1. Write an equation of **a line** with slope 3 that passes through the point $(2, 1)$ in point-slope form. Then, write an equation of this line in slope-intercept form.

2. Write an equation of the **tangent line** to the graph of $f(x) = x^2 + 2$ at the point $(1, 3)$ in point-slope form. Then, write an equation of this tangent line in slope-intercept form.

3. More with tangent lines.

   (a) For a given function f, describe how to find an equation of the tangent line to the graph of f at $x = a$.

   (b) Now, write an equation of the tangent line to the graph of f at $x = a$.

4. Find the zeroes of the following functions.

   (a) $f(x) = x^2 - 4$

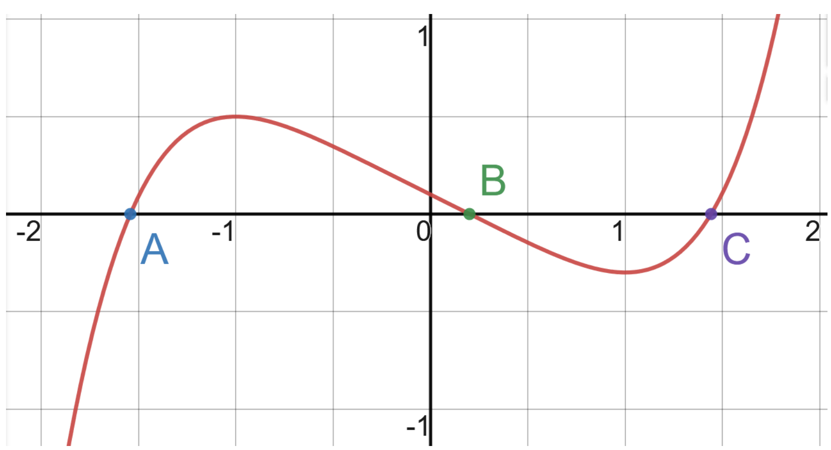
   (b) $g(x) = 3x^2 + 7x - 2$

   (c) $h(x) = x^3 + x^2 - 2x$

5. Consider the function, $f(x) = \frac{1}{10}x^5 - \frac{1}{2}x + \frac{1}{10}$

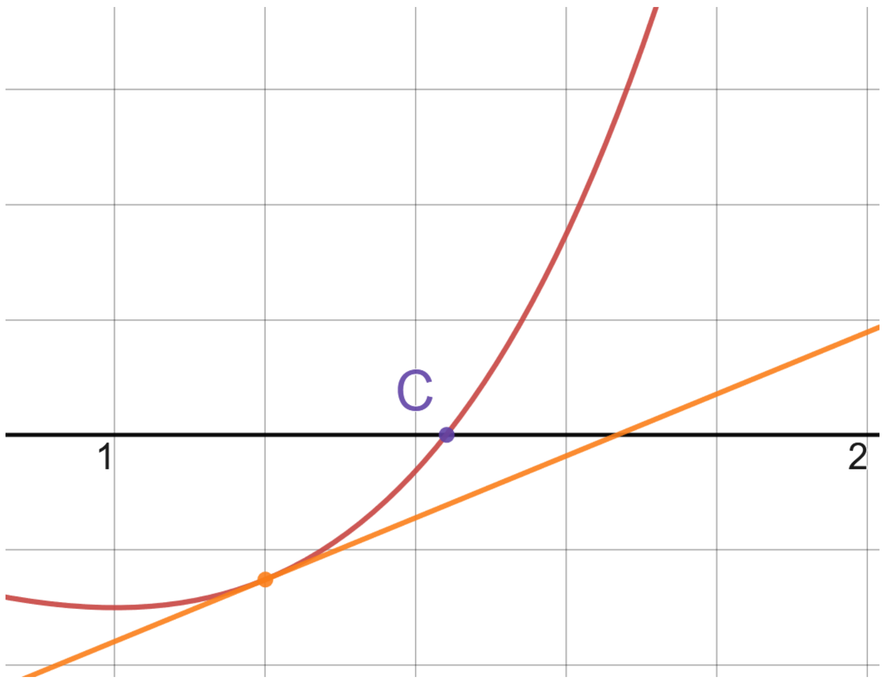
   (a) Nnamdi has excellent algebra skills, yet he tries to find the zeroes algebraically and gets stumped. Explain why he is having trouble.

   (b) Nnamdi decides to graph f to find the zeroes. The zeroes are indicated on the graph as A, B, and C. Estimate the value of C.



Recall the function from Problem 5 on the Pre-Activity, $f(x) = \frac{1}{10}x^5 - \frac{1}{2}x + \frac{1}{10}$, for which Nnamdi wanted to find the zeroes of the function. Nnamdi initially thinks that $x = 1.2$ is a good estimate of the zero, C, but when he zooms in on the graph he realizes that C is further to the right. He starts to experiment with linear functions to try to find a better estimate for C.

1. Nnamdi zooms in on the graph and sketches the tangent line at $x_0 = 1.2$ (see graph below).



   (a) Label the x-intercept of Nnamdi's tangent line as x_1.

