Abstracts for the MAA Undergraduate Student Poster Session

2021 Joint Mathematics Meetings
Virtual Online Event
January 7–8, 2020

MAA
MATHEMATICAL ASSOCIATION OF AMERICA

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Organized by

Eric Ruggieri
College of the Holy Cross

and

Chasen Smith
Georgia Southern University

MAA

Dear Students, Advisors, Judges and Colleagues,

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Welcome to what is sure to be a very unique MAA Undergraduate Student Poster Session. Thank you for your willingness to try something new and your flexibility when things dont go exactly as planned. This years poster session contains 200 posters and 325 student presenters, representing a wide array of mathematical topics and ideas, and these posters showcase the vibrant research being conducted as part of summer programs and during the academic year at colleges and universities from across the United States and beyond. The session offers a great opportunity for interaction between students and professional mathematicians and is incredibly rewarding for all who participate.

The new format of this years poster session required a lot of extra work during a challenging semester. A lot of people stepped up to attend extra planning meetings and volunteer for new tasks. We cant thank all of you enough.

The judges that come to your presentations are professional mathematicians from institutions around the world. They are advisors, colleagues, new Ph.D.s, and administrators. Many of the judges signed up when they registered for the conference, but there are also a number of judges who volunteered in the last few days. Their support is vital to the success of the session and we thank them. We also want to thank the moderators of our sessions, a new role this year due to the virtual format of the poster session. The moderators are members of the MAA staff, Pi Mu Epsilon (PME), and the MAA Committee on Undergraduate Students (CUS). It shows just how important this session is to so many groups of people.

We are also helped by the members of the MAA Committee on Undergraduate Students (CUS) in some way or another. They are: Emily Cilli-Turner (University of La Verne), James Collins (University of Mary Washington), Paul Fishback (Grand Valley State University, PME), Janine Janoski (King's College), Emille Davie Lawrence (University of San Francisco), Aihua Li (Montclair State University), Sara Louise Malec (Hood College), Rhonda McKee (University of Central Missouri, KME), Andy Niedermaier (Jane Street Capital), Pam Richardson (Westminster College), Peri Shereen (California State University Monterey Bay), Hortensia Soto (MAA Associate Secretary), and our committee chair, Violeta Vasilevska (Utah Valley University). Violeta was instrumental in arranging extra planning meetings and seeking out volunteers for various roles. There are many details of the poster session that begin with putting out the advertisements, making sure the online submission system works properly, and this year, setting up a new online platform that are attributed to Hortensia Soto, Deveney Smith (MAA), and Melissa Colton (AMS). Preparation of the abstract book is a time-consuming task. Thanks to Beverly Ruedi for doing the final production work on the abstract book.

Our online submission system and technical support is key to managing the ever-growing number of poster entries we receive. Thanks to MAA staff, especially Deveney Smith, for her work managing the system this year. Actually, Deveney probably had her workload increase more than anyone else due to the new format of the poster session. Saying thank you here doesnt feel like enough.

We would also like to thank James Collins, Emille Davie Lawrence, Pam Richardson, Peri Shereen, and Doug Ensley (Shippensburg University) for their help in authoring the judging form, as well as Sara Malec, Peri Shereen, and Violeta Vasilevska for writing documentation to help acclimate the judges to our new virtual format. Finally, we would like to thank Pam Richardson, Sara Malec, and Emily Cilli-Turner for their willingness to help distribute the judges feedback to students after the conference ends.

Thanks to all the students, judges, volunteers, and sponsors. We hope you have a wonderful experience at this years poster session!

Eric Ruggieri
College of the Holy Cross

Chasen Smith
Georgia Southern University

Organized by the MAA

Committee on Undergraduate Students

The MAA gratefully acknowledges the support of

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Their generosity on behalf of the 2021 Undergraduate Student Poster Session enables students to interact with peers and role models in the mathematical sciences during the largest mathematics meeting in the world.

Titles, Authors, Advisors and Abstracts

2. A classification of 4-point regular triangle-free graphs using planar algebra techniques

Jingqiao Tang Denison University
Grey McCarthy Denison University
Advisor(s): Joshua Edge, Denison University

In this talk we classify a certain class of highly symmetric graph using planar algebra techniques. In a recent paper by Edge, he classified all spin models of singly-generated Yang-Baxter planar algebras and demonstrated a relationship between these spin models and highly symmetric graphs. Using this framework, we classify all 4-point regular triangle-free graphs. In particular, all 4-point regular graphs are one of the following graph types: a complete bipartite graph, a collection of bars, a collection of vertices without edges, or the pentagon. In our work we give a more straightforward and accessible proof to this already known result, which was integral to the classification of finite simple group.

3. On Two-sided Matching in Infinite Markets

Yunseo Choi Phillips Exeter Academy, Research Science Institute at MIT
Advisor(s): Scott Kominers, Harvard University

Matching is a branch of economic theory that has seen real-life applications in the assignment of doctors to medical residencies, students to schools, and cadets to branches of military services. Although standard matching models are finite, economic theorists often lean on infinite market models as approximations of large market behaviors. While matching in finite markets has been studied extensively, the study of infinite matching models is relatively new. Here, we lift a number of classic results for one-to-one matching markets, such as group strategy proofness, comparative statics, and respect for unambiguous improvements, to infinite markets via the compactness theorem of propositional logic. In addition, we show that two versions of the lattice structure of finite markets carry over to infinite markets. At the same time, we prove that other results, such as weak Pareto optimality and strong stability property, do not hold in infinite markets. These results give us a clearer sense about which matching results are the most canonical. This project was completed as a part of MIT's Research Science Institute 2020.

4. Parallel Time Integration for Constrained Optimization

Dylan King University of Cambridge
Caroline Hills University of Notre Dame
Matthew Torrence Gettysburg College
Michael Kielstra Harvard University
Advisor(s): Susana Serna, Institute for Pure and Applied Mathematics

While parallel computation is becoming increasingly important, many problems in scientific computing follow a time-evolution model making it difficult to solve such problems in parallel across the temporal domain. The Multi-Grid Reduction In Time (MGRIT) algorithm, developed at Lawrence Livermore National Laboratory (LLNL), solves differential equations with a method designed specifically to utilize extreme numbers of processors by parallelizing across time. The Tri-diagonal MGRIT (TriMGRIT) algorithm, also developed at LLNL, is a generalization of MGRIT which parallel-in-time-solves constrained optimization problems. These seek to choose a control function which minimizes an objective functional under a differential-equation constraint. We consider two such problems: applying torque to a pendulum to bring it to a gentle stop and moving a crowd of people from one distribution into another. Optimization problems are ubiquitous and parallelization (through methods like TriMGRIT) allows them to be solved on a scientific computing scale. This research was conducted as part of the Research in Industrial Projects for Students program at the Institute for Pure and Applied Mathematics.

5. Classification of tight contact structures on a solid torus

Jessica Zhang Proof School
Advisor(s): Zhenkun Li, Stanford University

It is a basic question in contact geometry to classify all non-isotopic tight contact structures on a given 3-manifold. If the manifold has boundary, this classification is based on the dividing set on the boundary. In this paper, we completely

classify the tight contact structures for the solid torus with boundary. In particular, we are able to write down a closed formula for the number of non-isotopic tight contact structures on the solid torus in terms of the dividing set on the boundary. To do this, we use “bypass induction,” a new technique involving certain half-disks called bypasses. Previously, only a few special cases for the solid torus were known by Honda, and only one other manifold with boundary was completely classified, namely the 3-ball. This project was done as part of the PRIMES-USA research program.

6. Analyzing, Predicting, and Mitigating Defect Formation in Metal Additive Manufacturing

Victor Feagins The University of Texas at San Antonio
Julie Fucarino Wellesley College
Justin Haenel The University of Vermont
Advisor(s): Shay Deutch, University of California, Los Angeles

Recent advances in metal additive manufacturing have made this technique a promising direction in industry for printing metal parts, such as for biomedical devices and automobile machinery. Despite its ability to efficiently print highly specific parts, this process is prone to errors during printing, causing pores to form in the metal part. These pores weaken the metal by increasing the likelihood of internal cracks forming. With current techniques implemented by HRL Laboratories, these defects are only able to be detected after printing,which impedes any efforts to correct these errors in real time. In order to provide a real-time quality control framework, we developed a machine learning model that inputs the laser printing parameters and predicts whether a defect will form at a given location, which is informed by the X-ray images of the metal scanned after printing. This model provides an in-situ prediction for the formation of defects in real time so that corrections can immediately be made. In addition to building this predictive model, we provide a semi-automated process for preprocessing and registering the X-ray images with the in-situ laser data. Work done at UCLA, RIPS.

7. A Reinforcement Learning Approach to Packet Routing on a Dynamic Network

Christina Tran Harvard University
Anthony Meza University of California, Irvine
Huanxi Liu University of California, San Diego
Hwai-Liang Tung Brown University
Advisor(s): Thomas Merkh, University of California, Los Angeles

Finding optimal packet routing strategies for real world dynamic networks is a ubiquitous but largely open problem. In this work, we answer two main questions: (1) Can the reinforcement learning techniques Q-learning and Deep Q-learning be applied to the packet routing problem in order to reduce congestion and packet delivery times in comparison to the traditional shortest path algorithm? (2) Does this advantage hold for many different types of networks? To answer these questions, we develop a model for simulating packet routing on a dynamic network, in which a network’s adjacencies and edge weights change discretely. We then use this model to quantify each routing method’s performance across a wide variety of connectivity regimes, where network size, edge distribution, and network dynamics are systematically varied. Overall, Q-learning and Deep Q-learning consistently lower congestion and decrease packet delivery times compared to the shortest paths algorithm on all tested networks. From this, we offer general guidelines for optimizing routing across various network topologies, and aggregate method-specific results for each topological regime. This work was part of UCLA RIPS 2020.

8. Emulating Ray Tracing using Machine Learning

Sneha Sambandam UCLA
Yubing Cui University of Michigan
Yiran Jia University of Washington
David Jin California Institute of Technology
Advisor(s): Qiujing Lu, UCLA

Ray tracing can immerse audiences with incredibly realistic lighting effects, but is highly computationally expensive. This makes ray tracing poorly suited for real-time applications like video games, where speed is critical in rendering each frame. Thus, Advanced Micro Devices partnered with the Institute for Pure and Applied Mathematics’ RIPS program to explore an alternative. We present a model that generates images that appear to have been rendered by ray

tracing. Since no suitable dataset exists, we made a customized one using Blender software where each input image consists of a fixed background and an object with varied properties. Using a U-net-based generator with a loss function composed of Wasserstein generative adversarial network gradient penalty, VGG-22 loss, mean-square-error loss and a proposed shadow loss, our model generates perceptually realistic and accurate ray-tracing feature reconstruction of reflections and shadows while scoring high on popular evaluation metrics. To further improve results, we add a discriminator to the network and adversarial loss to the loss function to get our final proposed model: Ray Tracing Generative Adversarial Network.

9. Using A Point Flow Model to Rank the WHAC Women's Basketball Conference

Marcia Hawkins-Day Siena Heights University
Advisor(s): Nate Iverson, Siena Heights University

Sports teams are ranked in a variety of ways such as polling/voting, point accumulation, and win-loss records. This presentation is about using the flow of points to rank basketball teams. A unique ranking is guaranteed whenever the conditions of the Perron-Frobenius theorem are met. We expect this would be the case for most basketball conferences or divisions. As a case study, we rank the teams within the WHAC Women's basketball conference for the 2018–19 and 2019–20 regular seasons.

10. Measuring and Improving Result Quality Withinthe Privacy Framework of Google's Ads Data Hub

Chenyang Sun Williams College
Renyuan Ma The Ohio State University
Alex Christensen University of Arizona
Lauren Gernes Lewis University
Advisor(s): Susana Serna, University of California, Los Angeles

Ads Data Hub (ADH) is a Google platform that helps advertisers obtain data about their online ads. Such information includes data about demographics, views, ad sequences, and paths to conversion (i.e., a viewer taking an action that is valuable to their business). There is a vast amount of personal information collected in this process. As a result, maintaining the privacy of individuals who view the online ads of ADH customers is an important consideration. Google has implemented a framework in ADH to help protect this privacy. This research project measures how effective the current ADH framework is, as well as investigates how customers can obtain better results within it. This is done in part through the development of a simulation that models arriving user data and customer queries. Additionally, new privacy frameworks are explored, result quality is assessed, and comparisons are made via the development of a metric to evaluate customer success within each framework. This project was done at the Research Industrial Projects for Students (RIPS) program.

11. Predicting tumor response to radiotherapy based on pre-treatment parameter estimates

Tina Huang Lafayette College
Advisor(s): Allison Lewis, Lafayette College

Due to continuous improvement in medical technology, it is now possible for clinicians to collect information about tumor characteristics as the tumor evolves. As a result, a variety of cancer treatments have been developed. However, it remains difficult to predict the efficacy of a given treatment prior to administration. Additionally, the process of collecting information about the tumor may be invasive and expensive. Thus, the creation of a framework for predicting patient response to treatment using limited data collected prior to the treatment is invaluable to clinicians in designing targeted treatment protocols. In this study, we employ ODE models for tumor growth to simulate tumor dynamics and utilize synthetic data from a cellular automaton model for calibration. We investigate which model parameters drive a tumor's response to radiotherapy by clustering model simulations according to the final tumor volume after treatment and comparing the associated parameter distributions. Additionally, we develop a framework for determining the probability of observing complete tumor remission following radiotherapy based only on a patient's pre-treatment parameter values.

12. Graph Scattering Transform

Mason Nakamura Marist College
Sam Smith Clemson University
Rebecca Gjini Lehigh University
Emily Thompson Southwestern University
Advisor(s): Michael Perlmutter, University of California

Deep multilayered networks have successfully applied to a variety of machine learning tasks. In particular, Convolutional Neural Networks (CNNs) attain state-of-the-art performance in many tasks such as image classification. The scattering transform is a mathematical model of these CNNs which allows the use of predefined wavelet filters. We apply a variation of the scattering transform to the task of graph classification. The scattering transform produces a sequence of coefficients at each layer of the network which can be used to classify different classes of graph data. Specifically, we use the scattering transform to classify different models of random graphs. We also re-produce and improve upon the results of Gao, Wolf, & Hirn (2019) on social network data. In addition, we explore the use of principal component analysis, applied to the scattering coefficients, as a dimensionality-reduction and visualization tool. Finally, we use Numba, a high-performance compiler, to improve the running time of our implementation. This was a project done as part of the SURIEM/REU program at Michigan State University.

13. COVID-19 Pandemic Volterra Integral Equation Model

Emma Dehetre Roger Williams University
Advisor(s): Yajni Warnapala, Roger Williams University

The objective of this research project is to find a numerical solution through the Galerkin Method for the Volterra Integral Equation Model. The non-homogenous Volterra Integral Equation of the second kind is used to capture a broader range of disease distributions. Volterra Integral equation models are used in the context of mathematics, public health, and evolutionary biology. The mathematical model of this integral equation will yield convergence results for the COVID-19 data for South Korea, Italy, and Washington State. The modeling of these countries/states will be done using the Galerkin method, a type of numerical approximation using the Gaussian Quadrature nodes. Inspired by the COVID-19 pandemic, the model will include the number of initially infected individuals, the rate of infection, contact rate, death rate, fraction of recovered individuals, and the mean time an individual remains infected.

14. Reconstructing Rooted Trees From Their Strict Order Quasisymmetric Functions

Jeremy Zhou Phillips Academy
Advisor(s): Yongyi Chen, Massachusetts Institute of Technology

Determining whether two graphs are isomorphic is an important and difficult problem in graph theory. One way to make progress towards this problem is by finding and studying graph invariants that distinguish large classes of graphs. Stanley conjectured that his chromatic symmetric function distinguishes all trees, which has remained unresolved. Recently, Hasebe and Tsujie introduced an analogue of Stanley's function for posets, called the strict order quasisymmetric function, and proved that it distinguishes all rooted trees. In this paper, we devise a procedure to explicitly reconstruct a rooted tree from its strict order quasisymmetric function by sampling a finite number of terms. The procedure not only provides a combinatorial proof of the result of Hasebe and Tsujie, but also tracks down the representative terms of each rooted tree that distinguish it from other rooted trees.

15. The Degeneration of the Hilbert Metric on Ideal Pants and its Application to Entropy

Marisa O'Gara University of Michigan- Ann Arbor
Marianne DeBrito University of Michigan- Ann Arbor
Andrew Nguyen University of Michigan- Ann Arbor
Advisor(s): Giuseppe Martone, University of Michigan- Ann Arbor

Entropy is a single value that captures the complexity of a group action on a metric space. We are interested in the entropies of a family of ideal pants groups Γ_T, represented by projective reflection matrices depending on a real parameter $T > 0$. These groups act on convex sets Ω_{Γ_T} which form a metric space with the Hilbert metric. It is known that entropy of Γ_T takes values in the interval $\left(\frac{1}{2}, 1\right]$; however, it has not been proven whether $\frac{1}{2}$ is the sharp lower bound. Using Python programming, we generate approximations of tilings of the convex set in the projective plane and estimate the entropies of these groups with respect to the Hilbert metric. We prove a theorem that, along with

the images and data produced by our code, suggests that the lower bound is indeed sharp. This theorem regards the degeneration of the Hilbert metric on the convex set Ω_{Γ_T}.

16. Convex Partitioning of Polygon Structures

Gabriel Loos Georgia Southern University
Advisor(s): Hua Wang, Georgia Southern University

Partitioning geometric structures into convex shapes is a classic problem with many applications. It is well known that any polygon can be divided into $n-2$ triangles. It has also been shown that orthogonal polygons can be divided into convex quadrilaterals. These results were applied to the art gallery problem, leading to some beautiful consequences in optimization. We consider generalizations of orthogonal polygonal shapes and 'optimal partitions". Some observations of broad interests are established, along with some systematic approach of convex partitioning polygonal structures. In particular, we will discuss related bounds and theories for some hexagonal structures.

17. Using Mathematical Models to Predict the Impact of the Coronavirus in the Chicagoland Area

Joseph Drozek Lewis University
Advisor(s): Brittany Stephenson, Lewis University

The rapid spread of the novel coronavirus SARS-COV-2 (aka COVID-19) has created great socioeconomic distress for individuals across the world. A striking number of cases of the SARS-COV-2 coronavirus in the United States, and more particularly the state of Illinois, has highlighted the need to better understand how to reduce its spread while a vaccine is developed. In our paper, we present a basic compartmental model of COVID-19 transmission using data from the Chicago area. We implement the Ordinary Least Squares Method to help estimate our infection parameters. To further improve our model, we introduce quarantine to determine the impact of proper contact precautions on reducing the spread of the virus. From this, we use a combination of the data available to us and the estimated parameters to predict how well different disease prevention scenarios will lower the transmission of the coronavirus. We will perform a rigorous sensitivity analysis of the various parameters involved within the model to determine which, if any, significantly modify our results. This work was completed during the Lewis University Summer Undergraduate Research Experience (SURE).

18. A Markov Chain Model for Predicting College Baseball

Megan Vesta Lewis University
Advisor(s): Amanda Harsy, Lewis University

Ranking sports teams can be a challenging task and using straight win percentage can be misleading at times. Among the many mathematically inspired sports ranking systems, linear algebra methods are among the most elegant and simple. In this research, we focus on applying a Markov chain method to predict the future results of NCAA Division 1 College Baseball. In particular, we investigate whether win streaks can help predict the final standings for college baseball.

19. A Mathematical Model of Interspecies Behavioral Patterns

Catherine LoGrande Elon University
Advisor(s): Crista Arangala, Elon University

Honeybees and zebrafish are two species which generally move and make decisions as a group. The two species have no way of interacting with one another under ordinary circumstances, however, they are capable of inter-species collective decision making with the use of robots to influence their movement. This project demonstrates a mathematical model of the two species coming to a consensus on their decision of movement when presented with a binary decision.

20. Infectious Disease Model- COVID-19- Volterra Integral Equation

Kate Gilbert Roger Williams University
Advisor(s): Yajni Warnapala, Roger Williams University

This research, inspired by the COVID-19 pandemic, investigates the feasibility of obtaining good convergence results for a model of the Volterra integral equation over the surface of a sphere. The Galerkin Method is used to numerically

solve the exterior boundary value problem. This model accounts for the number of initially infected individuals, suscept individuals, removed individuals, number of contacts per person, the recovery rate, and the total population. This model specifically looks at COVID-19 in South Africa and Brazil for the first 200 days of the pandemic. The model takes into account the geography of the countries and uses Green's Theorem. The numerical results of this research are expected to find good convergence for this model as well as limitations of the model such as the assumption for the number of contacts, and the lack of inflection points at this time.

21. Covering All Our Bases: Minimum Dominating Sets of Grids

Isaac Reiter Kutztown University of Pennsylvania
Advisor(s): Ju Zhou, Kutztown University of Pennsylvania

Dominating sets are a very interesting topic in graph theory. In Fundamentals of Domination in Graphs, the authors provide a comprehensive introduction and analysis of the subject. Authors such as Stephen Hedetniemi and Jason Hedetniemi have heavily contributed to this field. Given a graph, a dominating set D is a subset of the vertices such that every vertex is either in D or adjacent to a vertex in D. Of particular interest are minimum dominating sets. A minimum dominating set is a dominating set of the smallest possible cardinality. In other words, minimum dominating sets use the optimal number of vertices to dominate a graph. For a particular graph, the minimum number of vertices required to form a dominating set is referred to as the domination number. In his poster, the presenter will discuss his research on determining the domination numbers of particular grids. Through intentionally chosen cases and carefully worded mathematical induction, he proved formulas that provide the domination numbers for all $3\times N$ and $4\times N$ grids. Occasionally, he made use of operations research to confirm his results. The presenter will provide these formulas along with the associated proofs.

22. Quantum Jacobi Forms and Sum Of Tails Identities

Andrew Tawfeek Amherst College
Noah Solomon Amherst College
Elizabeth Pratt Amherst College
Advisor(s): Amanda Folsom, Amherst College

Modular forms are functions on the upper half complex plane that transform in a beautifully symmetric way. Around a decade ago, Don Zagier discovered a wide swath of functions in physics, topology, and elsewhere that transform like modular forms, up to a "suitable" analytic error. He called these quantum modular forms; in our work this summer, we discovered new families of their two-variable analog, called quantum Jacobi forms. Our main tools in this search were sums of tails identities. We found several new identities, which relate some q-hypergeometric series of interest to known quantum Jacobi forms. Via asymptotic arguments, we were able to show that these q-hypergeometric are indeed quantum Jacobi forms, and write down their transformations explicitly. These families add to the very small but growing pool of examples of quantum Jacobi forms.

23. Modeling the Effect of Vaccination, Testing, and Social Distancing on the COVID-19 Pandemic and Its Implications for In-Person Education in Santa Clara County Using a Compartmental Model with Variable Infection Rate

Caden Lin The Harker School
Advisor(s): Ying-Jen Yang, University of Washington

The emergence of SARS-CoV-2 has led to the global COVID-19 pandemic. To reduce spread, schools worldwide have closed, putting 1.6 billion children out of classrooms. This study examines the effect that testing, quarantine, and a vaccine have on containing COVID-19, identifies which measures are most significant, and investigates the implications for reopening schools. We analyze possible outbreaks in Santa Clara County, California, and translate the findings to general outbreaks. A compartmental model specific to SARS-CoV-2 was derived, which expands on the SIR model with categories for individuals who are asymptomatic, tested-positive, quarantined, vaccinated, or dead. The results show that a vaccine is the most impactful, followed by quarantine, and finally, testing. Also, the availability of a vaccine is more important than its efficacy. To limit spread, quarantine should be prioritized until a vaccine is created, after which maximum distribution of the vaccine should occur. However, because it is also important to restore education for those without access to virtual learning, we argue that schools can safely reopen if other measures are properly enforced.

