women and minorities are encouraged to apply. UTC is an Equal Opportunity Employment/Affirmative Action/Title IX Section 504 Institution.

EASTERN ILLINOIS UNIVERSITY

Department of Mathematics
Charleston, IL 61920

A tenure-track position starting fall semester 1991 is anticipated. Doctorate in mathematics or mathematical sciences (including appl. math., computing, math. ed., or stat.) is required. Excellence in teaching is expected and potential for scholarly activities is desired. Rank is open. Full consideration given to applications received by January 1, 1991. Applications from minorities and women are encouraged. Contact: Ira Rosenholtz, Chairperson, at the above address.

CALIFORNIA STATE UNIVERSITY HAYWARD

Dept. of Mathematics & Computer Science

The Department invites applications for entry-level, tenure-track assistant professor position in mathematics beginning fall 91. Applicants should hold PhD degree in mathematics, be committed to excellence in teaching, exhibit competence and potential to engage in professional activities including research and publication. All areas of specialization will be considered, including mathematics education. The department enrolls more than 700 majors in its four degree programs: BS in mathematics, BS in computer science, and MS in both disciplines. Please send resumé and 3 references to: Mathematics Faculty Search Committee, Dept. of Mathematics & Computer Science, California State University, Hayward, Hayward, CA 94542-3092. Applications received by Jan. 1, 1991 will be assured full consideration. Applications will be accepted until position is filled. Position #91-92 MATH-TT-1. CSUH is an Equal Opportunity/Affirmative Action Employer.

TEIKYO LORETTO HEIGHTS UNIVERSITY

Denver, Colorado

Two positions at the asst. professor level will be available in August 1991 to teach college algebra and/or calculus. A PhD in mathematics, mathematics education, or related discipline is required; teaching experience in a multicultural setting is desirable. The applicant should have an international perspective and be sensitive to the special needs of Japanese and other international students studying mathematics in English. Send letter of application, cv, and three letters of reference to: Dr. W. Hodson, Mathematics Search Committee, Teikyo Loretto Heights University, 3001 S. Federal Blvd., Denver, CO 80236 by January 12, 1991. Interviews will be conducted at the January MAA meeting. Equal Opportunity Affirmative Action Employer.

HAMILTON COLLEGE

Department of Mathematics
and Computer Science
Clinton, NY 13323

Two-year, tenure-track position. PhD and prior teaching experience desirable. Duties involve teaching five courses per year at a small, highly selective, 4-year liberal arts college. Excellence in teaching required. To apply, send curriculum vitae and three letters of reference (at least one about teaching) to: Richard Bedient, Chair; (315) 859-4138. Women and members of minorities are encouraged to apply; Hamilton College is an Equal Opportunity/Affirmative Action Employer.

National MAA Meetings

16–19 January 1991 74th Annual Meeting, San Francisco, California (Board of Governors, 15 January 1991)
8–11 August 1991 67th Summer Meeting, Orono, Maine (Board of Governors, 7 August 1991)
8–11 January 1992 75th Annual Meeting, Baltimore, Maryland (Board of Governors, 7 January 1992)