   (b) Write an equation of Nnamdi's tangent line in point-slope form and find the value of x_1.

2. Taking inspiration from Nnamdi's idea, Mari decides to sketch another tangent line to the graph of $f(x)$ at the point $(x_1, f(x_1))$. She claims that the x-intercept of her tangent line will be closer to the zero C than x_1.

   (a) Do you agree with Mari's claim? Explain why or why not.

(b) Sketch in Mari's tangent line. Label the x-intercept of her tangent line as x_2.

(c) Write the equation of Mari's tangent line in point-slope form and find the value of x_2.

3. Amy uses both Mari's and Nnamdi's ideas to find a point, x_3, even closer to the zero C.

   (a) What do you think she did? Explain.

   (b) Find the value of x_3.

4. Fill in the following table with the values of x_1, x_2, and x_3 that you found above. Describe what you notice about these values.

x_0	x_1	x_2	x_3
1.2			

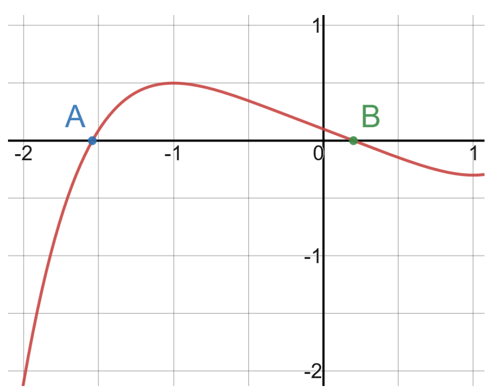
5. The iterative process Amy follows from the work of Mari and Nnamdi is called Newton's method. To apply Newton's method, the process of "finding a tangent line at the point on the graph corresponding to the guess for the zero, finding its x-intercept, and using this x-intercept as the next guess for the zero" is repeated. These x-intercepts (usually denoted x_0, x_1, x_2, x_3, $\ldots$) provide successive approximations of the value of a zero of a function.

   (a) Describe this process graphically.

   (b) Describe this process algebraically. Write out a formula to find x_{n+1}, the x-intercept of the tangent line created from the previous guess, x_n.

   (c) How do you know when to stop this iterative process? That is, when is your approximation of a zero "good enough?"

6. Reconsider $f(x) = \frac{1}{10}x^5 - \frac{1}{2}x + \frac{1}{10}$. Nnamdi now wants to use Newton's method to approximate the zero, A. He wonders what will happen if he uses the following initial guesses: $-0.5, -1, -1.5$, and -2.



(a) Without doing any calculations, which zero of f do you expect each of these initial guesses to lead? Explain your reasoning. Use the graph above to graphically show (by drawing tangent lines) what happens when you apply Newton's method using these initial guesses.

i. $x_0 = -0.5$

ii. $x_0 = -1$

iii. $x_0 = -1.5$

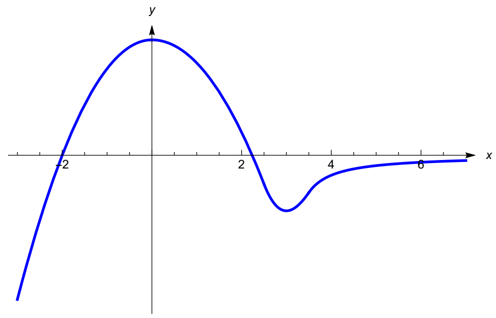
iv. $x_0 = -2$

(b) Use Newton's method with all four initial guesses to calculate a zero of f. Give your answer to three decimal places, when applicable.

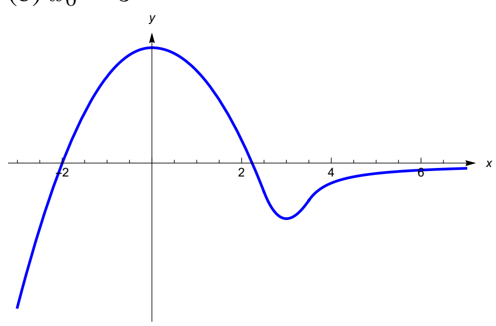
(c) Summarize to Nnamdi what you observe in the graph of f that indicates what zero you will approximate given your initial guess.

1. The graph of $y = f(x)$ is shown here. Use the initial guesses given to determine which successfully lead to an approximation of a zero of the function f when using Newton's method. For each initial guess, graphically (by drawing tangent lines) support your conclusion based upon using Newton's method and explain your reasoning.