24. Irreducible Characters for Verma Modules for the Orthosymplectic Lie Superalgebra $\mathfrak{osp}(3|4)$

Honglin Zhu Phillips Exeter Academy, MIT PRIMES
Advisor(s): Arun Kannan, MIT

We introduce the fundamentals of the representation theory of Lie algebras. Then, we present the problem of finding Jordan-Hölder multiplicities of Verma modules of basic Lie superalgebras. We outline the strategy used to solve this problem in the case of the orthosymplectic Lie superalgebra $\mathfrak{osp}(3|4)$.

25. Bilateral Trade Flow Prediction Models Enhanced By Wavelet and Machine Learning Algorithms

Evdokia Kottou Western Connecticut State University
Tyler Grubelich Western Connecticut State University
Advisor(s): Xiaodi Wang, Western Connecticut State University

International economy promotes a sense of global interdependence and offers mutual benefits to countries around the world through the system of imports and exports. In this study, we have utilized economic indicators as inputs in an algorithm scheme that is based on Machine Learning methods combined with Wavelet Transforms to predict the bilateral trade flow between pairs of trading countries. Utilizing this methodology will allow countries to strike more successful trade deals, and increase a nation's estimated Gross Domestic Product (GDP) per capita. This will ultimately help reduce poverty as governments reallocate funds to serve struggling populations and target the 2030 United Nations Agenda (UNA).

26. Modeling Mongolian Tent Graphs in Self Assembling DNA Using Graph Theory and Linear Algebra

Miles Mena Lewis University
Advisor(s): Amanda Harsy, Lewis University

Self-assembly is a term used to describe the process of a collection of components combining to form an organized structure without external direction. The unique properties of double-stranded DNA molecules make DNA a valuable structural material with which to form nanostructures, and the field of DNA nanotechnology is largely based on this premise. By modeling complexes with discrete graphs, efficient self-assembly of nanostructures becomes a mathematical puzzle which can be solved using methods from graph theory and linear algebra. This research shares the results of applying these techniques to Mongolian Tent Graphs.

28. Colored HOMFLY Polynomials of Genus-2 Pretzel Knots

William Qin MIT PRIMES
Advisor(s): Yakov Kononov, Columbia Univeristy

HOMFLY polynomials are one of the major knot invariants being actively studied. They are difficult to compute in the general case but can be far more easily expressed in certain specific cases. We examine in particular a type of knot called a pretzel knot, and in particular use a method reminiscent of normal polynomials, an analogue of finite differences, to help with this. We find a way to significantly decrease computation time of HOMFLY polynomials of genus-2 pretzel knots with certain large parameters, and conjecture some properties that could lead to a more general algorithm to quickly compute HOMFLY polynomials.

29. Routing on Convex Pieces of Grids

RJ Barnes Harvey Mudd College
Na'ama Nevo Colorado College
Hanna Yang MIT
Advisor(s): Hannah Alpert, University of British Columbia

Consider some graph G with one token placed on each vertex. The routing number of a graph G, denoted $rt(G)$, is defined as the minimum number of steps, consisting of swaps along disjoint edges, needed to get from any configuration of tokens to another. The routing number is known for some graph types including paths and trees. We now

consider subgraphs of the infinite square lattice, enclosed by some polygon. It is known that a subgraph contained within a rectangle R, aligned with the axes with width $w(R)$ and height $h(R)$, satisfies $rt(R) \in O(w(R) + h(R))$. Our main theorem is an extension of this estimate: for the subgraph contained in any convex polygon P, we have $rt(P) \in O(w(P) + h(P))$. This result has applications to fields such as parallel computing and motion planning. This research was conducted at the MathILy-EST REU. The program was funded by NSF grant 1851842.

30. Relay Protocol for Approximate Byzantine Consensus

Matthew Ding Westford Academy/MIT PRIMES
Advisor(s): Hanshen Xiao, Massachusetts Institute of Technology

This paper presents a novel algorithm for approximate Byzantine consensus, called Relay-ABC. The algorithm allows machines to achieve approximate consensus in the presence of b byzantine failures. The algorithm relies on our novel usage of a relayed messaging system and unforgeable signatures. This allows the strict necessary network conditions of traditional approximate byzantine consensus algorithms to be circumvented. Specifically, previous algorithms required all honest users to have at least $b + 1$ honest neighbors. Our algorithm achieves consensus on networks where honest users have as little as a single honest neighbor. Additionally, we provide theoretical guarantees of validity and convergence for Relay-ABC by utilizing transition matrices. Previous methods use a state vector to model the states of all machines at any given iteration. We extend these methods by expanding each state vector to model a set of D iterations, where D is a diameter of the network. This allows us to accurately model the delays of messages inherent within the relay system. We prove that the product of the sequence of stochastic matrices converges to a matrix with identical rows.

31. Bijections Between n-ary and Binary Labeled Totally Symmetric Medial Quasigroups and Abelian Groups

Benjamin Young Case Western Reserve University
Advisor(s): Harold Connamacher, Case Western Reserve University

We demonstrate a bijection between the set of all labeled (i.e. not up to isomorphism) n-ary medial (also known as entropic or abelian) totally symmetric quasigroups and the set of labeled n-ary abelian groups over a finite set with k elements. This is achieved by proving bijections between the following pairs of sets of labeled algebraic structures over k elements: n-ary and binary totally symmetric medial quasigroups, binary totally symmetric medial quasigroups and binary abelian groups, and finally binary abelian groups and n-ary abelian groups.

32. Go Big or Go Home

Yawen Zhang Elizabethtown College
Advisor(s): Timothy McDevitt, Elizabethtown College

Many people say that "you should play to win instead of playing not to lose." However, we show that is not the best strategy in Penney's game, which is a probability game involving sequences of coin flips where the second player always has an advantage over the first player. However, if both players choose their sequences simultaneously, then game theory paradoxically shows that a conservative player who "plays to win or tie" wins more often than an aggressive player who "plays to win". Interestingly, that result breaks down for two coin flips with a biased coin with probability $1/\varphi$ of being heads, where φ is the golden ratio. In that case, both players win equally often.

33. Upho Posets

Joshua Guo MIT PRIMES/Newton South High School
Karthik Seetharaman MIT PRIMES/Massachusetts Academy of Math and Science at WPI
Ilaria Seidel MIT PRIMES/Commonwealth School
Advisor(s): Yibo Gao, Massachusetts Institute of Technology

Upper homogeneous finite type (upho) posets are a large class of partially ordered sets with the property that the upper principal order filter at every vertex is isomorphic to the whole poset. Well-known examples include k-array trees, the grid graphs, and the Stern poset. Very little is known about upho posets in general. We construct upho posets with Schur-positive Ehrenborg quasisymmetric functions, whose rank-generating functions have rational poles and zeros. We also categorize the rank-generating functions of all planar upho posets. Finally, we prove the existence of an upho poset with uncomputable rank-generating function.

34. A Quantitative Analysis on Bitcoin Perpetual Inverse Futures Contract

Yue Wu Carnegie Mellon University
Advisor(s): Ariel Zetlin-Jones, Carnegie Mellon University

The perpetual inverse futures contract is a recent and popular cryptocurrency derivative on crypto exchanges. Though the contract is perpetual, exchanges implement a liquidation mechanism that terminates positions that no longer fulfill maintenance requirements. Understanding liquidation will help investors optimize the use of leverage for maximizing profitability and minimizing risks. I will present a quantitative description of the wealth process for holding such contracts on BitMEX exchange, statistical properties of funding rate & index price, and formulas that relate liquidation to leverage as a stopping time problem. I will also discuss modeling methods such as regression, stochastic calculus, and simulation. For example, in the theoretical scenario, I used the martingale method to derive PDEs and solved for theoretical liquidation probability; in the simulation scenario, I extracted parameters from historical data to formulate geometric Brownian motion and mean-reverting process models for index price & funding rates, and ran simulations to build regression models for expected liquidation. I will explain how the findings match and give a financial explanation of the models.

35. Asymptotics for Iterating the Lusztig-Vogan Bijection for GL_n on Dominant Weights

Yuxiao Wang PRIMES-USA, Princeton International School of Mathematics and Science
Advisor(s): Calder Morton-Ferguson, Massachusetts Institute of Technology

This talk will introduce Lusztig-Vogan Bijection, which was conjectured by Lusztig and Vogan and proved by Bezrukavnikov. It is a bijection between the set of dominant weights for any reductive group G over an algebraically closed field and the set of irreducible G-equivalent bundles on nilpotent orbits, critical for the representation theory. We iterate the algorithm of computing the Lusztig-Vogan Bijection Type A (GL_n) on dominant weights, which is proposed by Achur and simplified by Rush. We will illustrate the asymptotic behavior between the number of iterations for an input and the length of the input, and a recursive formula to compute the slope of the asymptote. This serves as another contribution to understanding the Lusztig-Vogan Bijection from a combinatorial perspective and a first step in understanding the iterative behavior of the Type A Lusztig-Vogan Bijection.

37. The Center of the q-Weyl Algebra Over Rings with Torsion

Quanlin Chen MIT PRIMES-USA, Princeton International School of Mathematics and Science
Advisor(s): Calder Morton-Ferguson, Massachusetts Institute of Technology

We compute the centers of the Weyl algebra, q-Weyl algebra, and the "first q-Weyl algebra" over the quotient of the ring $\mathbb{Z}/p^N\mathbb{Z}[q]$ by some polynomial $P(q)$. Through this, we generalize and "quantize" part of a result by Stewart and Vologodsky on the center of the ring of differential operators on a smooth variety over $\mathbb{Z}/p^n\mathbb{Z}$. We prove that a corresponding Witt vector structure appears for general $P(q)$ and compute the extra terms for special $P(q)$ with particular properties, answering a question by Bezrukavnikov of possible interpolation between two known results.

38. Dynamical Design of Hybrid Methods for Sparse Systems

Yihua Xu Georgia Institute of Technology
Advisor(s): Rachel Kuske, Georgia Institute of Technology

Although small underdetermined problems in compressive sensing are substantially well understood, large, inconsistent, nearly sparse systems have not been investigated with as much detail. In this dynamical study, two commonly used sparse recovery optimization algorithms, Linearized Bregman and Iterative Shrinkage Thresholding Algorithm are compared. The dependence of their dynamical behaviors on the threshold hyper parameter and different entry sizes in the solution suggests complementary advantages and disadvantages. We found that utilizing LB together with large thresholding could help predict large entries quickly, while ISTA with a much lower thresholding is better designed for predicting small entries without chatter. These results prompted the creation of a hybrid method which benefits from favorable characteristics from both optimization algorithms such as less chatter and quick convergence. The Hybrid method is proposed, analyzed, and evaluated as outperforming and superior to both LB and ISTA, principally due to the Hybrid's versatility when processing diverse entry sizes with different parameters. This project is done as part of REU at Georgia Tech.

39. Higher Moments for Lattice Point Discrepancy of Convex Domains and Annuli

Xiaorun Wu Princeton University
Advisor(s): Nicholas Marshall, Princeton University

Given bounded convex domain $\Omega \subseteq \mathbb{R}^2$, let $f(\vec{x}, R)$ be the number of lattice points in the scaled shifted domain $R\Omega - \vec{x}$ for $R > 0$ and $\vec{x} \in \mathbb{T}$ minus the area of $R\Omega$:

$$f(\vec{x}, R) = \#\{(j,k) \in \mathbb{Z}^2 : (j - x_1, k - x_2) \in R\Omega\} - R^2|\Omega|.$$

Define the p-th moment of discrepancy function as $\int_{\mathbb{T}^2} |f(\vec{x}, R)|^p d\vec{x}$. Huxley's 2014 paper showed that the fourth moment is bounded by $\mathcal{O}(R^2 \log R)$. In this paper, my contribution is two-fold: First I present a simple, direct proof of Huxley's 2014 results; Second, I establish estimates for the moments of lattice point discrepancy of annuli of any fixed thickness $0 < t < 1$ for $p \geq 2$.

40. Graph Universal Cycles of Permutations and Set Partitions

Dorothea Rugg Haverford College
Advisor(s): Anant Godbole, East Tennessee State University

Universal cycles are cyclic strings of elements which encode various combinatorial objects, such as permutations, in different ”windows” along the code. They were first developed by Chung, Diaconis, and Graham in 1992. Previous work has attempted to create universal cycles for permutations and set partitions with varying degrees of success, with some objects unable to be placed in a universal cycle altogether. Graph universal cycles are extensions of universal cycles developed by Brockman, Kay, and Snively in 2010. They use graphs rather than letters or numbers to encode different combinatorial objects within the cycle. Our work shows the existence of graph universal cycles for permutations and set partitions as well as for specific subsets of these objects. This research was conducted during the ETSU-UPR REU 2020 and supported by NSF-DMS REU-1852171.

41. Surfaces with Braided Boundaries in Blow Ups of $D^2 \times D^2$

Madelynn Roche Vanderbilt University
Andrew Nguyen University of Michigan
Sabrina Traver King's College
Advisor(s): Tom Mark, University of Virginia

On this poster, we will describe our research project that we began working on during an REU at the University of Virginia. Our problem is aimed at studying certain braided surfaces in a blowup of the 4-ball by adapting graphical methods developed by Seiichi Kamada in his paper, “Graphic descriptions of monodromy representations” and in his book *Braid and Knot Theory in Dimension Four* We devised a new method with which we could analyze the problem using cross sections of the four-dimensional surface and their monodromy representations. We conjecture that using this method we can show non-uniqueness of spanning surfaces for a particular braid, and further that these there may be infinitely many of such surfaces. This research was supported through the University of Virginia under NSF grant DMS-1839968.

42. Rational Powers of Monomial Ideals

Ethan Partida University of Minnesota Twin Cities
Josiah Lim Brown University
Ethan Roy University of Texas Austin
Advisor(s): Alexandra Seceleanu, University of Nebraska-Lincoln

This poster will focus on the exponentiation of monomial ideals. The typical exponentiation operation only allows for natural powers, we will expand this to allow real powers. A real power of a monomial ideal generalizes the integral closure of a monomial ideal, and highlights many interesting connections, such as to the theory of convex polytopes. Our research hopes to lay a groundwork for future work on the subject. In particular, we provide multiple algorithms for computing rational powers of monomial ideals and lay down a framework for the analytic study of rational powers of monomial ideals. An important result from this is, given any monomial ideal I, the function taking real numbers to their corresponding real power of I is a step function whose jumping points are rational. This work was done in collaboration with 2020 Polymath REU.

43. Velocity Inversion using the Quadratic Wasserstein Metric

Srinath Mahankali Stuyvesant High School
Advisor(s): Yunan Yang, New York University

Full-waveform inversion (FWI) is a method used to determine properties of the Earth from information on its surface. We use the squared Wasserstein distance (squared W_2 distance) as an objective function to invert for the velocity as a function of position in the Earth, and we discuss its convexity with respect to the velocity parameter. In one dimension, we consider multiple velocity models as a function of position, and we show the convexity of the squared W_2 distance with respect to the velocity parameter on the interval from zero to the true value of the velocity parameter when the source function is a probability measure. Furthermore, we consider a two-dimensional model where velocity is linearly increasing as a function of depth and prove the convexity of the squared W_2 distance in the velocity parameter on large regions containing the true value. We discuss the convexity of the squared W_2 distance compared to the convexity of the squared L^2 norm, and we discuss the relationship between frequency and their convexity. We also discuss multiple approaches to optimal transport for non-probability measures by first converting the wave data into probability measures.

44. Influence of Protein Eos and Environmental Signals on T Cell Differentiation

Elizabeth Duncan Virginia Tech
Advisor(s): Lauren Childs, Virginia Tech

Immune responses are the bodily mechanisms used to counter foreign invaders that pose a potential threat to our health. T cells and their sub-lineages, such as T helper cells or T central memory cells, play a critical role in adaptive immune responses by actively destroying infected cells in addition to signaling other cell types in the immune response. We use differential expression analysis to analyze the effects of the polarizing concentrations of the protein Eos on the regulation of a T cell sub-lineage, T helper 1 cells. Additionally, we develop a system of nonlinear ordinary differential equations that describes the relationships between Eos and other commonly associated genes and proteins. Our results show that high concentrations of Eos positively regulate T helper cell genes such as Prdm1 and Ifng. Conversely, selective deletion of Eos results in decreased expression of T helper 1 cells' associated factors. With this information, we position ourselves to better understand the complexities of immune responses. This insight may allow us to predict and intervene in such behaviors to control our bodies' responses to pathogens that cause diseases and conditions.

45. The Stembridge Equality for Skew Dual Stable Grothendieck Polynomials

Fiona Abney-McPeek University of Chicago Laboratory Schools
Serena An Brookings High School
Jakin Ng Ithaca High School
Advisor(s): Adela YiYu Zhang, Massachusetts Institute of Technology

The Schur polynomials s_λ are essential in understanding the representation theory of the general linear group. They also describe the cohomology ring of the Grassmannians. For $\rho = (n, n-1, \ldots, 1)$ a staircase shape and $\mu \subseteq \rho$ a sub-tableau, the Stembridge equality states that $s_{\rho/\mu} = s_{\rho/\mu^T}$. This equality provides information about the symmetry of the cohomology ring. The dual stable Grothendieck polynomials g_λ, developed by Buch, Lam, and Pylyavskyy, are variants of the Schur polynomials and describe the K-theory of the Grassmannians. The goal of our project is to prove the Stembridge equality for the skew dual stable Grothendieck polynomials. Using the Hopf algebra structure of the ring of symmetric functions and a generalized Littlewood-Richardson rule, we show that that $g_{\rho/(k)} = g_{\rho/(k)^T}$, a special case of the Stembridge equality for the skew dual stable Grothendieck polynomials.

46. Spectral Properties of a Non-Self-Adjoint Beam with Applications to Flutter

Kaitlynn Lilly University of Maryland, Baltimore County
Advisor(s): Jason Howell, Carnegie Mellon University

In this talk we focus on a simplified (1-D) partial differential equation beam model for aeroelastic flutter, a structural instability brought about by the presence of a flow surrounding the beam. We discuss the onset of instability in the problem, related to non-self-adjoint PDE terms. Various methods of simulating the dynamics are presented, and we empirically observe limit cycle oscillations for the solutions. Supported by the abstract theorem that the dynamical system of PDE solutions here has a global attractor, we aim to rigorously construct said periodic solutions, thereby ascertaining some structure of the attractor. By exploiting the 1-D structure of the problem and the explicit root formula for the quartic equation, we use separation of variables to construct eigenvalues and corresponding modes to the non-self-adjoint spatial problem (rather than using in vacuo modes to approximate the perturbed solution). Finally, our explicit eigenpairs yields a cubic-type temporal ODE which can be then be analyzed and solved.

47. Rodeo Algorithm for Quantum Computing

Kenneth Choi Ridgefield High School
Advisor(s): Dean Lee, Michigan State University

We present a recursive quantum computing algorithm that can construct any eigenvector of a quantum Hamiltonian residing within a selected energy interval $[E-\epsilon, E+\epsilon]$. To suppress the spectral weight of all other eigenvectors by a factor δ, the computational effort scales as $|\log\delta|/(\epsilon P)$, where P is the squared overlap of the initial state with the desired eigenvector. The method, named the rodeo algorithm, uses an auxiliary qubit to control the time evolution of the object Hamiltonian of interest, plus some tunable energy constant. This auxiliary qubit is also coupled to several other qubits that we collectively call the arena. By tuning the energy constant and adjusting the clock speeds of the object Hamiltonian and the arena, we can engineer the quantum state coupled to auxiliary qubit state $|1\rangle$ to have an enhanced spectral weight at energy E. By resetting or replacing the arena qubits with each cycle, we recursively converge to the desired eigenvector with exponential accuracy in the number of cycles. As an example, we consider a model Hamiltonian describing Anderson localization of electrons due to disorder in one dimension.

48. A first-pass statistical dashboard for categorizing diverse particle movement patterns

Riley Juenemann Tulane University
Advisor(s): Scott McKinley, Tulane University

At the intersection of nanoscience and biology lies the question of precisely how particles move within cells. Recent developments in imaging, including quantum dots and semiconductor nanoparticles, have made it possible for researchers to peek beneath the surface of living cells, tracking individual particles over significant timescales. In contrast to in vitro particle tracking experiments, wherein there are great controls on particle and environmental homogeneity, live-cell tracking features tremendous diversity in particle movement. In this research area, the use of mathematics has allowed for a better description of movement categorizations and quantitative methods to differentiate between them. We have developed a first-pass statistical dashboard to categorize disparate types of particle trajectories. Based on six statistical measures, K-means clustering was utilized to distinguish between free diffusion, anchored diffusion, directed transport, tracker error, subdiffusion, and skating diffusion. This automated categorization process proved to be successful on data simulated using stochastic differential equations and provided interesting results on the live-cell data.

49. Products of Differentiation, Multiplication, and Composition on Discrete Weighted Banach Spaces

Colin Jackson University of Wisconsin-La Crosse
Advisor(s): Robert Allen, University of Wisconsin-La Crosse

In recent years, discrete analogues to classical function spaces such as the Bloch space ($\mathcal{B}$), Hardy spaces (H^p), and weighted Banach spaces (H_ν^∞) have been developed. The study of operators on these spaces have included the multiplication, composition, and weighted composition operators. The discrete derivative has played a crucial role in the characterization of the fundamental concepts of boundedness and compactness of certain operators. In this work, we first study the discrete derivative as an operator acting on the discrete weighted Banach spaces. We characterize compactness, compute its operator norm, and show that it cannot be an isometry or a compact operator. Next, we study its interactions with multiplication and composition operators. We include some known results for completeness, then compute operator norms of products of these operators and characterize their properties, such as boundedness. The end goal is to understand the discrete derivative enough to develop discrete analogues of spaces such as the Hardy-Sobolev spaces (S^p).

50. Best-fit epidemiological models for COVID-19: New cases in Korea

Zixin Yin Dickinson College
Advisor(s): David Shanafelt, INRAE

The outbreak of COVID-19 has turned the year 2020 upside-down. While scientists have made progress in our understanding of the structure, transmission, and treatment of the disease, there remains a question as to the long-term

immunity of recovered patients. Our research attempts to answer this question by fitting model parameters of three epidemiological models (SIR, SEIR, and SIRS) to COVID-19 data from the Republic of Korea. We find that the SIRS model is the best-fit model, which suggests that—at least for the Republic of Korea during our study period—people obtain limited immunity post-infection.

