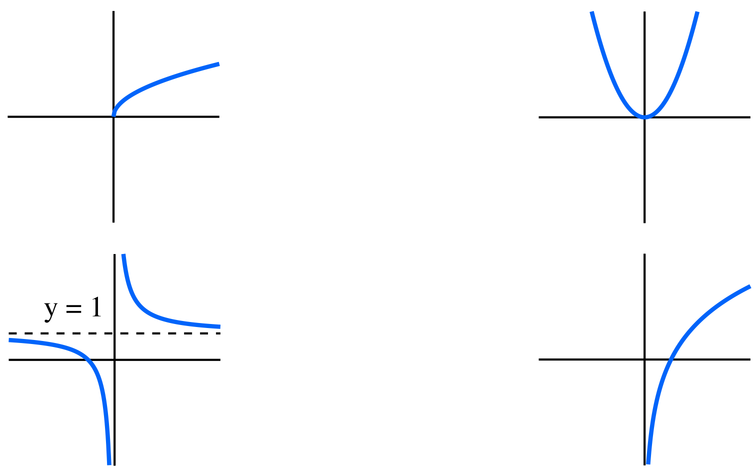
1. Given a function f, its ***inverse function*** (if it exists) is the function f^{-1} such that $y = f(x)$ if and only if $f^{-1}(y) = x$.

   (a) If we know that $f(\text{♫}) = \text{❤}$, what is $f^{-1}(\text{❤})$? What about $f(f^{-1}(\text{❤}))$?

   (b) If we know a function has an inverse function, what do we know about the properties, behavior, or graph of its inverse function? Create a list of these attributes.

2. Consider each of the functions below.



   (a) Which of the these functions has an inverse function? Explain how you can know without being given the defining expression.

   (b) What is the domain and range of each function? What is the domain and range of its inverse function (if it exists)?

1. Consider how Alex, Jordan, and Kelly found the inverse function of $f(x) = \frac{2}{3}x + 1$.

Alex's Work	Jordan's Work	Kelly's Work
$y = 2/3\,x + 1$ $x = 2/3\,y + 1$ $x - 1 = 2/3\,y$ $\frac{x-1}{2/3} = y$ $\boxed{\frac{3}{2}(x-1) = y}$	$f \circ f^{-1}(y) = y$ $f(x) = \frac{2}{3}x + 1$ So: $f(f^{-1}(y)) = \frac{2}{3}f^{-1}(y) + 1 = y$ $\frac{2}{3}f^{-1}(y) = y - 1$ $\frac{3}{2} \cdot \frac{2}{3}f^{-1}(y) = \frac{3}{2}(y-1)$ $\boxed{f^{-1}(y) = \frac{3}{2}(y-1)}$	$y = \frac{2x}{3} + 1$ $y - 1 = \frac{2x}{3}$ $3(y-1) = 2x$ $\frac{3(y-1)}{2} = x$ But $f(x) = y \Rightarrow$ $f^{-1}(f(x)) = f^{-1}(y) \Rightarrow$ $x = f^{-1}(y)$ So $\boxed{f^{-1}(y) = \frac{3(y-1)}{2}}$

Compare and contrast the key mathematical ideas used by Alex, Jordan, and Kelly to find the inverse function of $f(x) = \frac{2}{3}x + 1$. Make sure to identify which properties of inverse functions each student uses, if any.

2. Now consider two problems where a high school student used Alex's method of switching the variables and solving for the dependent variable to find the inverse function.

Find the inverse function of $T(C) = \frac{9}{5}C + 32$ where C is the temperature in Celsius and $F = T(C)$ gives the temperature in Fahrenheit.	Find the inverse function of $f(x) = \frac{2x+1}{x-1}$ for $x \neq 1$.
Let $F = \frac{9}{5}C + 32$ $C = \frac{9}{5}F + 32$ $C - 32 = \frac{9}{5}F$ $\frac{5}{9}(C-32) = F$	$y = \frac{2x+1}{x-1}, \quad x \neq 1$ $x = \frac{2y+1}{y-1}$ $(y-1)x = 2y+1$ $yx - x = 2y + 1$ $yx - 2y = 1 + x$ $y(x-2) = x+1$ $y = \frac{x+1}{x-2}, \quad x \neq 1$

(a) Describe why the student's work for the temperature function is problematic.

(b) Describe why the student's work for the rational function is problematic.

(c) What are the limitations of using Alex's method of switching the variables and solving for the dependent variable to find an inverse function? Would Jordan and Kelly have the same problem(s)? Explain.

3. We can use the properties of inverse functions to find their derivatives with respect to x.

   (a) Draw a right triangle that illustrates the relationship $y = \arcsin(x)$ for $0 < x < 1$.

   (b) Use the fact that $\sin(\arcsin(x)) = x$ and the chain rule to compute $\frac{d}{dx}\arcsin(x)$ in terms of x.

   (c) Use the fact that $e^{\ln(x)} = x$ and the chain rule to show that $\frac{d}{dx}.\ln(x) = \frac{1}{x}$

4. What key mathematical idea(s) from Problem 1 did we use to find the derivatives in Problem 3?

5. Let f be a function with inverse function g.

   (a) In the form of written sentences, describe how you would use the fact that $f(g(x)) = x$ and the chain rule to compute $\frac{d}{dx}g(x)$ for any function f with inverse function g.

   (b) Use the procedure you wrote above to compute $\frac{d}{dx}g(x)$ for any function f with inverse function g.

   (c) Why does the result in Problem 5(b) make sense given what we know about visualizing derivatives and the graphs of inverse functions?

6. What happens when we reverse the order of the composition in the previous problem before we differentiate? Can we still find a formula for $\frac{d}{dx}g(x)$? Why or why not?

1. A theater concludes that their total revenue for the week is a function of the number of tickets they sell. They use the equation $R(t) = 15t - 100$ to represent this relationship, where t is the number of tickets sold. Akira uses the method of "switch x and y and solve for y" to find the inverse function of the relationship described above.

   Let $r = 15t - 100$

   $$t = 15r - 100$$

   $$t + 100 = 15r$$

   $$r = \frac{1}{15}(t + 100)$$

   What two questions would you ask Akira to help them see the limitations of their work? Why would your questions be helpful?

2. Using the properties of inverse functions, find the inverse function of the composite function $h(x) = f(g(x))$, where both f and g are known to have inverse functions.

3. Find a formula for $\frac{d}{dx}\arccos(x)$ in terms of x. Compare this formula to $\frac{d}{dx}\arcsin(x)$ (from Problem 3(b) in the Class Activity). What do you notice about the two derivatives?

4. We call a point a in the domain of a function f a *fixed point* if $f(a) = a$.

   (a) Give an example of a continuous function with no fixed points.

   (b) Give an example of a continuous function with precisely one fixed point.

   (c) For some continuous function f, assume its inverse function exists and has a fixed point (i.e., $f^{-1}(b) = b$). Does f have a fixed point? If so, for what value of x?

1. A classmate is finding the formula of an inverse function and does not yet understand why the method of "switch x and y and solve for y" is problematic. Provide an example of a situation in which this approach is problematic and explain why.

2. Show the steps used to find a formula for $\frac{d}{dx}\arctan(x)$ in terms of x. Express $\frac{d}{dx}\arctan(x)$ as a rational function.