Sectional MAA Meetings

Allegheny Mountain West Virginia State College, Institute, West Virginia: 12 and 13 April 1991
Eastern Pennsylvania and Delaware University of Delaware, Newark, Delaware: 10 November 1990
Florida Eckerd College, St. Petersburg, Florida: 1 and 2 March 1991
Illinois Eastern Illinois University, Charleston, Illinois: 26 and 27 April 1991
Indiana Anderson University, Anderson, Indiana: 23 March 1991
Intermountain Ricks College, Rexburg, Idaho: 12 and 13 April 1991
Iowa Drake University, Des Moines, Iowa: Spring 1991
Kansas Southwestern College, Winfield, Kansas: 5 and 6 April 1991
Kentucky Northern Kentucky University, Highland Heights, Kentucky: 5 and 6 April 1991
Louisiana and Mississippi University of Mississippi, Biloxi, Mississippi: 1 and 2 March 1991
Maryland–District of Columbia–Virginia Towson State University, Towson, Maryland: 16 and 17 November 1990; Virginia Commonwealth University, Richmond, Virginia: Spring 1991
Metropolitan New York Columbia University, New York, New York: 4 and 5 May 1991 (50th Anniversary)
Michigan Calvin College, Grand Rapids, Michigan: 10 and 11 May 1991
Missouri The University of Missouri at Rolla, Rolla, Missouri: 5 and 6 April 1991
Nebraska Nebraska Wesleyan University, Lincoln, Nebraska: 26 and 27 April 1991
New Jersey Seton Hall University, South Orange, New Jersey: 10 November 1990
Northeastern Framingham State College, Framingham, Massachusetts: 16 and 17 November 1990
Northern California California State University at Hayward, Hayward, California: February or March 1991
Oklahoma and Arkansas Cameron University, Lawton, Oklahoma: 29 and 30 March 1991
Pacific Northwest Seattle Pacific University, Seattle, Washington: 20–22 June 1991
Rocky Mountain University of Northern Colorado, Greeley, Colorado: Spring 1991
Seaway State University of New York at Oneonta, Oneonta, New York: Spring 1991
Southeastern University of South Alabama, Mobile, Alabama: 5 and 6 April 1991
Southern California University of California at Irvine, Irvine, California: 10 November 1990
Southwestern New Mexico State University, Las Cruces, New Mexico: 5 and 6 April 1991
Texas Stephen F. Austin State University, Nacogdoches, Texas: 4–6 April 1991
Wisconsin University of Wisconsin, Oshkosh, Wisconsin: 26 and 27 April 1991

Other Meetings

9–10 November 1990 *Fourth Annual Southeastern Small College Computing Conference,* Lenoir-Rhyne College, Hickory, North Carolina. For further information, contact: Frank Cheatham, Campbellsville College, 200 West College Street, Campbellsville, Kentucky 42718; (502) 465-8158.
9–11 November 1990 *Third Annual International Conference on Technology in Collegiate Mathematics,* Ohio State University. Invited lectures, panel discussions, workshops, minicourses, and contributed paper sessions on the use of technology in collegiate mathematics teaching. Topics: precalculus, calculus, differential equations, fractals, statistics, linear algebra, and related issues. Send your early registration fee of $45.00 to: Franklin D. Demana and Bert K. Waits, 1990 Conference on Technology in Collegiate Mathematics, Department of Mathematics, Ohio State University, 231 West Eighteenth Avenue, Columbus, Ohio 43210.
7–9 January 1991 *SIAM Workshop on Automatic Differentiation of Algorithms: Theory, Implementation, and Application,* Hilton Hotel, Breckenridge, Colorado. Organizer: Andreas Griewank of Argonne National Laboratory. For further information, contact: SIAM Conference Coordinator, Dept CC0900, 3600 University City Science Center, Philadelphia, Pennsylvania 19104-2688; (215) 382-9800; siamconfs@wharton.upenn.edu. Fax: (215) 386-7999.
20 January 1991 An informal *Workshop on the Teaching of Calculus* will immediately follow the MAA's annual meeting in San Francisco, California. No advance registration necessary. For further information, contact: Gilbert Strang, Room 2-240, MIT, Cambridge, Massachusetts 02139.
28–30 January 1991 *Second ACM-SIAM Symposium on Discrete Algorithms,* Cathedral Hill Hotel, San Francisco, California. Organizer: Alok Aggawal of the IBM T. J. Watson Research Center. For further information, contact SIAM Conference Coordinator at address above (7–9 January 1991).
11–15 February 1991 *Twenty-Second Southeastern International Conference on Combinatorics, Graph Theory, and Computing,* Louisiana State University (LSU), Baton Rouge, Louisiana 70803. Invited speakers include: R. Brualdi, P. Erdös, C. Godsil, W. Pulleyblank, and R. Thomas. For further information, contact: J. G. Oxley of the Department of Mathematics at LSU.

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