51. Enumeration Degrees and Topology

Yanli Ren University of Wisconsin, Madison
Advisor(s): Mariya Soskova, University of Wisconsin, Madison

The project aims to study the interaction between topology and computability theory. The work focuses on incorporating Kihara, Pauly and Ng's paper *Enumeration Degrees and Non-metrizable Topology* into an online database of classes of enumeration degrees. (The database is available at http://zoo.ludovicpatey.com/.) In this project, four classes of enumeration degrees: *semicomputable* degrees, *co-dcea* degrees, *telograph-cototal* degrees and *cylinder-cototal* degrees, and the relations between each other are discussed. Alternative proofs of selected Kihara, Pauly and Ng's theorems are presented.

52. An Examination of Fontan Circulation Using Differential Equation Models and Numerical Methods

Vanessa Maybruck Kutztown University of Pennsylvania
Advisor(s): Brooks Emerick, Kutztown University of Pennsylvania

Certain congenital heart defects lead to the development of only a single pumping chamber, or ventricle, in the heart instead of the usual two ventricles. Individuals with this defect undergo a corrective, three-part surgery, the third step being the Fontan procedure, but as the patients age, their cardiovascular health often deteriorates. Using computational fluid dynamics and differential equations, we model Fontan circulation to investigate why the procedure fails and how Fontan failure can be maximally prevented. Borrowing from established literature on RC circuits, the differential equation models simulate systemic blood flow in a piecewise, switch-like fashion. Here, we develop numerical solvers for both ordinary and partial differential equations to model Fontan circulation, focusing on system stability and parameter analysis. We also develop a numerical model that simulates Fontan circulation for patients with cardiovascular conditions that modify vessel area. Overall, our goal is to accurately model blood flow and determine parameters of interest to decrease the diagnosis time of Fontan failure and/or improve the surgical technique to prevent the failure from occurring.

53. Combinatorics in Tarot

Stella Cunningham St. Edwards University
Advisor(s): Jason Callahan, St. Edwards University

Tarot is a deck of cards used for games and fortune telling. To read one's fortune with Tarot cards, a reader presents a spread of n cards. We use combinatorial methods to calculate how many spreads of n Tarot cards exist. In the Tarot deck there are nine types of constellations comprising disjoint subsets of the deck. We write a Python program to count how many n-card spreads contain at least three cards from any one of these constellations and then find the probability that an n-card spread contains such a subset of cards.

54. A Homomorphic Lattice-Based Algorithm for Differential Privacy

Drew Worden Western Connecticut State University
Advisor(s): Xiaodi Wang, Western Connecticut State University

In today's world the tension between the mass collection of people's data by corporations and the people's right to personal data privacy grows. To address these problems researchers have been developing different mechanisms that achieve differential privacy by minimizing the effect of each individual in the dataset. They do this by adding randomly distributed noise to the dataset which obscures the results enough to ensure privacy but not significantly enough to alter the statistical outcomes. An example is the Laplace mechanism which adds noise from the Laplace distribution. In the field of cryptography, homomorphic encryption is used to encrypt data while still being able to perform calculations on the encrypted data and then decrypt the result. In this research, we combine these two tools, one providing privacy, the other, security, then we can construct an algorithm that would protect a user's data from attacks directed towards differential privacy whilst being resistant to quantum-based brute force while retaining learnability through statistical analysis in machine learning environments.

55. An Empirical Study on Minimal Generators of Topological Generators

Lu Li Macalester College
Advisor(s): Lori Ziegelmeier, Macalester College

This work provides an empirical study of the computational cost and effectiveness of several common optimization procedures applied to homological generators in dimension one, to find uniform-weighted minimal generator and length-weighted minimal generator. We conduct these optimizations via standard linear programming methods, applying general-purpose solvers to optimize over column bases of simplicial boundary matrices. Our key findings are (i) the computational cost of optimizing a basis of generators exceeds the cost of computing such a basis, for each of the tested general-purpose solvers, (ii) optimization is generally effective in reducing the size of generators, though the extent of the reduction varies according to the distribution of the underlying data, and (iii) the space of L1 optimal solutions properly contains that of L0 optimal solutions in the majority of Vietoris-Rips complex studied, though none of these complexes had totally-unimodular boundary matrices in degree one, a sufficient condition found in other studies. (iv) the choice of linear solvers could greatly influence the computation time of optimizing cycles.

56. KAMILA Clustering for a Mixed-Type Data Analysis of Illinois Medicare Data

Heather Baldacci Butler University
Advisor(s): Rasitha Jayasekare, Butler University

The Centers for Medicare and Medicare Services (CMS) releases annual reports regarding nationwide Medicare coverage. CMS data provide an opportunity for an in-depth analysis of Medicare usage patterns within the United States that may provide insight into socioeconomic conditions in certain regions. To discover any potential patterns, the KAMILA (KAy-means for MIxed LArge data sets) clustering algorithm has been utilized within the most recent CMS data set from 2018. Due to the large size of the original data set, the focus of this research has been limited to Illinois Medicare data, grouped by the 102 counties in Illinois. The KAMILA algorithm extends the well-known k-means clustering algorithm to include mixed-type data by using a weighted semiparametric procedure. Therefore, it balances the contribution of continuous and categorical variables. The optimal number of clusters is decided in-part by the operator of the algorithm with respect to the number of cross-validation runs. Thus, through the application of the KAMILA algorithm, this research ultimately uncovers any underlying patterns of usage of different types of Medicare services in the state of Illinois.

57. Predicting the Identifiability of a Wound-Healing Model using Parameter Space Geometry and a Subset Selection Technique

Gloria Huang Western Kentucky University
Advisor(s): Richard Schugart, Western Kentucky University

The treatment of chronic wounds has long been a challenge to wound care professionals and presents a substantial economic burden to healthcare systems globally. To combat this issue, a mathematical model describing the interactions between matrix metalloproteinases (MMPs), their regulators (TIMPs), fibroblasts, and the extracellular matrix (ECM) was analyzed to find the most influential factors in the healing of diabetic foot ulcers. Using a differential equation model with de-identified patient data, the 3D geometry of parameter space was visualized for all combinations of the twelve model parameters to see how they affect the biological system. Knowledge of parameter identifiability can streamline treatment by allowing us to individualize patient care. This approach plots two parameters against the sum of squares to generate a 3D graph. By analyzing the minimum of the graph, we can conclude if a parameter can be uniquely determined. The identifiability of a parameter signifies its importance in healing. This research shows the regulators of MMPs (TIMPs) are the most influential parameters. A local sensitivity analysis was used in conjunction with eigenvalues to confirm results.

58. Ellipticity of Numerical Ranges of SomeTridiagonal Matrices, Their Generating Curves,and Polynomials

Muyan Jiang New York University Abu Dhabi
Advisor(s): Ilya Spitkovsky, New York University Abu Dhabi

Let A be an $n \times n$ complex matrix. One important characteristic of the matrix is its numerical range defined as $W(A) := \{x^*Ax : x \in \mathbb{C}^n, x^*x = 1\}$. A complete description of the numerical range of matrices with lower

dimensions such as 2×2 and 3×3 is well known, while the study of the numerical range of matrices with higher orders is more challenging and complicated. A matrix $A = (a_{ij})_{i,j=1}^{n}$ is *tridiagonal* if $a_{ij} = 0$ for $|i - j| > 1$. A tridiagonal matrix A is *reciprocal* if it has constant main diagonal $a_{jj} = a$, for $j = 1, \ldots, n$ and off-diagonal pairs satisfying $a_{j,j+1} \cdot a_{j+1,j} = 1$ for $j = 1, \ldots, n-1$. This paper investigates the numerical range of tridiagonal reciprocal matrices of higher dimensions and its generating curves. Specifically, we provide a detailed description of homogeneous structures of the numerical range generating polynomial for those matrices. Based on this, we set up a necessary and sufficient criteria for generating curves of a higher dimensional reciprocal matrix to have elliptical components and particularly an outer ellipse, which guarantees an elliptical numerical range of it.

59. Higher dimensional gap problems

Juan Ramirez University of Houston
Advisor(s): Alan Haynes, University of Houston

Generalizations of the three-gap theorem have been recently studied by Alan Haynes and Jens Marklof. In this project, we present an analogous version with respect to the maximum metric. We involve higher dimensional toral rotations and then provide upper bounds in all dimensions. Then we conclude by showing that in dimensions two and three, our bounds are strict.

60. Throttling for standard zero forcing on directed graphs

Emily Cairncross Oberlin College
Benjamin Kitchen Williams College
Emily Lopez University of California - Santa Barbara
Advisor(s): Josh Carlson, Williams College

Zero forcing is a coloring process performed on a directed graph with vertices initially colored either blue or white. The *color change rule* is applied repeatedly: a blue vertex u forces a white vertex w to become blue if w is the only white out-neighbor of u. *Throttling* is a process that gives the optimal balance of resources and time when performing zero forcing. In particular, the *throttling number* of a directed graph minimizes the sum of the number of vertices initially blue and the number of time steps needed to color every vertex blue. In this poster, we will present recent results regarding throttling on directed graphs. We first give a characterization theorem for directed graphs that is analogous to one that exists for undirected graphs, proving that given a positive integer t, there exist finitely many distinct simple digraphs with throttling number at most t. We also define the *orientation throttling interval* (OTI) of a simple graph G as the range of throttling numbers achieved by all possible orientations of G and present some of its important properties. This work was done at the Williams College SMALL REU.

61. Product Expansions of q-Character Polynomials

Andrew Huang Conestoga High School/MIT PRIMES-USA
Adithya Balachandran High Technology High School/MIT PRIMES-USA
Simon Sun Bergen County Academies/MIT PRIMES-USA
Advisor(s): Nir Gadish, MIT

We consider certain class functions defined simultaneously on the groups $\mathrm{Gl}_n(\mathbb{F}_q)$ for all n, which we also interpret as statistics on matrices. It has been previously shown that these simultaneous class functions are closed under multiplication, and we work towards computing the structure constants of this ring of functions. We derive general criteria for determining which statistics have nonzero expansion coefficients in the product of two fixed statistics. To this end, we introduce an algorithm that computes expansion coefficients in general, which we furthermore use to give closed form expansions in some cases. We conjecture that certain indecomposable statistics generate the whole ring, and indeed prove this to be the case for statistics associated with matrices consisting of up to 3 Jordan blocks. The coefficients we compute exhibit surprising stability phenomena, which in turn reflect stabilizations of joint moments as well as multiplicities in the irreducible decomposition of tensor products of representations of finite general linear groups.

62. On binomial coefficients associated with Sierpiński and Riesel numbers

Sofya Bykova Cornell University
Ashley Armbruster Frostburg State University
Advisor(s): Joshua Harrington, Cedar Crest College

A Sierpiński (resp. Riesel) number is an odd integer k such that $k \cdot 2^n + 1$ (resp. $k \cdot 2^n - 1$) is composite for all $n \in \mathbb{N}$. Finch, Harrington and Jones considered Sierpiński numbers of the form $k = x^r + x + c$ for $r, c \in \mathbb{N}$. We investigate the existence of Sierpiński numbers and Riesel numbers as binomial coefficients through applications of covering systems. In particular, we show that the set of values of r for which there exist infinitely many Sierpiński numbers of the form $k = \binom{x}{r}$ has asymptotic density 1. A similar study is also performed on Riesel numbers. Additionally, generalizations to base a-Sierpiński numbers and base a-Riesel numbers are considered. This research was done as part of an REU at Moravian College.

63. Isomorphic Polynomials: When is a Polynomial Isomorphic to an Even Polynomial?

Hanna Noelle Griesbach Elon University
Advisor(s): Chad Awtrey, Elon University

We quickly learn in calculus how important polynomials are. They allow us to analyze parts of functions while only needing basic arithmetic properties to construct. We can express the roots of polynomials radically up to degree 4, but not necessarily for degree 5 and beyond. The benefit to solving a polynomial radically is that there is an exact answer and no precision error. Galois Theory is motivated by whether a polynomial can be solved radically or if its roots can only be approximated. For certain polynomials, like even polynomials, there are efficient ways to find whether the function is solvable by radicals. We explored when a polynomial can be transformed into an even polynomial for easier root computation. This transformed polynomial is characterized as isomorphic. By looking at how the symmetries moved and grouped roots of even versus non-even polynomials, we found that even polynomials possess a symmetry that move the roots to their reflections. We concluded that a non-even polynomial is isomorphic to an even polynomial if and only if it contains a symmetry that does not equal the reflection or the identity but when composed with itself two times yields the identity.

64. Saturation of Newton Polytopes of Cluster Variables from Surfaces

Amal Mattoo Harvard College
Advisor(s): Lauren Williams, Harvard University

Cluster algebras are algebraic structures equipped with distinguished generating sets, called clusters, whose elements are cluster variables. Given a surface with marked points, a cluster algebra can be defined with clusters corresponding to triangulations of the surface. The cluster variable corresponding to any arc can be written as a Laurent polynomial with respect to any triangulation, defining a Newton polytope. Kalman (2013) studied these Newton polytopes in the case of polygon surfaces (Type A) and conjectured that they are always (1) saturated and (2) their only lattice points are their vertices. We prove both conjectures and extended the results to once-punctured polygons (Type D), giving precise conditions for when (1) and (2) hold. We also find similar results for different choices of frozen variables, including principle coefficients and no frozen variables. Examining cluster algebras from other surfaces, such as the annulus, the torus, and polygons with multiple punctures, we find that saturation and lack of non-vertex lattice points are no longer the norm, so results of the form we found in Type A and Type D are unlikely to extend further.

65. The Effects of Nitrogen Dioxide and Particulate Matter 2.5 on the Infection Rate of Respiratory Diseases

David Giron New Jersey City University
Advisor(s): Zhixiong Chen, New Jersey City University

Changes in the environment can cause adverse effects on human health in a variety of ways. Due to the onset of COVID-19, research was conducted on the effects of atmospheric pollution from both nitrogen dioxide (NO_2) and particulate matter 2.5 (PM 2.5) on the respiratory system. The focus on correlating the atmospheric concentration of the above two pollutants with the risk of developing several respiratory diseases. The diffusion-advection partial differential equation to model the transport of both NO_2 and PM 2.5 in the atmosphere above different cities. By comparing the conditions of these cities and previously reported data from exposure to the pollutants and the diseases

contracted as a result of the atmospheric conditions, potential factors are discussed for the symptoms experienced by some risk groups.

66. Introducing Three Best Known Binary Goppa Codes

Jan L. CarrasquilloLpez University of Puerto Rico at Cayey
Axel O. GmezFlores University of Puerto Rico at Ro Piedras
Christopher Soto Queens College, City University of New York
Advisor(s): Fernando L. Piero Gonzlez, University of Puerto Rico in Ponce

The current best known $[239, 21]$, $[240, 21]$, and $[241, 21]$ binary linear codes have minimum distance 98, 98, and 99 respectively. In our research, we introduce three binary Goppa codes with Goppa polynomials $(x^{17}+1)^6$, $(x^{16}+x)^6$, and $(x^{15}+1)^6$. The Goppa codes are $[239, 21, 103]$, $[240, 21, 104]$, and $[241, 21, 104]$ binary linear codes respectively. These codes have greater minimum distance than the current best known codes with the respective length and dimension. In addition, with the techniques of puncturing, shortening, and extending, we find more derived codes with a better minimum distance than the current best known codes with the respective length and dimension. This research was conducted in The Puerto-Rico/East Tennessee REU in Combinatorics, Probability, and Algebraic Coding Theory supported by NSF-DMS REU-1852171.

68. The Hilbert Series of $O(2)$; Deriving the formula for a generic weight vector and implementing the Mathematica algorithm to compute it.

Lawton Walker Rhodes College
Advisor(s): Christopher Seaton, Rhodes College

Let $O(2)$ be the 2×2 orthogonal group, the group of reflections and rotations. Let τa denote the irreducible representation of $O(2)$ induced from the circle representation with weight a. Define $V = \tau a_1 + \tau a_2 + \cdots + \tau a_n$ to be an arbitrary representation of $O(2)$. When the a_i's are distinct we say our representation is generic and degenerate otherwise. The Hilbert Series of the invariants and covariants of V counts the respective invariants and covariants of each degree after we expand our Hilbert Series at $t = 0$. We use the Molien-Weyl theorem for an arbitrary representation V, to solve two complex integrals. The resulting Hilbert Series can be interpreted in terms of shifting a power series. Thus, we implement a Mathematica algorithm that performs a partial fraction decomposition on our integral along with the power series interpretation to yield the Hilbert series for any given weight vector. Next, we compute a similar integral to determine the Hilbert Series of the covariants of $O(2)$. Finally, we compute the first coefficient of the Laurent series expansion at $t = 1$ of our Hilbert Series of the invariants.

69. Meromorphic functions with the same preimages at several finite sets

Kenta Suzuki Cranbrook Kingswood Upper School, MIT PRIMES
Advisor(s): Michael Zieve, University of Michigan

Let p and q be nonconstant meromorphic functions on $\mathbb{C}^m$. We show that if p and q have the same preimages as one another, counting multiplicities, at each of four nonempty pairwise disjoint subsets $S_1, \ldots, S_4$ of $\mathbb{C}$, then p and q have the same preimages as one another at each of infinitely many subsets of $\mathbb{C}$, and moreover $g(p) = g(q)$ for some nonconstant rational function $g(x)$ whose degree is bounded in terms of the sizes of the S_i's. This result is new already when $m = 1$, and it implies many previous results about the extent to which a meromorphic function is determined by its preimages of a few points or a few small sets, in addition to yielding new consequences such as a classification of all possibilities when two of the S_i's have size 1.

70. Geometric Methods in Computer Vision

Brooke Dippold Longwood University
Ian Klein Carleton College
Jose Agudelo North Dakota State University
Alex Kokot University of Notre Dame
Advisor(s): Irina Kogan, North Carolina State University

In high school geometry, two shapes are congruent if they are related by rotations, translations, and reflections. These transformations comprise the Euclidean group, but it is only one way to define congruence. Congruence can be defined

by a variety of transformation groups. Motivated by applications in automated assembly, image recognition, and arm movements, we studied the special Euclidean and equi-affine notions of congruence. One method for curve recognition relies on invariant properties of curves, like curvatures and their derivatives. We have reconstructed curves from their curvatures using power series for analytic curvatures and Picard iterations for non-analytic curvatures. In application, curves are given approximately, so we examine relationships between closeness of invariants and the closeness of the curves reconstructed from these invariants. This project was done through an REU at North Carolina State University.

71. Data assimilation for parameter estimation of a single-compartment Type 1 diabetes ODE model

Maya Watanabe University of Massachusetts Amherst
Christina Catlett Scripps College
Daniel Shenker Johns-Hopkins University
Rachel Wander Harvey-Mudd College
Advisor(s): Christina Edholm, Scripps College

The onset of type 1 diabetes, an autoimmune disease characterized by an inability to regulate blood glucose, is believed to be determined by the interactions of immune cells in response to a catalyst during weaning. It is difficult to collect immune cell population data, but mathematical modeling and data assimilation, allows us to realistically simulate these cell interactions. We explore methods to estimate parameters for a single-compartment ODE model of pancreatic immune cell response during type 1 diabetes onset in mice based on glucose measurements (Shtylla et al., 2019; Li et al., 2015). We evaluate and compare the performance and biological feasibility of Markov Chain Monte Carlo (MCMC) methods, Particle Swarm Optimization (PSO), and Unscented Kalman Filters (UKF). While MCMC showed a preference towards population-level data sets, and the UKF was better adapted to the noise present within glucose data of a single subject, PSO proved versatile for both types of datasets. Our work also suggests that model composition influences the ideal parameterization strategy based on the scale of the model and number of parameters. Our research was part of the Harvey-Mudd College REU.

72. Mathematical Modeling of the Stock Market

Kylie Hannafey Georgia Southern University
Advisor(s): Hua Wang, Georgia Southern University

Through the behavior of past data, we use Game Theory to model our findings, predict decisions made by businesses, and understand what scenarios will produce a stable stock market. In particular, we will provide a thorough analysis of the stock market behavior between the three leading competitors in technology: Apple, Microsoft, and Google. To begin, we will use the website Nasdaq, a detailed online record of the stock market, to record the daily price of stock and share volume for each company. We will analyze the trend of the data through common statistical models. The findings from this analysis will be used to construct our Game Theory model, which we analyze through Nash equilibriums. As a result, we should be able to evaluate the stability of the stock market and discuss the relationships between our companies. More specifically, we expect to find the impact changes in the stock market have on each business and predict the behavior, or the "best next move," they should have.

73. Autoencoder-Based ECG Anomaly Detection

Thomas Dunn University of Central Oklahoma
Advisor(s): Emily Hendryx, University of Central Oklahoma

Electrocardiogram (ECG) data can provide a wealth of information about a patient's health. However, even experienced clinicians struggle to distinguish normal from anomalous EGCs in cases when the differences are subtle. In addition, the sheer volume of data and low frequency of anomalous beats make automated ECG analysis appealing. We present an autoencoder-based model as an initial screening tool for ECG beats. An autoencoder (AE) is a neural network model which learns to compress input data into a low dimensional vector representation and then reconstruct an approximation. The goal for such models is to learn to reconstruct the input data as well as possible, which requires learning an effective data representation. An AE trained to reconstruct one class of data (normal beats) will have a high error when trying to reconstruct data from another class (anomalous beats). This reconstruction error discriminates between normal and anomalous beats for further assessment by clinicians or analysis through additional methods. We

investigate the efficacy of AE-based classification by comparing our model performance on the MIT-BIH Arrhythmia Database against other classification methods.

74. Inferring RNA state using Hidden Markov Models

Alexandra Nosarzewska University of Kentucky
Advisor(s): David Murrugarra, University of Kentucky

RNA state inference determines which nucleotides of an RNA sequence are paired or unpaired in the secondary structure of an RNA. Data-directed RNA secondary structure predictions can be improved by RNA state inference. We used a hidden Markov Model (HMM) to perform RNA state inference. We trained an HMM using publicly available RNA structure data. We used the maximum likelihood approach to estimate the transition probabilities from paired to unpaired nucleotides and vice versa as well as for the emission probabilities. We used different algorithms for state inference such as the Viterbi, the forward, and the backward algorithms. HMM is based on the Markov property in which the future state depends only on the current state. This presents a challenge since the HMM will not give information on nonlocal dependencies. However, HMM is a simple and interpretable model that provides insights about the rate of accuracies in the predictions as opposed to more sophisticated methods. This poster will present results of tests for different classes of RNA (e.g. 5S rRNA, tRNA) and will discuss some challenges such as overfitting and the availability of data.

75. Additive Decomposition of Matrices with Applications to Total Acquisition in Graphs

Doel Rivera Pontifical Catholic University of Puerto Rico
Advisor(s): Anant Godbole, East Tennessee State University

The question of graph acquisition, recently studied in a series of papers by Doug West and his REGS students, led to the conjecture that each diameter two graph has acquisition number 2 or lower. In 2015, Rose McCarty led an REU team that showed, in unpublished work, that the acquisition number of a diameter 2 graph is 3 or lower provided that the following auxiliary result, on zero-one matrices, could be established. Conjecture: Each integer $n \times n$ matrix with row and column sums equal to n can be decomposed as $A + B$, where the row and column sumsets of A, B are, respectively $\{1, 2, \ldots, n\}$ and $\{0, 1, \ldots, n-1\}$. Although the conjecture remains unproven, here we present four methods, from different areas of mathematics, which we have attempted. These are the Combinatorial Nullstellensatz, a Graph theoretic approach, the Lovász Local Lemma and a Linear Programming approach. This work was completed at the Puerto-Rico/East Tennessee State REU in Combinatorics, Probability and Algebraic Coding Theory.

