

Let g_1 and g_2 be two distinct elements of G . Then:

$$g_1 \cdot_G g_2 = g_2 \cdot_G g_1 \quad \leftarrow G \text{ is abelian}$$

$$f(g_1 \cdot_G g_2) = f(g_2 \cdot_G g_1) \quad \left. \begin{array}{l} \searrow \\ \swarrow \end{array} \right\} f \text{ is a homomorphism}$$

$$f(g_1) \cdot_H f(g_2) = f(g_2) \cdot_H f(g_1) \quad \left. \begin{array}{l} \searrow \\ \swarrow \end{array} \right\} f(g_1) \text{ and } f(g_2) \text{ are in } H$$

$$h_1 \cdot_H h_2 = h_2 \cdot_H h_1$$

So, H is also an abelian group.