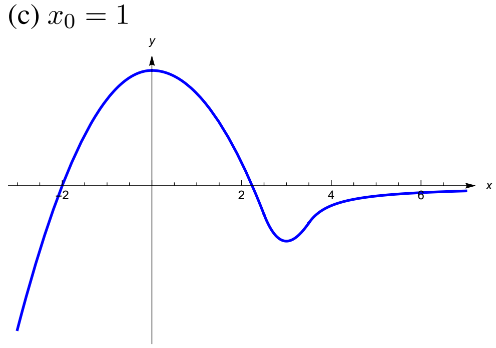
(a) $x_0 = 0$



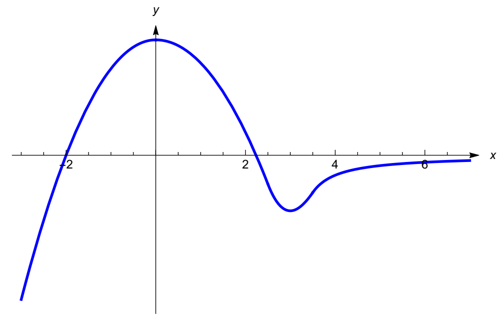
(b) $x_0 = 3$



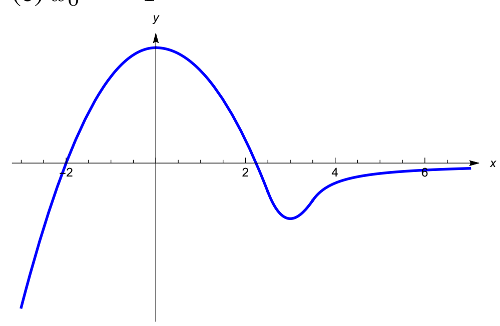
(c) $x_0 = 1$



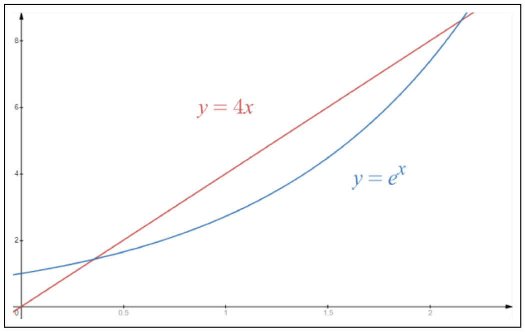
(d) $x_0 = 4$



(e) $x_0 = -1$



2. Consider the system of equations given below.



$$y = e^x$$
$$y = 4x$$

   (a) Explain how you could use Newton's method to approximate the two solutions to the system of equations.

   (b) Choose any initial guess and calculate one iteration of Newton's method for each solution. Record your approximations up to six decimal places.

3. Madalena is trying to use Newton's method to find the zeroes of the function $f(x) = x^3 - 2x + 2$. After making sure that $f'(1) \neq 0$, she chooses $x_0 = 1$. Then, she computes the first few iterations of Newton's method and begins a table of values:

x_0	x_1	x_2	x_3
1	0	1	

   Madalena sees that this pattern will continue and comes to you for help.

   (a) Show that Madalena's computations for x_1 and x_2 are correct.

   (b) Create a graph of f and sketch tangent lines to explain why Newton's method has failed.

   (c) Consider the following questions that you might ask Madalena:

      i. Explain how the question below might help you assess what Madalena understands about Newton's method:

      Given a graph of f and an initial guess x_0, how could you find x_1 without making the calculations you have already tried?

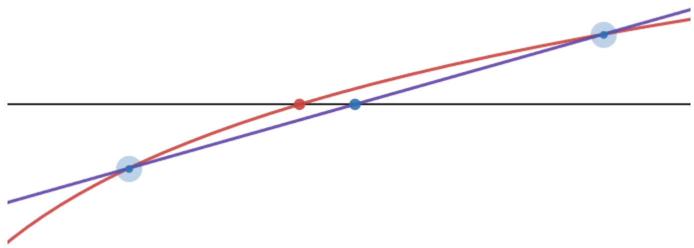
      ii. Explain how following up with the next question might help Madalena to advance in her understanding of Newton's method:

      Considering your graphical explanation of Newton's method, how could two iterations of Newton's method have the same value?

      iii. Explain why the question below might not help Madalena:

      What is the formula for Newton's method?

4. Terrance, a high school algebra student, is using his TI-84 graphing calculator to find the zero of a function. To do so, the calculator requires him to choose a left bound (a point on the graph to the left of the zero), a right bound (a point on the graph to the right of the zero), and a guess (a point on the graph very close to the zero). Terrance thinks that the calculator is using the bounds he has chosen to construct a secant line, which it uses to approximate the zero. He draws the following example to illustrate his idea.



   (a) Under what circumstances would Terrance's "secant method" fail to approximate a zero? Create a graph of one such example.

   (b) Using your understanding of Newton's method (*but language that a high school algebra student would understand*), explain to Terrance why the calculator might need a left bound, right bound, and guess.

1. Use Newton's method with initial guess $x_0 = 3$ to calculate the first three approximations of a zero of the function $f(x) = x^2 - 5$. Be sure to use at least six decimal places.

2. Jack and Raven are working together to find the zeroes of the function $f(x) = x^3 - 3x + 1$ using Newton's method. Jack suggests they begin with an initial guess of $x_0 = 1$. Raven says that won't work.

   (a) Why do you think Raven claims Jack's initial guess will not work? Using tangent lines, explain what Raven may have noticed.

   (b) Write two questions Raven can ask Jack to help him revise his initial guess. Explain how Raven's questions might help Jack.