76. Random Strategies in the Iterated Prisoners Dilemma

Guangya Wan University of Illinois, Urbana Champaign
Xiaorui Wang University of Illinois, Urbana Champaign
Advisor(s): AJ Hildebrand, University of Illinois, Urbana Champaign

The Prisoner's Dilemma is a two-player game in which each player (prisoner) can either "corporate" or "defect". In the Iterated Prisoners Dilemma (IPD), two players repeatedly play the Prisoners Dilemma game against each other. In our research, we investigate a class of strategies for the IPD, called "memory one" strategies, that is, probabilistic strategies that depend only on the actions (cooperate or defect) by each of the two players during the previous round of the game. Such strategies are characterized by a vector $\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$, where p_{CC} is the probability of cooperating if both players cooperated in the previous round, and p_{CD}, p_{DC}, p_{DD} are defined similarly. Given that a player plays a memory one strategy $\mathbf{p}$, we are interested in the probability that the player wins against a player employing a random memory one strategy $\mathbf{q}$, selected uniformly among all vectors in $[0, 1]^4$. In particular, we characterize the strategies $\mathbf{p}$ that win, with probability 1, against a random strategy $\mathbf{q}$.

77. Involution Derangement Graphs

Veronica Dobbs Marquette University
Advisor(s): Anant Godbole, East Tennessee State University

The derangement graph on all permutations is defined as the graph whose vertex set is the set of all permutations on $[n]$ with two vertices being adjacent if they have no position in common. Much is known about this graph, but specifically as a Cayley Graph whose vertex set is a group. Permutations have plenty of interesting subsets, which

makes us question, what happens if we restrict the vertex set to one of these subsets? We examine the involution derangement graph, which is a subgraph of the permutation derangement graph. The involution derangement graph is the derangement graph whose vertex set is the set of all involutions on $[n]$, with an involution being a permutation whose inverse is itself. We study and compare classic graph theory properties between the two graphs, such as degree, diameter, chromatic number, independence number, and more. This work was completed at the Puerto-Rico/East Tennessee State REU in Combinatorics, Probability and Algebraic Coding Theory.

78. Optimal solutions and ranks in the max-cut SDP

Hyunwoo Lee Interlake High School
Daniel Hong Skyline High School
Alex Wei Interlake High School
Advisor(s): Diego Cifuentes, Massachusetts Institute of Technology

The max-cut problem is a classical graph theory problem which is NP-complete. The best polynomial-time approximation scheme relies on *semidefinite programming* (SDP). We study the conditions under which graphs of certain classes have rank 1 solutions to the max-cut SDP. We apply these findings to look at how solutions to the max-cut SDP behave under simple combinatorial constructions. Our results determine when solutions to the max-cut SDP for cycle graphs are rank 1. We find the solutions to the max-cut SDP of the vertex sum of two graphs. We then characterize the SDP solutions upon joining two triangle graphs by an edge sum.

79. The Influence of "Bots" and Gaslighters on Twitter and their Effect on the General Perception of Current Events

Luc Telemaque New York City College of Technology
Advisor(s): Nadia Benakli, New York City College of Technology

The goal of this project is to analyze and visualize Twitter[1] conversations based on the current event, the COVID-19 Pandemic. Social network analysis enabled us to identify the influencers and followers of these conversations, who are often 'bots' and gaslighters. Text analysis was used to grasp an understanding of twitter users' sentiments about the disease. The primary tool used for this study, Netlytic, is a community supported text and social networks analyzer.

80. An Unstructured Mesh Approach to Nonlinear Noise Reduction

Jonah Botvinick-Greenhouse Amherst College
Aaron Kirtland Washington University in St. Louis
Megan Osborne University of Scranton
Advisor(s): Casey Johnson, Claremont Graduate University

In any type of data acquisition, the event of gathering undesirable noise along with desirable data is inevitable. To denoise signals originating from smooth, chaotic attractors, the Air Force Research Laboratory (AFRL) adapted the time-delay embedding theory of Takens' Theorem (1981) and the causation-detecting method of Convergent Cross Mapping (CCM) to develop a grid-based denoising technique. Given a clean signal from such a dynamical system, AFRL's technique attempts to denoise a corrupted signal observed from the same system. To improve this grid-based method, we implement an unstructured mesh based on triangulations and Voronoi diagrams that better distributes data over mesh cells and improves the accuracy of the reconstructed signal. Our method achieves statistical convergence with known test data and reduces synthetic noise on experimental signals from Hall Effect Thrusters (HETs) with greater success than the grid-based strategy. This project was done as a part of the virtual Research in Industrial Projects for Students (RIPS) program at the University of California - Los Angeles (UCLA) Institute for Pure and Applied Mathematics (IPAM).

81. The Effect of TIMP-1 levels, MMP-1 Levels, and Healing Times on the Wound Surface Area

Aarini Panzade Western Kentucky University
Advisor(s): Richard Schugart, Western Kentucky University

Matrix metalloproteinases (MMPs) are enzymes that degrade all kinds of extracellular matrix proteins during the wound-healing process. TIMP-1 is a tissue inhibitor of metalloproteinases (MMPs). In this work, we are investigating

how MMPs and TIMPs, and the ratio of MMPs to TIMPs affect the wound surface area and the healing time. The data used for this work was of sixteen patients with diabetic foot ulcers in which measurements were taken of MMPs and TIMPs during a 12-week period. A multilinear regression analysis was performed on the interaction terms of different variables and compared to the wound surface area and compared to the regression analysis done on just the individual variables. It was concluded that the linear regression analysis done by the interaction terms was more beneficial to the wound surface area than the one done on individual variables.

82. Game of Cycles: Extended

Heather Baranek University of Wisconsin - Eau Claire
Advisor(s): Shanise Walker, University of Wisconsin - Eau Claire

The study of graphs provides an answer to questions in a variety of disciplines through the process of modeling the problem onto a graph. The Game of Cycles is a newly developed impartial mathematical game, introduced by Francis Su in his book *Mathematics for Human Flourishing* in 2020. The two-player game is played on simple planar graphs in which players take turns marking edges using a sink-source rule. In this poster, we extend the game of cycles to more than two players and outline the strategies in winning a game on certain families of planar graphs. We also discuss further results of the two-player Game of Cycles.

83. What is the boundary of a finite field numerical range?

Brandon Collins University of Nebraska at Omaha
Gage Hoefer University of Nebraska at Omaha
Jonny Quezada University of Nebraska at Omaha
James Willson University of Nebraska at Omaha
Advisor(s): Patrick Rault, University of Nebraska at Omaha

Given a square matrix A, its numerical range is the image of a certain map from the unit sphere to the complex plane. This numerical range is the convex hull of a "boundary generating curve." We will discuss a finite field version of the numerical range, using a map $g_A : S \to \mathbb{Z}_p[i]$, where p is any prime congruent to 3 modulo 4 and S is the unit sphere in $\mathbb{Z}_p[i]^2$. In the modular situation of finite fields, where there is no concept of boundary, the aforementioned boundary generating curve has some very special properties. If A is a 2-by-2 matrix, then g_A is a two-to-one map (up to trivial multiples), except on the boundary generating curve, where it is a one-to-one map (up to trivial multiples). We will discuss the geometry of the image of g_A (the numerical range of A) using this boundary generating curve.

84. Flat-foldable origami vertices and 3-colorings of graphs

Xin Yan Western New England University
Advisor(s): Thomas Hull, Western New England University

Origami is the art of folding paper. We use a mountain-valley (MV) assignment to describe the state of a flat-folded piece of paper since creases on the paper after folding would either be mountains or valleys. Properly 3-coloring the vertices of a graph $G = (V, E)$ is an assignment of three colors to the vertices in V where no two adjacent vertices are of the same color. The relationship between MV assignments of flat-foldable origami crease patterns that contain only a single vertex and the 3-colorings of a corresponding graph will be discussed, following the prior work of Chiu, Hoganson, Hull, and Wu (2020) and improving their degree-4 vertex graph.We also consider a degree-6 vertex, for which no such corresponding graph is currently known.

85. True Parameters of Algebraic Geometry Codes using Groebner Bases

Austin Allen Carnegie Mellon University
Advisor(s): Fernando Pinero-Gonzalez, University of Puerto Rico at Ponce

Algebraic Geometry (AG) codes are a class of codes studied for their resemblance to Reed-Solomon codes and their improvement of the Gilbert-Varshamov bound. These codes are defined by their parameters, i.e. defined by their length, dimension, and minimum distance. A particular parameter that is of interest is a code's minimum distance. The theory of algebraic function fields often gives nonconstructive bounds on the dimension, minimum distance, and decoding of AG codes. In this work we propose Groebner bases techniques as a simpler method to constructively determine the dimension, minimum distance, and decoding algorithms of AG codes generated from the Giulietti-Korchmaros

curve and the Garcia-Guneri-Stichtenoth curve. We also expect to have explicit Locally Recoverable Codes (LRC's) generated from these curves. This work was completed at the Puerto-Rico/East Tennessee State REU in Combinatorics, Probability, and Algebraic Coding Theory.

86. Student Attitudes in First-Year Mathematics Courses

Stephanie Tran California State University, Monterey Bay
Advisor(s): Alison Lynch, California State University, Monterey Bay

In Fall 2018, California State University (CSU) eliminated remedial mathematics courses. After moving away from remedial mathematics courses, California State University, Monterey Bay's Mathematics and Statistics department wanted to better understand student attitudes in GE mathematics courses. To do this, the department distributed the Attitudes Toward Mathematics Inventory (ATMI) survey to students in Precalculus, Finite Mathematics, and Quantitative Literacy from Fall 2018 through Spring 2020. The ATMI instrument focuses on four constructs: self-confidence, value, enjoyment, and motivation. Each student was expected to take the ATMI survey at the beginning (pre) and at the end (post) of the math course. Both the Paired T-Test and Two Sample T-Test were used to see the change in student attitude from the beginning to the end of the course and to compare the post scores for each construct. In addition, a Regression Model was created to see if each construct correlates to student grade in the GE course. Variables in the model includes the pre and post score for each construct as well as class name, high school GPA, and age.

87. Universal Enveloping Algebras of Poisson Superalgebras

Thomas Lamkin Miami University
Advisor(s): Jason Gaddis, Miami University

Poisson algebras arise naturally from the study of Hamiltonian mechanics, and have since appeared in numerous areas of mathematics and physics such as quantum groups and algebraic geometry. Due to the development of supersymmetry theories, one has also witnessed the increased use of Poisson superalgebras, along with other super structures such as Lie superalgebras and supermanifolds. One method of studying Poisson algebras is to study their universal enveloping algebra, generalizing the construction for Lie algebras. In this poster, we present the results of our research on the universal enveloping algebra of Poisson superalgebras. In particular, we prove a PBW theorem for Lie-Rinehart superalgebras independent of Rinehart's PBW theorem, leading to a PBW theorem applicable to all Poisson (super)algebras; we show the universal enveloping algebra of a Poisson Hopf superalgebra (resp. Poisson-Ore extension) is a Hopf superalgebra (resp. iterated Ore extension) generalizing the analogous results for Poisson algebras; and we describe the universal enveloping algebra for examples such as quadratic polynomial Poisson superalgebras and Poisson symmetric superalgebras.

88. Bounds for Approximating Insolation on Exoplanets on Circular Orbits

Matthew Choy Cornell University
Advisor(s): Alice Nadeau, Cornell University

The distribution of incoming stellar radiation across a planet's surface (or insolation distribution) is a major driving force on a planet's climate. Although a planet's insolation distribution can be derived from first principles using its orbital parameters, it is still not known in general how a planet's rotation rate affects the distribution. In particular, the distributions for slowly rotating planets and rapidly rotating planets are known, but the transition between these states is not. For rapidly rotating planets, it is possible to approximate the distribution with a polynomial in the sine of the latitude and the axial tilt. This simple approximation makes it easier to model rapidly rotating planets. In this study, we numerically investigate the rate of convergence of the annual mean insolation distribution to the insolation distribution for a rapidly rotating planet as the rotation rate is increased. We consider the case of a planet on a circular orbit with no tilt to its rotational axis. We show that for our tested rotation rate range, the decay to rapid rotation insolation is proportional to the reciprocal of the rotation rate.

89. Groupoids and Solving the 9-Puzzle

Kayla Lehtola University of Minnesota - Twin Cities
Advisor(s): Kathryn McCormick, California State University Long Beach

Lloyd's 9-puzzle ($\mathcal{L}_{3\times3}$) is a 3×3 matrix of squares with uniquely labeled squares in eight of the nine indices. The groupoid $G(s_1, s_1)$ represents all of the distinct legal moves of labeled squares so that the empty square starts and

ends at s_1, the $(1, 1)$ index of the matrix. The goal of this project was to determine the upper limit of moves needed to solve the 3×3 Lloyd's puzzle in this case. This information can be used in an algorithm to solve the puzzle, and in estimating run time when coding a solution to the 9-puzzle.

90. Generalizations of Alder's Conjecture via a Conjecture of Kang and Park

Adriana Duncan Tulane University
Simran Khunger Carnegie Mellon University
Ryan Tamura University of California Berkeley
Advisor(s): Holly Swisher, Oregon State University

Let $\Delta_d^{(a,b)}(n) = q_d^{(a)}(n) - Q_d^{(b)}(n)$ where $q_d^{(a)}(n)$ counts partitions of n into parts with difference $> d$ and size $> a$ and $Q_d^{(b)}(n)$ counts partitions of n into parts $\equiv \pm b \pmod{d+3}$. In 1956, Alder conjectured that $\Delta_d^{(1,1)}(n) \geq 0$ for all positive n and d. This conjecture, which generalizes several partition identities including Euler's theorem and the first Rogers-Ramanujan identity, was proved in parts by Andrews in 1971, Yee in 2008, and Alfes et al. in 2011. In 2020, Kang and Park constructed an extension of Alder's conjecture related to the second Rogers-Ramanujan identity by considering the difference $\Delta_d^{(a,b,-)}(n) = q_d^{(a)}(n) - Q_d^{(b,-)}(n)$, where $Q_d^{(b,-)}(n)$ counts partitions of n into parts $\equiv \pm b \pmod{d+3}$ excluding the part $d+3-b$. Kang and Park conjectured that $\Delta_d^{(2,2,-)}(n) \geq 0$ for all positive n and d, and proved this when $d = 2^r - 2$ and n is even. We prove Kang and Park's conjecture for all $d \geq 62$ and present a more generalized conjecture for higher $a = b$, which we prove for infinite classes of n and d. This work was completed at the 2020 Oregon State REU.

91. Evolutes that don't Evolve

Daniel Havens Utah Valley University
Advisor(s): Alan Parry, Utah Valley University

We present results on curves whose evolutes are self-similar by rigid motion. Specifically, we derive a set of differential equations that model curves whose evolutes are exactly a translation of the original curve. Additionally we show restrictions on the set of curves which have an evolute that is a reflection of the original curve. We also give partial results for rotations similar to those obtained for translations.

92. Another Look at Croftian Sequences

Yilin Liu University of Pittsburgh
Advisor(s): Roxana Popescu, University of Pittsburgh

A sequence $(x_n)_{n\geq 1}$ with the property that given a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $\forall x \lim_{n\to\infty} f(x_n + x)$ exists, then $\lim_{x\to\infty} f(x)$ exists is called a Croftian sequence. A classical example of such a sequence is $(\ln(n))_{n\geq 1}$. Motivated by the work of J.F. Kingman, in this presentation we will discuss the algebraic stability of Croftian sequences under certain conditions as well as some new examples and counterexamples to demonstrate the necessity of these and other conditions.

93. A Logical, Mathematically-Sound Approach to the Solution of the Quantum Mechanical Harmonic Oscillator

Jacob Metcalfe Muhlenberg College
Advisor(s): Michael Huber, Muhlenberg College

The quantum mechanical harmonic oscillator is often the second quantum model taught to undergraduate physical chemistry students, yet the solution to the model is frequently skipped due to its complexity. This work presents a logical, mathematically-sound approach to the solution of the harmonic oscillator that only requires knowledge of differential and integral calculus, and allows students to see the origin of the Hermite polynomials that are present in the solution.

94. Not Your Normal Fibonacci Sequence

Montana Ferita Westminster College
Advisor(s): John Bonomo, Westminster College

The Fibonacci sequence is perhaps the most well-known sequence in the field of mathematics. In the Fibonacci sequence each element in the series is the sum of the previous two numbers. We will denote the first two numbers in the series as a and b, where a is less than or equal to b. Given a number n we seek to find the smallest positive value b such that n appears in the Fibonacci series starting with a and b. One can determine a and b given n by performing an exhaustive search for various combinations of a and b. We investigate quicker methods than this brute force approach for n values with certain properties.

95. Tic Tac Toe in Higher Dimensions

Ferdinand Gruenenwald QUEST University Canada
Advisor(s): Asia Matthews, QUEST University Canada

A friend introduced me to a multiplayer version of tic tac toe. Any number of participants play in nine standard 3x3 tic tac toe fields arranged in a 3x3 square giving a total of 81 cells in which the players can place their marks. Similar to standard tic tac toe, a player can score by marking three cells in a row that are equidistant to each other, also referred to as a winning triplet. I kept wondering; 1)What is the actual shape of the gameboard? 2)How many ways are there to score with a winning triplet? By constructing a mathematically precise version of higher dimensional tic tac toe, I show that the gameboard forms a 4D hypercube. Then, I continue to construct a matrix group describing the gameboard's symmetry properties. By exploiting the algebra, I define what it means for a sequence of three marked cells to constitute a winning triplet. Finally, the symmetry properties provide a way to classify winning triplets and reduce the complexity of our counting problem. After all, it is possible to generalize and find the general formula that describes the number of winning triplets in n-dimensional tic tac toe.

96. Wavelet-Based Machine Learning Algorithms for Handwriting Recognition

Lylah Bottelsen Western Connecticut State University
Carina Cardoso Western Connecticut State University
Advisor(s): Xiaodi Wang, Western Connecticut State University

In this research project, we will explore an effective method for developing handwriting recognition algorithms using a sample of handwritten digits. We will use the methods of (Discrete) Wavelet Transformations, Principal Component Analysis (PCA), Linear Discriminant Analysis (LDA) and Artificial Neural Networks (ANN) to accomplish these goals. First, we will use the combination of Wavelet Transformations, PCA, and LDA to extract the features of the handwritten digits in our sample, and to reduce the dimensionality of the matrices we will be working with. We will apply ANN train our model to recognize each digit. Our demonstration of the capabilities of Wavelet Transformations, PCA, LDA, and ANN with recognizing handwritten digits can be expanded to recognizing alphabetical figures, as well as complex mathematical figures and notations. This methodology can be used to help doctors by processing important documents, banks by processing checks and other signed forms, as well as mail services by processing zip codes, addresses, etc. for automatic mail sorting.

97. Stock Price Prediction Algorithm by Wavelet Based Machine Learning Methods

Brandon Roe western connecticut state university
Allen Yang University of Colorado
Advisor(s): Xaiodi Wang, Western Connecticut State University

Even though technology continues to make massive leaps forward, some problems still remain difficult to solve. Predicting the future values of stocks is one of these issues. The goal of this research is, while using a combination of existing methods, to find a new way to predict the values of various stocks. Specifically, so that it performs better than today's current standards. Using two known working methods in conjunction could ease the difficulty of the predictions. By using the Wavelet Transform and denoising stock signals, the data will be 'cleaned' and made easier to understand and predict. Then, by using Principal Component Analysis (PCA), we will be able to compress the data to identify its Principle Components. Finally, the altered data can be used in Wavelet based SVR-NN, in order to find future stock prices. By using the described method, any individual attempting to anticipate the market's future moves would be left with a prediction more accurate than the industry standard.

98. Integration of Proprioceptive Feedback for Forward Locomotion in Caenorhabditis Elegans Network Model

Anna Thomas Lehigh University
Matthew Clark Fisk University
Advisor(s): Xiaopeng Zhao and Tian Hong, University of Tennessee, Knoxville

We are interested in understanding the neuron dynamics within Caenorhabditis elegans during forward locomotion and the cause of sustained oscillations associated with this movement after removal of tail touch stimulus. In previous works by Kunert, Shlizerman, and Kutz, the differential equation model appeared sophisticated enough to drive oscillatory behavior in response to a stimulus to touch receptive PLM class neurons. However, in attempting to recreate and build upon this work, we were unable to produce the same sustained oscillatory behavior. The network lacked the structure required to produce a response that could be interpreted as forward movement. In dynamical systems theory, such a response is usually governed by a limit cycle resulting from a Hopf bifurcation . A revised model based on a bottom-up approach revealed that oscillation could be induced by the addition of proprioceptive feedback in the form of a sinusoidal stimulus applied to specific classes of motor neurons within the worm. This work was done as a part of the 2020 NIMBioS SRE Program.

99. The Kernel of Newform Dedekind Sums

Evuilynn Nguyen Rhodes College
Advisor(s): Matthew Young, Texas A&M University

Newform Dedekind sums are a class of crossed homomorphisms that arise from newform Eisenstein series. We initiate a study of the kernel of these newform Dedekind sums. Our results can be loosely described as showing that these kernels are neither “too big” nor “too small.” We conclude with an observation about the Galois action on Dedekind sums that allows for significant computational efficiency in the numerical calculation of Dedekind sums. This research was conducted as part of the 2020 Texas A&M University REU.

100. Forecasting Long-Term Government Bond Payments: An Application and Analysis of Machine Learning Algorithms

Alvin Josi Western Connecticut State University
Advisor(s): Xiaodi Wang, Western Connecticut State University

In this research we are using a selection of four economic indicators and applying machine learning and classical algorithms to accurately forecast the monthly yield of United States 10-year Treasury bonds. Due to the complexity of an accurate prediction, the objective produces a difficult test for the algorithms under review. This analysis is of particular interest for the crucial and paradigmatic role that the United States sector plays in the world economy. The algorithms are testing from 1986 to 2004 on a whole set of end-month United States 10-year Treasury bond yields and economic indicators. Using a combination of statistical and machine learning techniques as a learning paradigm, we search for the complicated mechanisms ruling and measuring changes in interest rates. Their performance is compared to the performance of two classical models; a statistical model for autoregressive integrated moving average (ARIMA) and an econometric model for error correction.

101. Graph coloring for the assignment of seats to fans

Lam Doan La Salle University
Advisor(s): Janet Fierson, La Salle University

Graph coloring, in which labels are assigned to elements of a graph in such a way that specified conditions are met, has many variations. We consider an application of graph coloring to the assignment of seats to sports fans. Each seat is represented by a vertex, and pairs of vertices associated with adjacent seats are connected by edges. Each team has a unique color. The vertices of the graph are colored, subject to certain constraints, to indicate the placement of fans in seats. For various families of graphs, we determine which collections of fans can be seated under the stated compatibility conditions, and we provide optimal assignments in the feasible cases. Fans supporting more than one team are also considered; this results in multi-colored vertices when seating arrangements are determined.

102. Characterizing Patients with Pulmonary Hypertension Using Dynamical Modeling

Mariam Kharbat North Carolina State University
Robert Sternquist North Carolina State University
Advisor(s): Mette Olufsen, North Carolina State University

Pulmonary hypertension (PH) is a deadly disease characterized by high blood pressure above 25 mmHg in the pulmonary arteries. For this study, we use mathematical modeling to simulate and analyze right heart catheterization data from 5 patients diagnosed with PH. To predict the flow, pressure, and volume of the cardiovascular system, we develop a compartment model analogous to an electric circuit. This closed-loop system estimates model parameters that minimize the difference between model predictions and the RHC data. Using parameter subset selection, sensitivity, and covariance analysis, we determine a set of identifiable parameters that are fitted specific to each data set. Results show that hypertensive patients have increased pulmonary vascular resistance and decreased compliance compared to normotensive control patients. By simulating PH treatments, we can achieve normotensive pressure values in the right ventricle and pulmonary artery, while also reducing the pressure in the right atrium and systemic arteries. Since PH is a chronic and progressive disease, recognizing the features of a hypertensive network can facilitate diagnosis and improve treatment strategies. [NCSU REU]

103. Interpreting the Relationship Between Non-Overlapping Confidence Intervals and Familywise Error Rate in One-Way Layout

Noah Prime Western Washington University
Ana Uribe Western Washington University
Advisor(s): Kimihiro Noguchi, Western Washington University

One of the rejection criteria utilized in hypothesis testing concerning the statistical significance of the mean difference is the “non-overlap” of their confidence intervals. By analyzing the balanced one-way repeated-measures layout, we construct an interpretable multiple linear regression model which well describes the relationship between the familywise error rate (FWER) and the corresponding confidence level used in the non-overlap criterion to determine the statistical significance, assuming homoscedasticity, compound symmetry, and multivariate normality. Then, we construct a similar model in the case of independent one-way layout, which is applicable for Tukey’s Studentized range test with various sample sizes. The usefulness of these models are demonstrated through a detailed error analysis and a real-life application.

104. The Sunflower Problem

Tolson Bell Georgia Institute of Technology
Suchakree Chueluecha Lehigh University
Advisor(s): Lutz Warnke, Georgia Institute of Technology

A sunflower with p petals consists of p sets whose pairwise intersections are all the same set. The goal of the sunflower problem is to find the smallest $r = r(p,k)$ such that every family of at least r^k k-element sets must contain a sunflower with p petals. Major breakthroughs within the last year by Alweiss-Lovett-Wu-Zhang and others show that $r = O(p\log(pk))$ suffices. In this poster, we review the history and significance of the Sunflower Problem and outline our improvement to $r = O(p\log(k))$. This bound was obtained during our 2020 REU at Georgia Tech under the supervision of Lutz Warnke.

105. U-Net Neural Network Reconstructions of Under-sampled Magnetic Resonance Imaging (MRI)

Rachel Roca Manhattan College
Advisor(s): Angel Pineda, Manhattan College

Magnetic Resonance Imaging (MRI) is a common and powerful diagnostic tool that allows for visualization of soft tissue in the body. However, MRI machines require a lot of time to fully acquire data. Therefore, research has been done to accelerate the process using a variety of methods. We will utilize convolutional neural networks, U-Nets, a kind of convolutional neural network architecture, to reconstruct under-sampled images from four and eight fold accelerations. We apply area under the receiver operating characteristic (ROC) curve (AUC) as a novel metric, besides the standard normalized mean squared error (NMSE), structural similarity (SSIM), and L1 to assess the quality of

reconstruction. A task-based metric approximating an ideal model observer is applied in order to appeal to the clinical usage of MRI. We apply these metrics to various forms of the U-Net implemented, by varying hyper-parameters. We preliminarily find that a more complicated model with dropout and greater number of channels does not appear to make much of a difference in the reconstruction. We also observe that the AUC mostly aligned with the other metrics but varied less.

106. Math LIteracy

Brian Dawson State University of New York at Oswego
Advisor(s): Sarah Hanusch, State University of New York at Oswego

Math curriculum has changed from procedures to word-based problem solving where text rich, complex, multi-tier language structures are the norm. In this heavy content and context environment how do students navigate the complexities and subtleties to become successful in the Common Core problem solving environment. The purpose of this project is to explore literacy, in word based problems, in high school mathematics. While investigating the literature around word problems we focus on the following: all the crucial components that make up a word problem, how much literacy affects word problem solving, and how literacy has changed the dynamic of word problems over the last eight years. This project investigates the literacy requirements for word problems on the high school Regents examination in New York. In particular, I will report on the vocabulary requirements required on the free-response mathematics problems for the years 2010–2019, and how the vocabulary and literacy requirements have changed over this time period.

107. An Extension Theorem for Bicomplex Functions

Chloe Makdad Butler University
Advisor(s): William Johnston, Butler University

A new root and ratio test for bicomplex power series produces the Bicomplex Extension Theorem, where any complex power series with positive radius of convergence R extends, via a simple change of the complex domain variable z to the bicomplex variable Z, to a bicomplex function analytic in a four real dimensional ball with the same positive radius R. In this way, any complex analytic function has a natural extension to a bicomplex analytic function.

108. Classifying Non-Edge-to-Edge Tilings of the Sphere by Regular Polygons

Peter Hollander Williams College
Liza Jacoby Williams College
Cameron Edgar Williams College
Advisor(s): Colin Adams, Williams College

In the Euclidean plane, there exist beautiful infinite families of tilings using exclusively regular polygons. When we move to the surface of the sphere, we still define regular spherical polygons to be ones which are equiangular and equilateral, though contrary to Euclidean space, the surface of the sphere is tiled with only finitely many such polygons. We consider tilings of the sphere by regular spherical polygons with three or more edges, gluing the tiles edge-to-edge. These tilings include the prisms and antiprisms projected onto the sphere, and as reported by Pappus of Alexandria, Archimedes listed thirteen additional possible edge-to-edge tilings using regular spherical polygons. The classification of tilings of the sphere by regular spherical polygons has since been left incomplete, as non-edge-to-edge tilings—that is, where the edges of two adjacent polygons do not perfectly align—were left unconsidered. However, we complete the classification of tilings of the sphere using regular spherical polygons with three or more edges, thus completing the work of Archimedes and Johnson and providing new beautiful families of spherical tilings.

109. Unexpected Chaos in a Billiard with Focusing Boundaries

Camryn Colonna Fairfield University
Hailey Spinella Fairfield University
Advisor(s): Mark Demers, Fairfield University

Mathematical billiards are important models of dynamical systems from mathematical physics in which point particles collide elastically with fixed boundaries. Chaotic dynamics emerge when the boundary of the billiard table is dispersing or when it contains focusing arcs at sufficient distance to allow a defocusing effect to occur. This project studies a type

of billiard known as an asymmetric lemon billiard, comprised of focusing boundaries which seem to violate the usual defocusing condition. Numerical evidence is obtained showing that chaotic dynamics nevertheless occur for a large range of parameter values, extending beyond the range to which analytic proofs apply. This work was completed at Fairfield University during Summer 2020, and was supported by a grant from the National Science Foundation.

110. Calculating Monodromy Groups of Modular Curves

Vanessa Sun Macaulay Honors College at Hunter College, City University of New York
Fabian Ramirez Sonoma State University
Advisor(s): Edray Goins, Pomona College

It is well known that all compact connected Riemann surfaces X of genus at least two are quotients of the extended upper-half plane $\mathbb{H}^*$ by a discrete subgroup Γ of $PSL_2(\mathbb{R})$. For example, when Γ is a classical congruence subgroup such as $\Gamma_0(N)$, $\Gamma_1(N)$ or $\Gamma(N)$, then the Riemann surfaces, namely $X_0(N)$, $X_1(N)$, and $X(N)$ are well-known. By projecting to the "j-line" $X(1) \simeq \mathbb{P}^1(\mathbb{C})$, we have a morphism $\beta : X \to \mathbb{P}^1(\mathbb{C})$ branched above 0, 1, ∞. In this project, we consider the monodromy groups $\mathrm{Mon}(\beta)$ and automorphism groups $\mathrm{Aut}(\beta)$ of such Belyĭ maps. This project was completed during the Mathematical Sciences Research Institute Undergraduate Program.

111. Calculating Components of Chebyshev-like Maps

Amanda Lee Pepperdine
Advisor(s): Joshua Bowman, Pepperdine

Chebyshev-like maps are higher-dimensional analogues of Chebyshev polynomials in one variable. The components of these maps, $T_{d,k}$, where d is the degree and k is the coordinate, can be computed recursively. A key step of this calculation is obtaining a certain set of coefficients. Such elements can be acquired through an operation called plethysm, which is a kind of composition law for symmetric functions. We use SageMath to write programs which calculate the $T_{d,k}$ given d and k. Thus, we make it more computationally feasible to find explicit formulas for Chebyshev-like maps of dimension greater than or equal to 2.

113. The Minimum Number of Multiplicity 1 Eigenvalues among Real Symmetric Matrices whose Graph is a Linear Tree

Wenxuan Ding College of William & Mary
Advisor(s): Charles Johnson, College of William & Mary

Let T be a linear tree, and let $\mathcal{S}(T)$ denote the set of real symmetric matrices whose graph is T. If $U(T)$ is the minimum number of eigenvalues with multiplicity 1 among matrices in $\mathcal{S}(T)$ and T' is a linear tree resulting from the addition of 1 vertex to T, we show that $|U(T') - U(T)| \leq 1$. We also determine the exact set of possible values of $U(T) - U(T')$, depending upon the manner in which the vertex is added to T to get T'. These results are then used to give a new bound for $U(T)$, the diameter bound, and to improve an existing bound, $2 + D_2(T)$.

114. Completing and Extending Shellings of Vertex Decomposable Complexes

Michaela Coleman The University of Tulsa
Nathan Geist Duke University
Advisor(s): Anton Dochtermann, Texas State University

A pure d-dimensional simplicial complex Δ is said to be shellable if there exists an ordering of its facets with nice intersection properties. Shellability has important applications to the underlying combinatorial, topological, and algebraic properties of Δ. We define the new notion of "shelling completable", which says Δ can be realized as the initial shelling order of the d-skeleton of a simplex. The well-known Simon's conjecture then posits that any shellable complex is shelling completable. We prove that vertex decomposable complexes are shelling completable, thus establishing Simon's conjecture for a large class of complexes. In fact, if Δ is vertex decomposable, there exists a $(d+1)$-subset F such that $\Delta \cup F$ is again vertex decomposable. We explore applications to matroids, shifted complexes, and k-vertex decomposable complexes. Finally, for a d-dimensional complex Δ on at most $d + 3$ vertices, we show that the notions of shellable, vertex decomposable, shelling completable, and extendably shellable coincide. This research was conducted during the 2020 Texas State University REU.

115. To and From 2-Generated Groups and Origamis

Elisa Rodriguez Ursinus College
Sarai Gonzalez Eastern University
William Sablan University of Guam
Advisor(s): Edray Goins, Pomona College

In this research, we focus on the geometric construction of origami in detail. Initially, we construct various origami, by considering different examples of 2-generated groups. Conversely, we begin with an arbitrary collection of squares, glued together to form an origami, and determine the corresponding transitive subgroup of S_n. This research was completed during MSRI-UP Summer 2020.

116. Computation of Monodromy Groups for Toroidal Belyĭ Maps

Rebecca Lopez Marist College
Advisor(s): Edray Goins, Pomona College

A Belyĭ map $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a rational function with at most three critical values; we may assume these values are $\{0,\ 1,\ \infty\}$. A Dessin d'Enfant is a planar bipartite graph obtained by considering the preimage of a path between two of these critical values, usually taken to be the line segment from 0 to 1. Such graphs can be drawn on the sphere by composing with stereographic projection: $\beta^{-1}\big([0,1]\big) \subseteq \mathbb{P}^1(\mathbb{C}) \simeq S^2(\mathbb{R})$. Replacing $\mathbb{P}^1$ with an elliptic curve E, there is a similar definition of a Belyĭ map $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$. The corresponding Dessin d'Enfant can be drawn on the torus by composing with an elliptic logarithm: $\beta^{-1}\big([0,1]\big) \subseteq E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$. In this project, we use *Mathematica* to write code which takes an elliptic curve E and a Belyĭ map β to return information about the monodromy group by employing solving a system of first-order differential equations. This research was done at the MSRI-UP 2020 under Dr Edray Goins.

117. Social Inequality on the Affine Wealth Model

Marquell Williams Carthage College
Advisor(s): Haley Yaple, Carthage College

Capitalism delivers prosperity for a wide array of individuals, but how exactly the market distributes its wealth remains largely unanalyzed. This paper investigates the impacts of social inequality on the Affine Wealth Model (AWM) of wealth distribution. The AWM is a mathematical simulation modeling wealth concentration resulting from free market agent-based exchanges over time. In this study, we expand the AWM by adding economic disadvantage for a random subgroup of the population to resemble the economic effects of social inequality. We compare wealth inequality in this system to those without random disadvantage. We find that imbalanced power dynamics between economic agents accelerate extreme wealth concentration when compared to systems without social inequality.

118. On the Odd Order Composition Factors of Finite Linear Groups

Alexander Betz Stony Brook University
Ting Gong University of Notre Dame
Advisor(s): Yong Yang, Texas State University

The order of a finite group is perhaps the most fundamental quantity in group theory. Accordingly, the question of bounding the order of a finite group is a very natural one and has long been a subject of vigorous research. In 1993, Manz and Wolf showed that one can bound the order of certain solvable linear groups by the vector spaces on which they act. We generalize this result to obtain a bound on the product of the odd order composition factors of an arbitrary finite linear group. This research was conducted under an NSF-REU grant at Texas State University, during the Summer of 2019.

119. Finite Permutation Groups with Few Orbits under the Action on the Power Set

Anthony Ter-Saakov Boston University
Max Chao-Haft Harvey Mudd College
Advisor(s): Yong Yang, Texas State University

Our project investigates the natural action of a permutation group $G \leq S_n$ on the powerset $\mathcal{P}(\{1,\ldots,n\})$. In particular, we seek to classify permutation groups based on their orbits under this action, which we call set-orbits. Permutation

groups with exactly $n+1$ set-orbits are called set-transitive (being analogous to transitive groups) and were fully classified in a 1955 paper by Beaumont and Peterson. Pushing this further, we develop a general method for characterizing the permutation groups with $n+r$ set-orbits for arbitrary $r > 1$, and apply the computer algebra system GAP to obtain explicit classifications for small r. This research was conducted under an NSF-REU grant at Texas State University, during the Summer of 2019.

120. On Updating and Querying Submatrices

Jason Yang Belmont High School
Advisor(s): Jun Wan, MIT

Given a d-dimensional matrix, an *update* adds all elements in a given submatrix with a given number v, and a *query* returns the minimum of all elements in a given submatrix. The **update-query problem** consists of finding the fastest algorithms to perform both of these operations with arbitrary inputs. For $d = 1$, the segment tree with lazy propagation is a well-known data structure that performs updates and queries in $O(\log N)$ time. However, for $d \geq 2$, we show that if min-plus matrix multiplication cannot be computed in $O(N^{3-\varepsilon})$ time for any $\varepsilon > 0$, then either updates or queries cannot both run in $O(N^{1-\varepsilon})$ time for any $\varepsilon > 0$, or preprocessing cannot run in polynomial time. In addition, for specific variants of the update-query problem where there exist $O(\text{polylog}(N))$ time algorithms for updates and queries, we present an algorithm that is slightly simpler than those of previous works.

121. Downscaling for Ice Fog

Jonathan Lindbloom Southern Methodist University
Shailee Manandhar University of Southern Mississippi
Evan Lipton Yale University
Junqing Yan Dartmouth College
Advisor(s): Anne Gelb, Dartmouth College

The research project seeks to choose the most accurate method from three candidate methods for identifying ice fog, a natural phenomenon occurring in regions of Alaska that can severely affect traffic and transportation safety, as well as activities on the Air Force bases in those regions. In each of these methods, we constructed a relationship between prediction temperature from the Global Forecast System and observed temperature to downscale the GFS. The methods we considered are the following: a simple Bayesian regression, normalizing flows that consists of a chain of transformations to take us from the reference to the target, and transport maps with approximation using polynomial expansions to capture the complicated distribution of the target with a well-understood distribution for reference. To determine the accuracy of each method, we used uncertainty quantification techniques. Specifically, we tested out a variety of parameters with a number of neurons, layers, and learning rate. We also calculated the Brier score and created control plots to visualize the result.This project was done as part of the 2020 Dartmouth REU.

122. Modeling signal detection in Magnetic Resonance Imaging (MRI) using human observer models

Alexandra G. O'Neill Manhattan College
Emely L. Valdez Manhattan College
Advisor(s): Angel R. Pineda, Manhattan College

Human observer studies are a clinically relevant way to evaluate the quality of magnetic resonance images but they are expensive and time consuming. Mathematical models of observer performance can reduce the time and cost. Using varying anatomical backgrounds and a signal known exactly (SKE), we carried out a two alternative forced choice experiment (2AFC) with three trained human observers. Human observers needed to pick which of two images had the signal. With 4x acceleration and a total variation constraint (TV), we found that human observer results remained approximately constant across different regularization constraints but severely decreased at larger values. We used a non-prewhitening eye filter (NPWE) and sparse difference-of-Gaussians (SDOG) channelized Hotelling observer with internal noise to model human observer performance. These model observers tracked the performance of the human observers as the regularization was increased but over-estimated the percent correct for large amounts of regularization. To our knowledge this is the first time that model observers have been used to track human observer detection for undersampled MRI.

123. Modeling Resistance Emergence in Aedes albopictus on a Heterogeneous Landscape

Leah Rolf NC State University
Advisor(s): Alun Lloyd, NC State University

In the United States, Aedes albopictus is considered a nuisance mosquito due to its peridomestic habitat and propensity to feed on humans. However, in much of the world, they are known to be vectors of diseases such as dengue, Zika, and chikungunya. There are two major methods to eliminate Ae. albopictus: adulticides and larval habitat reduction. However, it is feared that resistance to treatment will emerge in Ae. albopictus mosquitoes in the US. Resistance emergence occurs rapidly and needs to be explored further within Ae. albopictus. We use a population genetics model with multiple patches to model a metapopulation of mosquitoes as they move within a landscape of treatments of different strengths and placement. Using this framework, we look at how treatments affect resistance of the mosquitoes. We also examine the effects of fitness cost on our metapopulation's size and genetic makeup. The goal of the model is to help determine effective control strategies that reduce populations without giving rise to resistance.

124. Voting on Cyclic Orders

Abraham Holleran Gordon College
Josephine Noonan Gordon College
Advisor(s): Karl-Dieter Crisman, Gordon College

We studied voting on cyclic orders, focusing on those with four candidates per order. We explored voting systems in which voters choose aspects of their preferred order in an intuitive way. This allows a single ballot to vote for more than one order, reducing the number of ties we get and allowing the voter to specify what they see as the most important aspect in an order. In each system, we explored different kinds of ties and their symmetries through permutations. Some of these ties map to the net of a cuboctahedron. Permuting the ties forms groups that also interact with the cuboctahedron. This was completed as part of a REU at Gordon College.

125. Mathematical Understanding of Red Blood Cell Dynamics

Benjamin Brindle Lehigh University
Advisor(s): Miranda Teboh-Ewungkem, Lehigh University

Red blood cells are one of the most important components of life in humans. Loss of red blood cells has consequences, such as anemia and hypovolemic shock. Such a loss could be the result of bloodletting, parasitemia, or viral infection. Red blood cell dynamics within a human involve several stages of precursor cells before a red blood cell fully matures to an erythrocyte. After blood loss, a feedback mechanism contingent on loss and level of erythrocytes causes the production of more precursor cells to return the blood dynamics to equilibrium. We model this process using a system of nonlinear, deterministic, ordinary differential equations. Functions describing this feedback, the stem cell recruitment, and the erythrocyte loss are chosen to examine the system dynamics. Some parameter choices cause a Hopf bifurcation, while others lead to death, stable steady states, or limit cycles. Numerical methods are used to display bifurcation diagrams and transient dynamics. Methods of mathematical analysis such as nondimensionalization and proofs of invariance, positivity, boundedness, and uniqueness for arbitrary functions are given. Numerical results are performed for specific functions.

126. The Trouble with Traffic

Clayne Williams Eastern New Mexico University
Advisor(s): Brian Pasko, Eastern New Mexico University

This project creates an original simulation in python which accurately and effectively models real world traffic and analyzes the effect on traffic by variable road conditions and vehicle attributes. Varied road conditions include closed lanes, closed lane length, and overall vehicle density on the road. The effects of these conditions on the overall speed and flow of the road system are recorded and studied. Individual vehicle attributes, such as maximum speed, following time, and following distance, are also varied as to mimic real-world situations, and the effects of the vehicle's characteristics on the traffic system's speed and flow are recorded and analyzed. Notable results include the forward and backward propagation of slowdown from a closed lane, and the effect of varying proportions of vehicles with large deviations from the average speed of the traffic system. Insights into what may cause traffic jams in the real-world are discovered, as well as situations in which the efficiency of the traffic system is improved compared to other combinations of varied attributes of the road and the vehicles on the road.

127. Monomials, Convex Bodies, and Optimization

William Frendreiss Texas A&M University
Caleb Fong University of St. Andrews (UK)
Advisor(s): Alexandra Seceleanu, University of Nebraska, Lincoln

In commutative algebra, there are various notions of taking a power of a monomial ideal, namely the ordinary power, the symbolic power, and the rational/real power. These notions and the relationships between them are a topic of active research. In our poster, we will present novel results concerning some algebraic invariants of these ideals. In particular, we establish a lower bound on the Waldschmidt constant of a monomial ideal, an invariant which can be defined using a symbolic polyhedron. We construct a new polyhedron and are able to extrapolate and obtain an analogous invariant. We will also present some variants of the long-standing packing problem for squarefree monomial ideals, in the context of linear programming and polyhedral optimisation. Some interesting consequences of the packing problem will also be shown. These results were obtained during the inaugural Polymath REU, a fully online REU that took place in the summer of 2020 in lieu of traditional REU programs canceled due to current events.

128. Structure of Number Theoretic Graphs

Lee Trent Rose-Hulman Institute of Technology
Advisor(s): Joshua Holden, Rose-Hulman Institute of Technology

Questions about the structure imposed on a set of objects by the relationships between them can be reframed as graph theory questions. In particular, one considers the graph whose vertices are elements of that set and whose edges indicate the relationship of interest, and investigates properties of that graph, such as its hamiltonicity. In Matt Parker's "Things to Make and Do in the Fourth Dimension," the question of whether the integers 1 through n can be ordered linearly so that the sum of each pair of adjacent elements is a square is investigated briefly. For n up to and including 14, n from 18 to 22, and $n = 24$, it can't be done. For all other n less than 200, it can be done, as shown by example. It's suspected but not proved that it can be done for all n greater than 25. This presentation will further discuss relevant properties of the graphs underlying this problem, as well as graphs generated by other number theoretic properties.

129. Vivani's n Vertices

Franklin Maxfield Utah Valley University
Advisor(s): Alan Parry, Utah Valley University

We present extensions of Vivani's Theorem to all regular convex polygons, all regular convex polyhedra, and most regular convex polytopes. In the case of two of the platonic solids we show this extension produces two identities for the golden ratio. We also show that the proof for each of these cases is essentially the same. Finally, we show that there are considerable complications extending Viviani's Theorem to the four dimensional 24-cell.

130. Machine Learning Models for Predicting the Onset of Depressive Disorders

Minhwa Lee The College of Wooster
Advisor(s): Marian Frazier, The College of Wooster

Depressive disorders are the most common mental health concern all over the world. In this study, we investigate what societal factors, such as socio-demographic information and accessibility to healthcare services, impact the onset of depressive disorders among US citizens in 2018. Then, we implement statistical and computational methods to extract factors and predict the disorders based upon them. The entire process of the study are as follows: presenting a logistic regression model, explaining mathematics behind two different supervised machine learning algorithms - decision trees with CART and support vector machines - and predicting the onset of the disorders by using all three methods. Lastly, we evaluate the performance of the three models by comparing accuracy of prediction, recall, AUC, and F1 score of each model.

131. Controlling Covid-19: An Epidemiological Model

Logan Rose Marshall University
Advisor(s): Anna Mummert, Marshall University

Coronavirus disease 2019 is a highly infectious disease caused by the novel coronavirus (SARS-CoV-2). It was first reported in Wuhan, China in late December 2019 and was declared a pandemic in March 2020. As of October 29,

2020, there have been over 44 million cases reported worldwide with over 1.17 million deaths (WHO). In late August, many colleges across the U.S. reopened with control measures in place. Some were able to successfully contain the virus while others experienced massive outbreaks. We model the spread of COVID-19 across a campus with a relatively small population ($N = 4000$). We consider a differential equations model with Susceptible-Exposed-Infected-Asymptomatic-Quarantined-Recovered-Dead classes. We parameterized the model using real world values. Then we find a formula for the transmission rate based on the Basic Reproduction Number, R0. R0 was estimated to be 2.35 (Smith). Next, we develop an Optimal Control problem to minimize the Asymptomatic and Infected populations and the cost to implement the following control strategies: mask wearing and increased testing. Adjoint equations are derived and the effectiveness of each control measure is analyzed.

132. Rigid Folding Equations for Degree-6 Origami Vertices

Johnna Farnham Western New England University
Aubrey Rumbolt Western New England University
Advisor(s): Thomas Hull, Western New England University

Rigid origami describes when a material, like paper, is folded along straight crease line segments while keeping the regions between the creases planar. It has recently been used in micro and macro technologies such as nano-robots and large-scale solar farms. Previous work in rigid origami has found explicit equations for the folding angles of a flat-foldable degree-4 origami vertex. We extend this work to the symmetric degree-6 case and waterbomb base crease pattern by using 2nd-order Taylor expansions and linear algebra. In addition, we enumerate the different viable rigid folding modes of these crease patterns using a combination of Polya enumeration, numerical computation, and our rigid origami results. We also made use of Ghassaei's online origami simulator to approach the problem from a strictly computational vantage point.

133. A computational investigation of a continuum model for flocking dynamics

Luke Galvan University of Nebraska-Lincoln
Advisor(s): Adam Larios, University of Nebraska-Lincoln

Throughout 2016–2017, R. Shvydkoy and E. Tadmor published a series of papers proving well-posedness for a hydrodynamic flocking model in 1D. Flocking can be described as the collective behavior of a large number of interacting agents satisfying i.) the alignment of velocities, ii.) cohesion of agents, and iii.) separation throughout time. The continuum flocking model can be thought of as the hydrodynamical-limit of the discrete Cucker-Smale (CS) model. The CS model, in 2007, was proven to emit flocking solutions in a discrete setting. This study introduces a novel method for computationally investigating 2D nonlocal nonlinear continuum models. Leveraging the Fourier transform and using a singular kernel for the nonlocal operator, the computations involved efficiently compute the convolution operator to preserve the time and power needed to investigate this system in 2D and higher. In 2D and higher, the lack of a conserved quantity prevents a proof for well-posedness. However, candidates for conserved quantities are able to be numerically tested from this investigation. This work was supported by UNL's UCARE (Undergraduate Research and Creative Activities) program.

134. Can recommendation letters predict college success?

Benjamin Siegel United States Military Academy
Advisor(s): COL Joseph Lindquist, United States Military Academy

Recommendation letters are routine components of college admission, but the volume of letters prohibits objective analysis, so admissions personnel rarely evaluate individual letters. We relate text mining properties of recommendations to college outcomes at the United States Military Academy (USMA). A database housing seven years of USMA admissions data and performance variables was compiled. The database consisted of applicant recommendations and traditional admissions variables such as standardized test scores and high school GPA. Detailed USMA performance variables such as academic GPA and USMA-specific outcome variables such as character evaluations were also included. Using the statistical package, R, sentiment, word frequency, and word count were calculated for the recommendations. These variables were used in regression models to predict college outcomes. The average word count predicted class rank with an $R^2 = 0.090$ ($p < .0001$). The regression model including average word count, average "I" count, and sentiment predicted class rank with an adjusted $R^2 = .099$ ($p < .0001$), suggesting automated text analysis can be used to objectively inform admission decisions.

135. Uniformly Sparse Graphs and Matrices

Yuqiao Li College of William & Mary
Advisor(s): Charles Johnson, College of William & Mary

We introduce the uniformly spars (US) graphs and the US matrices associated with them. We can perform diagonal equivalence (DAE) on US matrices, and we provide several theorems related to DAE and other characteristics of US graphs. Diagonal equivalence simplifies the calculation of the minimum rank, so in the square case, the determination of maximum geometric multiplicity (M) of an eigenvalue is straightforward. In addition to M, we also discuss two other key parameters: the path cover number (P) and the zero forcing number (Z). Important examples and counterexamples are also shown in our poster to give insight into the minimum rank problem. Part of this work was done in the 2019 Matrix REU at William and Mary.

136. Topological data analysis of pulmonary arterial networks

Natalie Johnston Vassar College
Miya Spinella University of Massachusetts Dartmouth
Ian Livengood North Carolina Agricultural and Technical State University
Advisor(s): Mette Olufsen, North Carolina State University

Cardiovascular disease (CVD) is the world's leading cause of mortality, annually claiming about 17.9 million lives. CVD conditions include structural and functional problems, diseased vessels, and blood clots, causing patients to experience heart failure and eventually leading to death. One incurable CVD is pulmonary hypertension (PH), defined as blood pressure above 25 mmHg in the main pulmonary artery and accompanied by vascular remodeling. This study uses topological data analysis (TDA) to analyze pulmonary arterial networks from control and PH mice. The networks are segmented and skeletonized from micro-CT images. Mice with PH had thicker vessels, causing their segmented networks to falsely appear to contain more arteries than control networks. To obtain networks of comparable size, we applied tree-pruning algorithms based on vessel radius and Strahler order. TDA is used to compute persistent homology of the networks. Results from a height filtration show total degree 0 persistence is lower in PH networks than in control. The alpha-complex filtration and bottleneck distances between persistence diagrams will also be discussed. This work was completed as part of an REU at NCSU.

137. Pellquadratic and Jacobsthalqudratic Numberss

Lauren Spensiero Gonzaga University
Advisor(s): Tomas Guardia, Gonzaga University

With relation to the introductory research by Guardia, Jimenez, and McCurdy on the Fiboquadratic numbers and their properties in the mid-evil board game Rithmomachia, this new research creates an alternative extension of the Rithmomachia board in terms of Pellquadratic and Jacobsthalquadratic numbers. We have found that the new extensions uphold the definitions of Greek Number Theory, and have similar properties as the Fiboquadratic numbers.

138. Strong Proper Connection

Emma Miller Moravian College
Rob Lorch Grinnell College
Jonathan Moore Rowan University
Advisor(s): Caitlin Owens, DeSales University

A path is properly edge colored if adjacent edges are colored with different colors. The strong proper connection number of a graph is the smallest number of colors required to edge color the graph such that there exists at least one properly colored shortest path between each pair of vertices in the graph. We will discuss our results on the strong proper connection number for special classes of graphs and graph operations. These results answer some questions previously posed on the topic. This research was a part of the Computational and Experimental Mathematics REU at Moravian College funded by the National Science Foundation.

139. Understanding the effect of adaptive mutations on the three-dimensional structure of RNA

Justin Cook Duquesne University
Advisor(s): Lauren Sugden, Duquesne University

The human genome contains over 300 million variable positions, or single-nucleotide polymorphisms (SNPs), constituting the genetic diversity seen around the world. Variation within a DNA sequence affects its downstream RNA sequence and potentially its protein product. RNA forms 3-D structures via intramolecular base pairing, which can directly affect the synthesis of protein by sequestering or exposing the binding sites of translational machinery; thus an understanding of the RNA structural change elicited by an SNP can further medical knowledge and offer insight into potential treatment approaches. We aim to identify quantitative metrics for measuring the change in computationally predicted structure ensembles, and to apply those towards an understanding of the effects of specific mutations in the human genome. As a preliminary study, we investigate a collection of SNPs previously shown to harbor signatures of positive selection (Sugden et al., 2018). We identify one mutation which induces structural change, providing a hypothesis for the factors motivating adaptation at this locus. We hope to further understand the effects of this mutation and others throughout the genome.

140. Error-Correcting Output Codes (ECOC): Hadamard Transform

Jordan Jiosi Rowan College at Burlington College (RCBC)
Advisor(s): Jonathan Weisbrod, Rowan College at Burlington College

Error-correcting output codes (ECOCs) are used in machine learning to represent class labels as codewords constructed from an ECOC matrix. The ECOC matrix includes relatively large minimum row and column Hamming distances. Classification of an object is performed by matching its corresponding output codeword to the class codeword of nearest Hamming distance. Anomalies are detected and encoded as new concepts based on output codewords of the unseen objects and their Hamming distances with respect to recognized objects. This paper describes how the properties of the Hadamard Transform can be used to achieve better-than-standard error correction. NSF, CURM (Center for Undergraduate Research in Mathematics) Grant: DMS-1722563

141. Post-Inhibitory Rebound-like Behavior in Networks of Pulse-Coupled Integrate-and-Fire Neurons

Lane Lewis University of Arizona
Advisor(s): Calvin Zhang-Molina, University of Arizona

A fundamental challenge in computational neuroscience is to build biologically realistic models while controlling the model's complexity and dimensionality so that it remains tractable for analysis. Many large-scale computational models rely on networks of integrate-and-fire model neurons (IF). Unlike Hodgkin-Huxley-type model neurons, an IF neuron alone cannot produce rich nonlinear effects such as post-inhibitory rebound (PIR) and bursting. Here we show that a pair of pulse-coupled IF neurons can give rise to a PIR-like effect, which we define as a brief increase in a neuron's firing rate following the termination of a strong inhibition. We derive the conditions under which such a PIR-like effect occurs. In addition, we investigate the population dynamics of a larger network of IF neurons in response to inhibitory inputs. This work suggests that a careful choice of the IF model parameters may help preserve some biologically-relevant nonlinear behaviors.

142. A New Method to Compute the Hadamard Product of Two Rational Functions

Ishan Kar Saratoga High School
Advisor(s): Ira Gessel, Brandeis University

The Hadamard product (denoted by $*$) of two power series $A(x) = a_0 + a_1x + a_2x^2 + \cdots$ and $B(x) = b_0 + b_1x + b_2x^2 + \cdots$ is the power series $A(x) * B(x) = a_0b_0 + a_1b_1x + a_2b_2x^2 + \cdots$. Although it is well known that the Hadamard product of two rational functions is also rational, a closed form expression of the Hadamard product of rational functions has not been found. Since any rational power series can be expanded by partial fractions as a polynomial plus a sum of power series of the form $\frac{1}{(1-ax)^{m+1}}$, to find the Hadamard product of any two rational power

series it is sufficient to find the Hadamard product:

$$\frac{1}{(1-ax)^{m+1}} * \frac{1}{(1-bx)^{n+1}} = \frac{(1+ax)^m * (1+bx)^n}{(1-abx)^{m+n+1}}$$

The Hadamard product of negative powers of quadratic polynomials have also been derived.

143. Caterpillar Game Chromatic Number

Paige Beidelman University of Mary Washington
Advisor(s): Jeb Collins, University of Mary Washington

The graph coloring game is a game played with two players, Alice and Bob, such that they alternate to properly color a graph G, meaning no adjacent vertices are the same color. Alice wins if every vertex is properly colored with k colors, otherwise Bob wins when a vertex cannot be colored using k colors. While strategies for winning this game may seem helpful, more interesting is the least number of colors needed for Alice to have a winning strategy, which is called the game chromatic number, $\chi_g(G)$. Tree graphs have a game chromatic number of at most 4, so what are the criteria for a tree T with $\chi_g(T) = 4$? To answer this question we classified segmented caterpillar graphs with at least one vertices of degree 2, 3, and 4, for which the game chromatic number have not yet been explored.

144. Nontransitive Relations in Independent Random Variables

Pavle Vuksanovic University of Illinois at Urbana-Champaign
Advisor(s): AJ Hildebrand, University of Illinois at Urbana-Champaign

Motivated by classical nontransitivity paradoxes, we call an n-tuple $(x_1, \ldots, x_n) \in [0,1]^n$ *cyclic* if there exist independent random variables $U_1, \ldots, U_n$ with $P(U_i = U_j) = 0$ for $i \neq j$ such that $P(U_{i+1} > U_i) = x_i$ for $i = 1, \ldots, n-1$ and $P(U_1 > U_n) = x_n$. We call the tuple $(x_1, \ldots, x_n)$ *nontransitive* if it is cyclic and in addition satisfies $x_i > 1/2$ for all i. We consider the following questions: (1) What is the probability that a randomly chosen tuple in $[0,1]^n$ is cyclic (resp. nontransitive)? (2) Is there an algebraic characterization to determine whether a tuple is cyclic? Can we use this to find the maximal value π_n (introduced by Trybula in the 1960s) such that there exists a cyclic n-tuple with all of its coordinates $\geq \pi_n$?

145. Pattern Avoidance in Cyclic Permutations

Alexander Sietsema Michigan State University
Advisor(s): Bruce Sagan, Michigan State University

Pattern avoidance in permutations is an extensively studied field of enumerative combinatorics. We will discuss the classical version for linear permutations and then introduce a recent variant for cyclic permutations. Finally, we will present our new results counting and classifying cyclic avoidance sets for all pairs of length 4 patterns.

146. Effects of Mars' Chaotic Obliquity on Surface Ice Cover: A Budyko Model Approach

Anushka Narayanan Cornell University
Advisor(s): Alice Nadeau, Cornell University

Studies suggest that the evolution of Mars' rotational axis is chaotic and may have varied by as much as 60 degrees over time. This large and unpredictable range in the orientation of the planet makes Mars' climate history uncertain. A planet's climate largely depends on the characteristics that describe the planet's specific orbit around the Sun. It is known that for Mars, its planetary obliquity, which refers to the angle of the tilt of its rotational axis, has a prominent role in the extent of ice cover on its surface. Ice cover is particularly interesting due to the potential geological and biological implications. To observe how Mars' changing obliquity possibly impacted ice cover dynamics over time, we use an obliquity-dependent energy balance model examining incoming and outgoing energy. We will discuss our simulations of Mars' climate and the resulting ice regimes. We find as obliquity changes, there are large oscillations in polar ice cover. We see certain regions of parameter space where the model indicates stable oscillations between partial ice cover and ice-free states. The model also shows no stable partial ice cover on the surface with the poles being ice-free.

147. Logistic cumulative case functions and generalizations of the SIR model

Sydney Woods Wright State University
Colton Bennett Wright State University
Advisor(s): Ayse Sahin, Wright State University

The SIR model and the logistic growth model are both used in modeling epidemic outbreaks. Lega (2020) has shown that the cumulative case function for an epidemic that follows an SIR model is not a solution of the logistic differential equation. In this project we study the case of a logistic cumulative case function. We investigate what modification is necessary in an SIR model to fit the progression of the disease. In particular, in an SIR model the transitions from the susceptible, infected, and recovered populations are given by constants. We ask if replacing one of the transitions by a function of time is sufficient for the cumulative case function to be logistic.

148. Inner Distance in Latin Squares

Jieqi Di Boston College
Omar Aceval University of Texas, Austin
Mitchel OConnor Whitman College
Yewen Sun University of California, Santa Barbara
Advisor(s): James Hammer, Cedar Crest College

A Latin square of order n is an $n \times n$ array which contains n distinct symbols exactly once in each row and column. We define adjacent distance between two cells (containing integers) to be the absolute difference between entries in adjacent cells, and inner distance of a Latin square to be the minimum of adjacent distances in the Latin square. By first establishing an upper bound and then creating algorithms to construct a square with inner distance equal to this bound, we found the maximum inner distance of an $n \times n$ Latin square to be $\lfloor \frac{n-1}{2} \rfloor$. In this presentation we will discuss this result and similar results for Latin squares with additional constraints. This research was conducted at the REU at Moravian College on Research Challenges of Computational and Experimental Mathematics, with support from the National Science Foundation.

149. Isoperimetric Problems on the Real Number Line with Prescribed Density

Zariah Whyte Southwestern University
Evan Alexander Southwestern University
Advisor(s): John Ross, Southwestern University

Variations on the isoperimetric problem typically involve trying to find a region in some ambient space that has an "optimal measurement" but is subject to a "fixed constraint." In the classic example, mathematicians seek out an object with optimal/maximal area, subject to a fixed/constant perimeter). Oftentimes, the isoperimetric regions that optimize these quantities (such as the circle) have symmetric or otherwise "nice" geometry. There is often a deep relationship between the stated problem and the geometry of the solution. In our project, we examined isoperimetric regions on the real number line with prescribed densities. The densities added a weight that affected how "area" and "perimeter" were measured. We were interested in finding optimal configurations of 1, 2, or 3 regions (so-called single, double, or triple bubble problems). Specifically, we examined the number line with two densities of interest: $\mu(x) = |x|$ and $\mu(x) = ||x| - 1|$. With these densities, we found isoperimetric solutions to the single and double region problem for $\mu(x) = ||x| - 1|$ and the triple region problem for $\mu(x) = |x|$.

150. A Mathematical Model of COVID-19: Effects of Seasonality and Integration of Shield Immunity

Rosa Flores Dixie State University
Marcy Judd Dixie State University
Addesyn Marshall Dixie Sate university
Advisor(s): Vinodh Chellamuthu, Dixie State University

SARS-CoV-2 (COVID-19) has continued to spread across the globe. There are many unanswered questions regarding the dynamics of COVID-19. In this project, we have addressed a couple of crucial factors that influence COVID-19 dynamics, 1) shield immunity effectiveness in protecting against COVID-19 and 2) correlation of disease transmission to the seasonal changes within the local area. We developed a modified SEIR compartmental model to simulate the

dynamics of seasonality with a temperature dependent transmission rate and a shield immunity effectiveness variability. Furthermore, we did a comparative study between two regions to show the seasonal variability in relation to transmission rate. Our simulation results suggest that the confirmed cases of COVID-19 increases as the temperature fluctuates within an optimal bound. Moreover, our simulated results showed the effectiveness of shield immunity in relation to different possible scenarios.

151. On Class Numbers, Torsion Subgroups, and Quadratic Twists of Elliptic Curves

Talia Blum Massachusetts Institute of Technology
Caroline Choi Stanford University
Alexandra Hoey Massachusetts Institute of Technology
Kaya Lakein Stanford University
Jonas Iskander Harvard University
Advisor(s): Ken Ono, University of Virginia

The Mordell-Weil groups $E(\mathbb{Q})$ of elliptic curves influence the structures of their quadratic twists $E_{-D}(\mathbb{Q})$ and the ideal class groups $\mathrm{CL}(-D)$ of imaginary quadratic fields. We define a family of homomorphisms $\Phi_{u,v} : E(\mathbb{Q}) \to \mathrm{CL}(-D)$ for particular negative fundamental discriminants $-D := -D_E(u, v)$, which we use to simultaneously address questions related to Goldfeld's Conjecture on ranks of quadratic twists, the Cohen-Lenstra heuristics, and Gauss's class number problem. This research was conducted at the University of Virginia REU.

152. On Quasisymmetric Functions with Two Bordering Variables

Alexander Zhang I did research through the MIT PRIMES program, but I attend Lower Merion High School
Advisor(s): Slava Gerovitch, MIT

We study a family of formal power series $K_{n,\Lambda}$, parameterized by n and $\Lambda \subseteq [n]$, that largely resemble quasisymmetric functions. This family was conjectured to have the property that the product $K_{n,\Lambda} K_{m,\Omega}$ of any two formal power series $K_{n,\Lambda}$ and $K_{m,\Omega}$ from the family can be expressed as a linear combination of other formal power series from the same family. In this paper, we prove the aforementioned conjecture and thus that the span of the $K_{n,\Lambda}$'s forms an algebra, generalizing past work by Grinberg. We also provide techniques for examining similar families of formal power series and a formula for the product $K_{n,\Lambda} K_{m,\Omega}$ when $n = 1$.

153. Investigating Pattern Formation & Phase Transitions using Applied Computational Topology

William Zhang Brown University
Advisor(s): Ian Wong, Brown University

Active matter systems, ranging from liquid crystals to populations of cells and animals, exhibit complex collective behavior characterized by pattern formation and dynamic phase transitions. However, quantitative classification is challenging for heterogeneous populations of varying size, and typically requires manual supervision. Here, we use topological data analysis, to analyze spatial organization in heterogeneous populations of interacting discrete agents. Using persistent homology, we extract topological barcodes that uniquely identify the spatial arrangement of agents by keeping track of clusters, loops, voids and higher dimensional holes at multiple scales. To measure similarity between two simulations, we compute a distance metric between their barcodes using the persistence image representation (Adams et al., JMLR, 18(8): 1–35, 2017). Our preliminary results suggest that topological classification outperforms classification based on order parameters by a significant margin, validated with fixed and time-varying population sizes. We envision this approach will be relevant to understand emergent behaviors of multiple interacting species in self-organizing systems.

154. Point at Infinity and a Presentation of the Generalized Skein Algebra

Matt Dreyer Cornell University
Advisor(s): Han-Bom Moon, Fordham University

We will describe our presentation of the Roger-Yang generalized skein algebras for punctured spheres. Specifically, we will prove that the algebra is a quotient space of the skein algebra for the punctured plane modulo a set of certain relations. This set of relations will be proven to correspond to passing a strand over the point at infinity. This research would have been performed at Fordham University, but instead was conducted online due to the pandemic.

155. Chromatic Number of Disks in the Plane

Kaylee Weatherspoon University of South Carolina
Advisor(s): Joshua Cooper, University of South Carolina

There is a lack of systematic approaches to the chromatic number of the plane problem outside of attempting to generate non-k-colorable graphs. We propose that the circumradius of a subgraph embedded in the plane is associated with its chromatic number. For r being the radius of a disk in the plane, we want to find the lower and upper bounds on r for the transition from chromatic number k to chromatic number $k+1$ where is an integer and $1 \leq k \leq 6$. We show that having multiple copies of a rigid subgraph admitting a forced pair on a disk can improve the lower bound on the transition from chromatic number 3 to chromatic number 4, and we expect that this forced pair strategy can be used to increase the lower bound on other chromatic number transitions. We are also developing a method to generate a comprehensive list of unit-distance embeddable subgraphs to use for improving the upper bounds on chromatic number transitions.

156. Relations of the Generalized Skein Algebras of Punctured Spheres

Farhan Azad Fordham University
Advisor(s): Han-Bom Moon, Fordham University

In this poster, I will talk about the presentation of skein algebra, which is a topological invariant of a surface constructed by using curves on the surface. The presentation of the generalized skein algebra of a punctured sphere has been computed recently. I will explain the basic relations that are used to calculate various arcs and loops on an n-punctured sphere and provide a visual understanding. Then, I will describe relations which generate the ideal for the presentation of an n-punctured sphere. This project was done at Fordham University during the summer of 2020.

157. Modeling Park Visitation Using Transformations of the Distance-Type Predictor Variables with LASSO

Ashley Hall Western Washington University
Advisor(s): Kimihiro Noguchi, Western Washington University

We examine three common transformations (identity, fourth-root, and log) to determine the most suitable transformation for evaluating the importance of certain common features surrounding the Twin Cities Metropolitan Area (TCMA) city parks on park visitation. The distances between these features and city parks are approximately exponentially distributed by noting that their relative locations closely follow the spatial Poisson process. Because a fourth-root transformation improves the normality of exponential random variables, via simulation, we verify that the fourth-root transformation is considered best for identifying significant predictor variables affecting park visitation. Specifically, we achieve that by comparing the probabilities of selecting fourth-rooted predictor variables to the untransformed and log-transformed predictor variables using the least absolute shrinkage and selection operator (LASSO) regression. Finally, we apply these three transformations to various TCMA distance-type predictor variables to demonstrate that the significance of distance to the nearest bus stop improves dramatically with the fourth-root or log transformation.

158. On the relativistic quantum mechanics of N-body electron-photon systems in $1+1$ dimensions

Lawrence Frolov Rutgers University
Adriana Scanteianu Rutgers University
Samuel Leigh Rutgers University
Xiangyue Wang Rutgers University
Marcus McLaurin Morgan State University
Advisor(s): A. Shadi Tahvildar-Zadeh, Rutgers University

Using a relativistic extension of Bohmian Mechanics known as Multi-time Wave Function formulation, we examine an N-body quantum mechanical system of photons and electrons that interact only upon contact, in one space dimension. For the case $N = 2$ we investigate the effects that various parameters in this theory, including momentum of the incoming photon and mass of the electron, have on the dynamics of the two interacting bodies, with the goal of gaining a qualitative understanding of Compton Scattering for this system. For $N = 3$ and the system consisting of two electrons separated by a photon, with interactions given by a no-crossing boundary condition, we construct the

solution of the initial-boundary value problem satisfied by the multi-time wave function of this system, and conduct a preliminary study of the particle trajectories.

159. Generators of the Generalized Skein Algebras of Punctured Spheres

Ryan Horowitz New York University
Advisor(s): Han-Bom Moon, Fordham University

The skein algebra is an algebraic invariant of a topological surface, constructed by using equivalence classes of curves on the surface. Recently, we obtained an explicit presentation of the skein algebra of punctured spheres. In this presentation, we will describe the generators for our presentation of the skein algebras of punctured spheres. This project was done at Fordham University during the summer of 2020.

160. Prime Graphs of Several Classes of Finite Groups

Dawei Shen Washington University in St. Louis
Advisor(s): Thomas Keller, Texas State University

The prime graph of a finite group G, also known as the Gruenberg-Kegel graph, is the graph with vertex set {primes dividing $|G|$} and an edge p-q if and only if G contains an element of order pq. Prime graphs encode information about element orders in a finite group. A natural question is what graphs can be realized as prime graphs of groups of a certain class. In 2015, *Gruber, Keller, Lewis, Naughton,* and *Strasser* constructed an orientation of complements of prime graphs of solvable groups to show that they are exactly the triangle free and 3-colorable graphs. By further studying this orientation, we characterize prime graphs of several other classes of groups, including solvable groups of n^{th}-power-free order for any $n \in \mathbb{N}$. We then generalize this orientation to allow groups that are not necessarily solvable and show a necessary condition on prime graphs of groups whose composition factors are cyclic or A_5. As a corollary, these groups satisfy *N. V. Maslova*'s conjecture from *Unsolved Problems in Group Theory*. This research was conducted at the REU at Texas State University during the summer of 2020.

161. Traffic Sign and Traffic Light Recognition with Convolutional Neural Networks

Geoffrey Thorpe Morehouse College
Advisor(s): Kaushik Roy, North Carolina Agricultural and Technical State University

Autonomous vehicles, or self-driving cars, are vehicles that are capable of safely operating with little to no human interaction. The traffic sign recognition systems in autonomous vehicles use convolutional neural networks (CNNs) to detect and classify traffic signs and traffic lights. Despite CNNs being proficient at image recognition and classification, they are subject to be deceived by adversarial images. These adversarial images cause the CNN to incorrectly classify images. The incorrect classification of traffic signs and traffic lights could lead to a plethora of safety issues on the road. This research is aimed at the creation of two CNN models. One model's purpose is to detect and classify traffic signs. The other models objective is to do the same for traffic lights. The data collected from the two models showcase how accurately they classify the traffic signs and traffic lights.

162. A Mathematical Model of COVID-19: Efficacy of Vaccination with Heterogeneous Populations

Brandon Payne Dixie State University
Cesar Vasquez Dixie State University
Advisor(s): Vinodh Chellamuthu, Dixie State University

Infections from the novel coronavirus disease 2019 (COVID-19) remain superfluous as it continues to spread profusely across the world. Currently, there is no available vaccine to protect against COVID-19. As scientists work to develop a vaccine, our goal is to explore scenarios for different levels of vaccine-effectiveness and varying proportions of vaccinated-populations in order to mitigate the spread of COVID-19. We develop a mathematical model to analyze the disease dynamics of COVID-19 in relation to vaccine-effectiveness. Furthermore, we performed a data fitting algorithm to estimate parameters within the model to best resemble current infection trends using data from the CDC. Our simulation results determine possible best-case scenarios at varying degrees of vaccine-effectiveness and proportions of vaccinated-populations. Moreover, to account for the disease's varying infection and mortality rates based on an individual's age, we further partition the population by age groups to determine which groups are most vital to vaccinate. Our simulation also identifies the minimal required vaccine-efficiency for a given proportion of vaccinated individuals.

163. Zeros of Polynomials Generated by a Bivariate Contiguous Relation

Jack Luong California State University, Fresno
Advisor(s): Khang Tran, California State University, Fresno

Bivariate recurrence relations are of interest due to their application in various combinatorial problems. For example, they can be used to count the number of rook paths from one corner of an infinite chessboard to another corner. We study the zero distribution of a table of polynomials obtained from the bivariate recurrence relation $H_{m,n} + H_{m-1,n} + H_{m,n-1} + zH_{m-1,n-1} = 0$ with initial conditions $H_{0,0} = 1, H_{-1,n} = H_{m,-1} = 0$ for all natural numbers m and n. Equivalently, this table of polynomials is generated by $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} H_{m,n}t^m s^n = 1/(1+s+t+zst)$. We show that all zeros of any polynomial in this table are real. We also consider the more general class of recurrence relations generated by $R(s,t,z)/(1+s+t+zst)$ where $R(s,t,z)$ is a polynomial in s and t. In particular, we give an upper bound on the number of complex zeros of $H_{m,n}$ that depends only on the degree of $R(s,t,z)$.

164. Visualizing Toroidal Belyi Maps

Deion Elzie California Polytechnic State University Pomona
Mikaela Nishida Pomona College
Cameron Thomas Morehouse college
Advisor(s): Duane Cooper, Morehouse College

A Belyĭ map $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a rational function with at most three critical values; assumed to be $\{0,\ 1,\ \infty\}$. A Dessin d'Enfant is a planar bipartite graph obtained by considering the preimage of a path between the critical values, usually taken as the line segment from 0 to 1. Such graphs can be drawn on the sphere by composing with stereographic projection: $\beta^{-1}([0,1]) \subseteq \mathbb{P}^1(\mathbb{C}) \simeq S^2(\mathbb{R})$. Replacing $\mathbb{P}^1$ with an elliptic curve E, there is a similar definition of a Belyĭ map $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$. In this project, we use *Mathematica* to write code which takes an elliptic curve E and a Belyĭ map β to return the Dessin d'Enfant of this map—both in two and three dimensions. Following a 2013 paper by Cremona and Thongjunthug, we make the elliptic logarithm $E(\mathbb{C}) \simeq \mathbb{C}/\Lambda$ explicit using a modification of the arithmetic-geometric mean. Using this, we focus on several examples of Belyĭ maps which appear in the L-Series and Modular Forms Database (LMFDB). MSRI-UP 2020.

165. Effects of Temperature on Population Mobility and Spread of COVID-19

William Clark Dixie State University
Advisor(s): Vinodh Chellamuthu, Dixie State University

The COVID-19 outbreak has caused a global pandemic and compelled many to search for deeper understanding of how ecological and sociological factors influence the disease's spread. Many studies have developed models and shown the effects of social distancing, the efficacy of masks, and the effect of ambient temperature on the number of cases reported. Yet numbers continue to rise despite the regulations being imposed, which leads researchers to ask why, and what factors contribute to the spread of COVID-19? We propose that a significant cause of the recent spikes in positive cases has been the mobility of the population and the seasonal change in temperature. This study aims to identify the effect that temperature has on the mobility of a population and the spread of COVID-19. We developed a mathematical model presenting the dynamics of COVID-19 cases in the state of Utah with the incorporation of social distancing, mask efficiency, and the relationship between temperature and the rate of infection. Our results suggest that the temperature effect has on a population's mobility plays a significant role in the Covid-19 disease dynamics and the attempt to mitigate its spread.

166. Class Number of Almost Pythagorean Triples

Auroni Hashim Muhlenberg College
Advisor(s): Byungchul Cha, Muhlenberg College

For a fixed integer D, an almost Pythagorean triple is an integer triple (a,b,c) of the form $a^2 + b^2 = c^2 + D$. This presentation will focus on triples with a negative D, which we call the discriminant of (a,b,c). I will refer to these almost Pythagorean triples with a negative discriminant as APTs. Counting the number of certain equivalence classes of these APTs is analogous to Gauss's exploration of his class numbers in regards to integral binary quadratic forms. The process of reducing an APT requires an algorithm like Gauss's reduction theorem. Using this algorithm, I compute the class numbers of some small discriminants.

167. Dessin d'Enfants from Cartographic Groups

Nicholas Arosemena Morehouse College
Yaren Euceda University of Minnesota
Ashly Powell University of the Virgin Islands
Advisor(s): Edray Goins, Pomona

A belyi map is a morphism $\beta : S \to \mathbb{P}^1(\mathbb{C})$ of degree N defined on a compact connected Riemann surface S which is branched above $\{0,\ 1,\ \infty\}$. The associated Dessin d'Enfant $\Delta_\beta = \big(B \cup W,\ E\big)$ is that bipartite graph whose "black" vertices are $B = \beta^{-1}(0)$, "white" vertices are $W = \beta^{-1}(1)$, and edges $E = \beta^{-1}\big([0,1]\big)$. In this project, we work in the opposite direction. Say that we are given a triple $\big(\sigma_0,\ \sigma_1,\ \sigma_\infty\big)$ of permutations $\sigma_0,\ \sigma_1,\ \sigma_\infty \in S_N$ such that (i) $\sigma_0 \circ \sigma_1 \circ \sigma_\infty = \mathbf{1}$ and (ii) $G = \langle \sigma_0,\ \sigma_1,\ \sigma_\infty \rangle$ is a transitive subgroup of S_N. There exists a unique bipartite graph $\Delta \hookrightarrow S$ which can be drawn on a Riemann surface in such a way that its cartographic group is G. This project focused on drawing such Dessin d'Enfants when the Riemann surface has genus 1 or greater by focusing on examples which appear in the L-Series and Modular Forms Database (LMFDB).

168. A conjecture for generating the $f(m)$ of a SYT

Gabriel Chavez California State University Monterey Bay
Advisor(s): Lipika Deka, California State University Monterey Bay

Is there a generating function for the $f(m)$, the number of standard Young Tableaux (SYT) of shape , and major index equal to $m(\text{mod } n)$? Here m is the major index of an SYT, and n is the number of boxes in the SYT of shape . This is a research question posed by MIT Professor Richard P. Stanley in a list of open combinatorial bijective proof problems. Stanley claims that the only information we need to generate $f(m)$ is the shape and the $\gcd(m,n)$. The result is known to be correct but a combinatorial proof is still open. This poster shows that there exists a generating function for the shape with one row or one column, and L-shape as well as other similar shapes. In addition, we provide a function to generate $f(m)$ for any shapes with a prime number of boxes. We also discuss some beautiful symmetries within the set of SYT with n boxes whose major index is equal $m(\text{mod } n)$. This helps to understand the structure of the set of SYT with n boxes, but of shape with major index equal to $m(\text{mod } n)$ that provides important information to find the generating function for any shape.

169. Calculating Individual and Population Parameter Values in the Healing of Chronic Wounds through Mixed-Effects Modeling

Diksha Satish Western Kentucky University
Advisor(s): Richard Schugart, Western Kentucky University

In previous work, four differential equations were used to model the relationships between matrix metalloproteinases (MMP-1), their inhibitors (TIMP-1), and the extracellular matrix (ECM) using averaged patient data during the diabetic foot ulcer healing process. The patient data was acquired from a study that collected data on the concentration of these three factors from sixteen patients over the course of twelve weeks. The aim of this study is to curve-fit individual patient data using mixed-modeling effects. Mixed-effects splits each parameter into two parameters. The population parameter is the same for each patient representing an average patient response. An individual parameter represents the individual patient response and varies. Mixed-effects was implemented in MONOLIX, where its Stochastic Approximation Expectation-Maximization Algorithm is used to calculate population parameters, from which individual parameters were generated. Through MONOLIX, all sixteen patients were successfully curve-fit. Future work will include applying the generated parameter values in a sensitivity analysis to identify those that most affect an individual patient's healing response.

170. Counting Conjugacy Classes of Elements of Finite Order in Exceptional Simple Lie Groups

Qidong He Colby College
Advisor(s): Tamar Friedmann, Colby College

Given a Lie group G and $m, s \in \mathbb{N}$, let $N(G,m)$ be the number of conjugacy classes of elements of order m in G, and $N(G,m,s)$ the number of such classes whose elements have s distinct eigenvalues. In string theory, one can rephrase the problem of counting certain classes of vacua in the string landscape in terms of the study of these quantities. We

demonstrate that Burnside's Lemma can be used in conjunction with the Smith decomposition to manually compute $N(G, m)$, where G is an exceptional simple Lie group. This provides a combinatorial, alternative method to that of Djokovic and extends previous results obtained by Friedmann-Stanley for the classical Lie groups. We apply the same technique on a partially ordered set of lattices, extracted systematically from a maximal torus of G, to develop an efficient algorithm for computing $N(G, m, s)$.

171. A Computational Investigation of Ionic Transport and Gating due to Electrical Stimulation Treatments

Joshua Abston Roger Williams University
Advisor(s): Edward Dougherty, Roger Williams University

Electrical stimulation therapies have been proven to alleviate symptoms and debilitating effects of neurodegenerative disease. The biomedical community continues to make strides in identifying the cellular-level consequences of electrical stimulation. One such modulation of neuronal physiology is the alteration of resting membrane potential and ionic flux. The precise mechanisms by which these therapies operate, however, are not fully understood. To gain insight, we have implemented a mathematical model of neuronal ion transport based on the well-known Hodgkin-Huxley Equations. Using numerical experiments with this model, we quantitatively investigate cellular electro-physiological changes from simulated electrical stimulations including resting membrane potential and subsequent ionic flux. In addition, we extend this work to include an investigation of the impact and interaction that both fast and slow ion channels, as well as their gating variables, have under electrical stimulation conditions. Preliminary results demonstrate that electrical stimulation yields a significant modification in the transport of ionic species thought to play a fundamental role in disease pathogenesis.

172. Volume Formulas and Fractal Lawns

Joseph Ingenito The College of New Jersey
Peter Tonuzi The College of New Jersey
Advisor(s): Edward Voskanian, The College of New Jersey

In the theory of Fractal Geometry and Complex Dimensions one studies the inner-Minkowski dimension of a fractal string, and the asymptotic behavior of the inner-tubular volume of the boundary of a fractal string. We present a formula for the inner-tubular volume of a product of $n = 2, 3, 4$ fractal strings, and we conjecture a formula for any n.

173. Chip-Firing, Firing lattices, and Block Stacking

Ram Goel High school
Olly Milshtein Reed College
Advisor(s): David Perkinson, Reed College

A *divisor* D on a graph G is an assignment of a number of chips to each vertex of G. According to the rules of chip-firing, if the number of chips at a vertex v is at least the number of edges incident on v, then the vertex may be *fired*, sending one chip to each neighboring vertex and yielding a new divisor E. In that case, we write $D < E$. Fixing an initial divisor D on a locally-finite infinite graph, the *firing-lattice* associated to D is the transitive closure of $<$ on the set of all divisors reachable from D through a sequence of vertex-firings. We focus on the case where G is an infinite tree and describe the resulting lattices in terms of stackings of blocks. Special cases include Young's diagram and the lattice of shifted shapes (integer partitions with unequal parts). We also find the rank generating function for a natural divisor on a tree consisting of a finite number of infinite rays attached to a central vertex.

174. Assessing the Impact of Advanced Visualization Media on Student's Engagement in Learning Volumes of Revolution in an Active Learning Environment

Andrew Stanciulescu California State University East Bay
Advisor(s): Jesus Oliver, California State University

Student engagement is an important part of creating and sustaining an effective academic environment. Evidence-based methods to boost student engagement include pedagogical techniques such as active learning and group problem-based learning. This experiment involved implementing visualization media in an active learning Calculus 2 classroom. Data was collected on student self-efficacy, task attraction, and effort regulation—constructs closely linked to student engagement. The visualization media in question is an 18' HoyluTM Huddlewall projection system that aids small groups in employing problem solving techniques. The methodology included parsing students into two groups: traditional lecture followed by the technology focused design and the technology focused design followed by traditional lecture. Students report a significantly positive user experience in working with the technology. Though no difference was found in the two approaches, the positive changes between the lecture and the lab remained statistically significant in both groups. These results serve to document the benefits of visualization media in conjunction with active and group problem-based learning.

175. Simulations of Transcranial Electrical Stimulation with Variable Tissue Conductivities

Elizabeth Wexler Roger Williams University
Advisor(s): Edward Dougherty, Roger Williams University

Transcranial electrical stimulation (TES) is a medical treatment that is known to help mitigate the symptoms associated with a variety of brain disorders. Mathematical modeling can augment these treatments by providing a means of prognosticating electrical current propagation, with the goal of fine-tuning treatment parameters for patient-specific anatomies and medical conditions. While TES simulations are effective, the fact that precise tissue conductivities for a particular patient are unknown and remarkably variable limits the effectiveness of these simulations. With the goal of enhancing TES simulation capabilities for patient-specific applications, we have implemented a mathematical model of TES with stochastic partial differential equations. Our model is based on Laplace's equation and numerically solved using the finite element method; iterative simulations of variable tissue conductivity values are then executed. Computational simulations are run on both a generic two dimensional domain as well as MRI-based head and brain meshes. Results demonstrate the utility of integrating head cavity tissue conductivity variability in models and simulations of TES.

176. Generalizing the Explorer-Director Game

Jonah Nan University of Michigan, Ann Arbor
Angela Li The Ohio State University
Mohamed Lotfi Wagner College
Elaine Danielson University of Florida
Advisor(s): Patrick Devlin, Yale University

Consider a two-player game on a graph G with a token starting at a vertex v. Each turn, player one—Explorer—specifies a distance, then player two—Director—moves the token to a vertex the given distance away. Explorer aims to maximize the number of distinct vertices visited, while Director seeks to minimize this number. Let $f(G, v)$ be the number of vertices visited throughout the game when both players play optimally. This game was introduced in 2008 by Nedev and Muthukrishnan, who found $f(G, v)$ for cycle graphs. We prove novel upper and lower bounds for $f(G, v)$ for any graph along with exact formulas for families of graphs such as trees and square lattices. This work was done as part of the Polymath REU program.

177. Pecan: An Automated Theorem Prover

Reed Oei University of Illinois Urbana-Champaign
Dun Ma University of Illinois Urbana-Champaign
Zhengyao Lin University of Illinois Urbana-Champaign
Yikai Teng University of Illinois Urbana-Champaign
Advisor(s): Philipp Hieronymi, University of Illinois Urbana-Champaign

Pecan is an automated theorem prover for reasoning about *automatic sequences*, which are sequences that can be recognized by some (typically finite) automaton. Automated theorem provers and automatic sequences have diverse applications: in computer science, they are commonly used for program verification; in mathematics, they have found uses in logic, number theory, and combinatorics. Pecan is capable of proving any statement expressed in terms of Büchi automata and first-order logic connectives. We have used Pecan to prove many theorems about a special class of automatic sequences called *Sturmian words*, and we are currently exploring extensions including deciding sentences involving linear inequalities with integer and quadratic irrational coefficients, and visualization of fractals defined by

Büchi automata. This work is done in the research project “Pecan: An Automated Theorem Prover” at the Illinois Geometry Lab in 2019–2020.

178. A Mathematical Modeling Approach to Cardiovascular Health and Interventions

Kristen Norray Roger Williams University
Advisor(s): Edward Dougherty, Roger Williams University

Cardiovascular diseases remain the leading cause of death for both men and women globally as well as in the United States. Unlike other conditions, there are many well-known strategies beyond pharmaceutical therapies for addressing cardiovascular health including lifestyle habits like exercise, diet, and stress reduction. For an individual partaking in several cardiovascular strengthening strategies, at times it is unclear which approach, or collection of approaches, are the primary drivers of positive outcomes. To investigate these ideas, we have implemented a differential equations based mathematical model of the cardiovascular system and circulation. Using a compartmental modeling based approach, major aspects including circulation related to systemic, pulmonary, arterial, and venous pathways are incorporated, as well as external influences such as exercise and stress. The model is utilized to conduct simulations that investigate the effects that various intervening strategies have on circulatory system functioning, for both healthy and diseased states. Preliminary results show the efficacy of intervention approaches and are correlated to medical knowledge.

179. Transit Time Compactness

Preston Ward Tarleton State University
Advisor(s): Scott Cook, Tarleton State University

Political gerrymandering is a complex and pressing threat to our system of government. At the heart of our difficulties to fairly divide ourselves into voting district lies a math problem—how do we measure fairness? Traditionally, it has been thought that districts drawn with “pretty” boundaries are more fair than “ugly” shapes. This lead to several widely-used measures of “compactness” that focus on geographic shape. However, appearance often deceives; pretty shapes can hide gerrymandering and ugly shapes can be fair. So, we present a new measure called “transit-time compactness” that focuses on the cohesiveness of the citizens living inside a district rather the prettiness of its shape.

180. A Longitudinal Analysis of Ecological Systems Valuation in The Lake Chad Basin.

Nicholas Grabill Michigan State University
Yifei Li Michigan State University
Advisor(s): Frederi Viens, Michigan State University

Nigeria’s Yobe river drains the northern portion of Africa’s landlocked Lake Chad basin, providing waters to millions of people and livestock. The region’s people live in extreme poverty. This belies the rich environment which the lake and its basin provide for the agricultural economy. We track agricultural production and prices per state, to the highest accuracy given the paucity/unreliability of data. We compare them to the basin’s variable hydrology by year, including statewide rainfall data, and a hydrology reconstruction for lake levels. We find statistical evidence that the Yobe river provides robust ecosystem services across its basin, much of this activity originating from systematic informal irrigation. We also find evidence that Nigeria’s Borno state profits particularly from recessional agriculture. The region has the world’s highest birth rates, and Borno state, with a population of 5 million, harbors the world’s most vicious/unpredictable terrorist group, Boko Haram, with more than 2 million internally displaced victims. Despite this reality, Lake Chad and the Yobe River provide for millions of varied disciplines who adapt to their dynamic environment.

181. Optimizing EMS Responses During Extreme Events

Yangxinyu Xie University of Texas at Austin
Advisor(s): Ngoc Tran, University of Texas at Austin

Optimizing ambulance allocation and routing is one of the most efficient ways for the EMS to save more lives at virtually no cost. However, current EMS software was developed under models that assume normal demands. They are unable to adapt to disasters such as the COVID-19 pandemic, where traffic patterns change, case clusters emerge, and hospitals rapidly reach capacity. This project aims to create such an optimal EMS routing strategy using real-time information. By design, the proposed system can rapidly adapt to changing situations and is robust to disruptions.

It guarantees that ambulances arrive at scenes the fastest and distribute patients optimally among care facilities. In this presentation, we outline the first two stages of this project. Specifically, we provide a detailed analysis of the spaciotemporal distribution of the Austin Travis County EMS calls, the impact of the pandemic on EMS incidents and the current performance of the EMS department. We also investigate a count regression model, based on CDC's Social Vulnerability Index data, that identifies socioeconomic factors for EMS call distributions during normal and disaster times.

182. A Cyclic Variant of the Erdős-Szekeres Theorem

James Schmidt Michigan State University
Advisor(s): Bruce Sagan, Michigan State University

We will discuss the Erdős-Szekeres Theorem, which states that every linear permutation of size at least $rs+1$ contains either an increasing subsequence of length $r+1$ or a decreasing subsequence of length $s+1$ or both. Then, we will give a proof of a cyclic variant of this theorem, which states that every cyclic permutation of size at least $rs+2$ contains either an increasing subsequence of length $r+2$ or a decreasing subsequence of length $s+2$ or both.

183. Counting Optimal Strategies in the Penney-Ante Game

Reed Phillips Rose-Hulman Institute of Technology
Advisor(s): A. J. Hildebrand, University of Illinois at Urbana-Champaign

In the Penney-Ante game, Player I chooses a head/tail string of a predetermined length. Player II, upon seeing Player I's choice, chooses another head/tail string of the same length. A coin is then tossed repeatedly and the player whose string appears first in the resulting head/tail sequence wins the game. The Penney-Ante game has gained notoriety as a source of counterintuitive probabilities and nontransitivity phenomena. For example, Player II can always choose a string that beats the choice of Player I in the sense of being more likely to appear first in a random head/tail sequence. It is known that Player II has a unique optimal strategy that maximizes her winning chances in this game, in the form of a best response to any given sequence chosen by Player I. On the other hand, for Player I there exist multiple equivalent optimal strategies. We investigate the number of optimal strategies for Player I, i.e., the number of head/tail strings of a given length that maximize the winning probability for Player I assuming optimal play by Player II. We derive a recurrence relation for these counts and use this to obtain a sharp asymptotic estimate.

184. A Bound on Tableau Stabilization Using Lattice Paths

Jacob David Phillips Exeter Academy
Christopher Wu Westlake High School
Advisor(s): Suho Oh, Texas State University

A standard Young tableau is obtained when the cells of a Young diagram are filled with integers so that the entries are increasing along each row and column. Rectification is a transformation on a skew-shape standard tableau that, by sliding the cells around like the pieces of a 15-puzzle, results in a straight-shape standard tableau. If one attaches copies of a skew tableau to the right of itself by concatenating corresponding rows, after some point the entries only experience horizontal displacement under rectification, a phenomenon called tableau stabilization. Our purpose is to improve the original upper bound on the stabilization function to the number of rows of the skew tableau. To prove this bound, we use Greene's Theorem, which provides the shape of a rectified tableau in terms of increasing subsequences. We then encode increasing subsequences as lattice paths and show that various properties of these lattice paths force them into an optimal configuration. This arrangement, within the context of Greene's Theorem, proves the desired result.

185. Filtering cohomology of ordinary and Lagrangian Grassmannians

Ajmain Yamin Stony Brook University
Gandhar Kulkarni BITS Pilani
Matthew Fritz Yu University of California, Berkeley
Yuanning (Andy) Zhang University of California, Berkeley
Advisor(s): Victor Reiner, University of Minnesota Twin Cities

This paper studies, for a positive integer m, the subalgebra of the cohomology ring of the complex Grassmannians generated by the elements of degree at most m. We build in two ways upon a conjecture for the Hilbert series of this

subalgebra due to Reiner and Tudose. The first reinterprets it in terms of the operation of k-conjugation, suggesting two conjectural bases for the subalgebras that would imply their conjecture. The second introduces an analogous conjecture for the cohomology of Lagrangian Grassmannians. This work was done at the Polymath REU.

186. Safety of Radon Concentrations in Wisconsin Homes: A statistical Investigation

Leah Koepke University of Wisconsin-Eau Claire
Advisor(s): Mohammad Aziz, University of Wisconsin-Eau Claire

The purpose of this study is to determine if the radon concentrations in Wisconsin homes is safe overall and provide a best estimate of the radon level. In answering these questions, we investigate the approximate underlying true distribution of radon concentrations for Wisconsin homes. Several parametric and nonparametric statistical techniques are used for the preliminary analysis. To further investigate the relationship between predictor variables and radon concentration, multiple linear regression and logistic regression analysis are performed. Measurement factors, such as whether the home has a basement and whether the measurement was made there, are also discussed. From our results, a recently developed skew-t distribution is found to closely approximate the distributions of radon concentrations compared to a national finding best fitting lognormal distribution. We also observe that radon concentration is high in the basement compared to other floors, and the radon problem is severe in southern Wisconsin compared to other regions of the state. Finally, radon level in Wisconsin homes is found at a dangerous level and radon test is recommended for homes in Wisconsin.

187. Explicit Constructions of Finite Groups as Monodromy Groups

Ra-Zakee Muhammad Pomona College
Eyob Tsegaye Stanford University
Advisor(s): Duane Cooper, Morehouse College

In 1963, Greenberg proved that every finite group appears as the monodromy group of some morphism of Riemann surfaces. In this work, we give two constructive proofs of Greenberg's result. First, we utilize free groups, which given with the universal property and their construction as discrete subgroups of $\mathrm{PSL}_2(\mathbb{R})$, yield a very natural realization of finite groups as monodromy groups. We also give a proof of Greenberg's result based on triangle groups $\Delta(m,n,k)$. Given any finite group G, we make use of subgroups of $\Delta(m,n,k)$ in order to explicitly find a morphism π such that $G \simeq \mathrm{Mon}(\pi)$. This work was done as part of the Mathematical Sciences Research Institute Undergraduate Program (MSRI-UP).

188. Using Elastic Deformation Functions for a Segmentation Tool in Medical Imaging

Helen Piltner Georgia Southern University
Advisor(s): Reinhard Piltner, Georgia Southern University

In order to extract areas of interest within medical images, we need algorithms that assist in marking their boundaries. Our artificial deformations facilitate the process of medical image segmentation, allowing us to identify crucial information about organs or lesions displayed in an image. We create an oval-shaped area within a region of interest by generating points with several mouse clicks, which are then connected with Catmull-Rom splines. This initial boundary is then deformed to closely approach the real boundary of interest. The Navier equations, which are equilibrium equations written in terms of partial derivatives of displacements, are used for the deformation analysis. In our case, we have elastic features in a 2D domain. When working with cross sections of objects in medical images, we use the 2D case of the Navier equations for the displacement components. The Kelvinlets functions from Pixar animation studios satisfy the Navier equations and produce localized deformations in the vicinity of selected points. We proceed to use a CT image from an abdominal aortic aneurysm (AAA). By deforming the initial boundary, we obtain the shape of the AAA's cross section.

189. On the Evolutes of Rectifying Curves with Constant Curvature

Yaser Monterrey Andrews university
Advisor(s): Yun myung Oh, Andrews university

A rectifying curve is a curve whose position vector always lies in its rectifying plane. An evolute of a curve comprises the collection of all centers of curvature. We show some results for the evolutes of rectifying curves with constant curvature in R^3. In particular, we determine completely an evolute's curvature and torsion.

190. Involutes and Secondary Involutes of Space Curves in Minkowski Three-Space

Devin Garcia Andrews University
Advisor(s): Yun Myung Oh, Andrews University

The involute of a space curve is a curve whose tangent vector is orthogonal to the tangent vector of the original curve. Though originally studied in Euclidean space, recently the properties of involutes of space curves in Minkowski three-space have been investigated. We study the properties of the involute of a rectifying curve showing the involute is similar to a plane curve. We also define the secondary involute of a space curve and determine some of its properties in Minkowski three-space.

191. Counting k-Naples Parking Functions Through Permutations and the k-Naples Area Statistic

Christo Keller University of Massachusetts Amherst
Eunice Sukarto University of California Berkeley
Advisor(s): Laura Colmenarejo, University of Massachusetts Amherst

We recall that the k-Naples parking functions of length n are defined by requiring that a car which finds its preferred spot occupied must first back up a spot at a time before proceeding forward down the street. Note that the parking functions are the specialization of k to 0. For a fixed $0 \leq k \leq n-1$, we define a function φ_k which maps a k-Naples parking function to the permutation denoting the order in which its cars park. By enumerating the sizes of the fibers of the map φ_k we give a new formula for the number of k-Naples parking functions as a sum over the permutations of length n. We remark that our formula for enumerating k-Naples parking functions is not recursive, in contrast to the previously known formula of Christensen et al. We give a formula for the sizes of the fibers of the map φ_0, and we provide a recurrence relation for its logarithmic generating function. Furthermore, we relate the q-analog of our formula to a new statistic that we denote $\mathtt{area}_k$ and call the k-Naples area statistic, the specialization of which to $k = 0$ gives the $\mathtt{area}$ statistic on parking functions. Work from AIM UP REU.

192. Bernoulli Numbers and Class Number of Cyclotomic Fields

Marcella Manivel Carleton College
Margot Shaya Carleton College
Gavin Peng Carleton College
Marco Bommarito Carleton College
Advisor(s): Caroline Turnage-Butterbaugh, Carleton College

The Bernoulli numbers (B_n) are a sequence of rational numbers given by the coefficients in the generating function of $\frac{te^{xt}}{e^t-1}$. A prime number p is said to be B-irregular if p divides the numerator of at least one of the nonzero Bernoulli numbers up to B_{p-3}. Loosely speaking, the class number of an algebraic number field can be thought of as a measure of the extent to which unique factorization holds in its ring of integers. We discuss the connection between Bernoulli numbers and the class number of cyclotomic fields.

193. Okounkov Olshanski Terms of generalized zigzag shapes

Sunita Bhattacharya UMass Amherst
Advisor(s): Alejandro Morales, UMass Amherst

The classical hook-length formula from the 1950s, counting the number of standard young tableaux of straight shapes is one of the beautiful results in Enumerative Combinatorics. Unlike the straight shapes, there isn't an elegant product formula for skew shapes yet. Okounkov and Olshanski found a positive formula for enumerating the standard young tableaux of skew shapes in 1996. Only recently in 2014, Naruse introduced another formula to count the number of standard young tableaux of skew shapes as a positive sum over excited diagrams of products of hook-lengths. We compare both these formulas by studying their number of terms on special cases like border strip, rectangle, hook and zigzag shaped tableaux. It is known that number of terms for the latter shapes are given by the Genocci and Catalan numbers respectively. We look at generalizations of this result for variations of zigzag shapes.

194. Clique Structure of Orthomodular Posets

Gregory Li Alabama School of Math and Science
Advisor(s): Ethan Sussman, MIT

Dacey established a class of simple, finite graphs, called Dacey graphs (or D-graphs), which maps bijectively to the set of finite orthomodular posets, posets where the join of orthogonal elements always exists and satisfy constructive properties. Orthomodular posets are important in the study of quantum logics and quantum experimentation. A graph is Dacey if the maximal cliques intersect coherently under Dacey's mapping. We begin an enumeration of D-graphs by presenting methods of partitioning D-graphs into equivalence classes based on the structures of maximal clique intersections and characterizing D-graphs which are edge-covered by m maximal cliques. This leads to a complete classification of D-graphs for $m \leq 4$ and examples of small orthomodular posets not commonly found in literature. We then explore the implications of applying the conditions of D-graphs as local conditions to show each D-graph is "locally" Dacey and investigate of bounds of such an approach.

195. Predicting the Utilization of Mental Health Treatment with Various Machine Learning Algorithms

Meera Sharma Western Connecticut State University
Sonok Mahapatra Western Connecticut State University
Adeethyia Shankar Western Connecticut State University
Advisor(s): Xiaodi Wang, Western Connecticut State University

Mental health is an essential and integral component of the overall health of a person. For 2017, it is estimated that 792 million people lived with a mental disorder. This is about 10% of the global population. Furthermore, in these unprecedented times of COVID-19, mental health challenges have been even further exacerbated. Diagnosis and treatment of people with mental health disorders remains challenged partly due to the public stigma attached to mental health. Many people are reluctant to seek help especially early in the detection phase. Big data and artificial intelligence can help alleviate that problem. Current research and efforts in this field suffer from challenges in data privacy, data security, high need of medical experts and diagnosis may not lead people to seek treatment.

196. Equality in the Eisenbud–Goto Conjecture for Certain Toric Varieties

Vijay Srinivasan University of Chicago
Alan Peng Massachusetts Institute of Technology
Preston Cranford Massachusetts Institute of Technology
Advisor(s): Christine Berkesch, University of Minnesota, Twin Cities

A projective variety can be thought of as the set of solutions in a projective space to a system of homogeneous polynomial equations. How can we measure how "complicated" such a variety is? A quantity called the Castelnuovo–Mumford regularity, defined in terms of free resolutions of the coordinate ring, is one such measure of complexity. In 1984, Eisenbud and Goto conjectured an upper bound on the Castelnuovo–Mumford regularity of a nondegenerate projective variety. It is an open question to determine whether this bound holds even for the special class of varieties called toric varieties. We will discuss combinatorial techniques that can be used to approach this question, and give a combinatorial characterization of when the Eisenbud"Goto bound is achieved for most toric varieties of codimension 2. This research was conducted at the 2020 University of Minnesotia, Twin Cities REU.

197. Data analysis and visualizations of drosophila behavioral phases

Xiaona Zhou New York City College of Technology, CUNY
Advisor(s): Boyan Kostadinov, New York City College of Technology, CUNY; Maria de la Paz Fernández, Barnard College, Columbia University; Orie T. Shafer, Neuroscience Initiative, ASRC, CUNY

We analyze drosophila behavioral data from the neuroscience labs of Prof. Maria de la Paz Fernandez, Barnard College of Columbia University, and Prof. Orie Shafer, Advanced Science Research Center, CUNY. The main objective of this project is to create algorithms for analyzing and visualizing the average activity of drosophila across a specified number of days and across all live flies, and use this analysis to calibrate a smoothing filter to be applied to the raw fly activity so that the drosophila behavioral phases can be computed and visualized. In addition, we investigate how to compute and visualize the onset and offset of behavioral phases as well as the trend-lines of mid-day fly activity. We are also developing an interactive Shiny App that contains all the data analysis and visualizations.

198. Counting Spiders on Trees

Don Edwards Morehouse College
Kobe Lawson Morehouse College
Advisor(s): Duane Cooper, Morehouse College

An ordered tree, also known as a plane tree or planar tree, is defined recursively as having a root and a ordered set of subtrees. One characteristic of ordered trees are spider legs leading to complete spiders. A spider leg is a path from the leaf to the root where a spider is a combination of spider legs that only intersects at the root. In this paper we will count spiders on various types of trees.

199. Optimality of Helices on Some 6-D Lie Groups

Yutong Qing University of Chicago
Luis Carlos Soldevilla University of Toronto
Advisor(s): Andy Borum, Cornell University

We consider a family of geometric optimal control problems on the six-dimensional Lie groups $SE(3)$, $SO(4)$ and $SO(1,3)$. We show that some extremals correspond to helices in the 3-D spaces $\mathbb{R}^3$, $\mathbb{S}^3$, and $\mathbb{H}^3$, and use necessary conditions for optimality to present an explicit parameterization of all of these helical extremals. Additionally, we prove that most of these helices are locally optimal for a finite length and we use Jacobi's sufficient condition to compute this critical length. Finally, we derive a scaling property relating the length at which helices lose optimality and the twisting rate along the helix, and we use this to compute and visualize the boundary between optimal and non-optimal helices. Project done as part of the SPUR Program at Cornell University

200. Robust and Efficient Phase Retrieval from Magnitude-Only Windowed Fourier Measurements

Penelope Fiaschetti Boston University
Jessica Bennett Brown University
Advisor(s): Aditya Viswanathan, University of Michigan - Dearborn

We propose and analyze a new generalization of an existing algorithm to reconstruct a complex vector (up to a global phase factor) from squared magnitude of its windowed discrete Fourier transform. This is also referred to as phase retrieval problem, since this process requires the recovery of critically important phase information from magnitude-only measurements. The proposed algorithm utilizes results from discrete Fourier analysis to linearize the governing equations and obtain a highly structured Fourier based linear system. The linear system of equations can be efficiently inverted using the fast Fourier transform algorithm. This provides relative phase information which we use to construct a special class of banded matrices, on which we perform spectral analysis to retrieve individual phase information. In addition to developing efficient reconstruction algorithm, we provide mathematically rigorous theoretical error bounds in the case of noisy measurements, and provide numerical simulations demonstrating that this algorithm is computationally efficient and able to recover data in the presence of noise. The research was done as part of REU at University of Michigan-Dearborn.

201. Generating rational distance sets on a parabola using Pythoagorean triplets

Sayak Bhattacharjee Indian Institute of Technology Kanpur (IITK)
Advisor(s): Santosha Pattanayak, Indian Institute of Technology Kanpur (IITK)

A rational point-distance set is a set of points with rational coordinates such that each of its pairwise distances are rational. We study the problem of finding N-point rational point-distance sets on the parabola $y = x^2$. Our approach establishes a correspondence between the solutions and sets of Pythagorean triplets, and studies the existence and nature of the solutions for general N from a linear algebraic point-of-view. Moreover, we show the density of the solutions (and show density for $N = 2$ and 3), which stands to be of interest in light of the Erdős-Ulam conjecture. Finally, we concentrate on the solutions obtained computationally for $N = 3$ and perform enumeration and classification. This leads to interesting novel results about the spatial distribution of the solutions on the parabola which in turn provides perspective on the structure of the set of Pythagorean triplets.

202. Permutation binomials of index $q^{e-1} + \cdots + q + 1$ over $\mathbb{F}_{q^e}$

Javier Santiago University of Puerto Rico at Río Piedras
Advisor(s): Ariane Masuda, New York City College of Technology

We denote the finite field of order q by $\mathbb{F}_q$. A polynomial in $\mathbb{F}_q[x]$ is a *permutation polynomial* over $\mathbb{F}_q$ if it induces a permutation of $\mathbb{F}_q$. Binomials of index $q^{e-1} + \cdots + q + 1$ over $\mathbb{F}_{q^e}$ with $e \geq 2$ are of the form $x^r(x^{q-1} + a)$ where $a \in \mathbb{F}_{q^e}^*$. In this paper we present several existence and nonexistence results for permutation binomials over $\mathbb{F}_{q^e}$ of this type. As a consequence, we obtain a complete characterization of such permutation binomials over $\mathbb{F}_{q^2}, \mathbb{F}_{q^3}, \mathbb{F}_{q^4}$, $\mathbb{F}_{p^5}$, and $\mathbb{F}_{p^6}$ where p is an odd prime. This extends previous results obtained by Li et al. and Liu for $e = 2$ and $e = 3$ with odd q, respectively.

203. Involutions of F_q Obtained by Binomials of the form $x^m(x^{\frac{q-1}{2}} + a)$

Lillian Gonzlez -Albino University of Puerto Rico, Río Piedras
Advisor(s): Ivelisse Rubio, University of Puerto Rico, Río Piedras

Polynomials over finite fields that permute the elements of the field are called **permutation polynomials**. Permutations over finite fields have many applications ranging from cryptography, and combinatorics to theory of computation. For many of these applications, it is important to find permutations with a small memory footprint that are easy to implement. A good option is to use involutions generated by polynomials; **involutions** are functions that are their own inverse. In 2017, Castro et al. gave explicit formulas for monomial involutions over $\mathbb{F}_q$ and their fixed points. In 2018, Zheng et al. characterized involutions of the form $x^m h(x^s)$ over $\mathbb{F}_q$, but an explicit formula for m and the amount of fixed points was not given. Our focus for this talk will be on explicit formulas for both involutions over $\mathbb{F}_q$, where q is odd, of the form $x^m(x^{\frac{q-1}{2}} + a)$ and for their fixed points.

204. Generalizing Differences of Bijections

Andres Ramos-Rodriguez University of Puerto Rico, Río Piedras
Advisor(s): Ivelisse Rubio, University of Puerto Rico, Río Piedras

In a recent article, Ullman and Velleman studied functions **a** from an abelian group G to itself that can be expressed as a difference of two bijections **b**, **c** from G to itself, and presented connections to directed graphs, juggling sequences, bus scheduling and Latin Squares. In this work we relax the condition that **b** and **c** are bijections and instead study which functions can be expressed as the difference of two functions whose image as a *multiset* is the same. We classify which sequences $\mathbf{a} : G \longrightarrow G$ can be a expressed in such a way. Also, given a function **a**, we show how to construct all possible **b**, **c** such that **b** and **c** have the same image as multiset and $\mathbf{a} = \mathbf{b} - \mathbf{c}$.

205. Maximum Distance Codes over $\mathbb{Z}_4$

Najalia Singh Valley Stream Central High School
Advisor(s): Ariane Masuda, New York City College of Technology of CUNY

The parameters of Error-Correcting Codes are important in the transmission of messages through noisy channels. A code with a large minimum distance increases its reliability as more errors can be detected and corrected. Over the past decades researchers have given special attention to codes over rings of order 4. An open problem is to find and classify maximum codes over these rings with respect to certain distances. I will present the computational explorations that I did in this direction, using the Lee and Euclidean distances in $\mathbb{Z}_4$.

206. Using Topology to Analyze Antarctic Firn Samples

Erin Cronce College of William & Mary
Advisor(s): Sarah Day, College of William & Mary

Firn refers to partially compacted snow whose micro-CT scans researchers can use to determine past climate patterns and atmospheric activity. However, one of the main challenges in the field is to distinguish the ice from the interconnected pore space in the scan. We implemented tools from Topological Data Analysis, a novel area in applied mathematics, in our study of several hundred cross-sectional two-dimensional scans of an Antarctic firn column. In this talk, we will discuss the use of topological methods such as thresholding and persistent homology in interpreting firn scans of different resolutions. These topological methods have proven effective in studying scans and extracting

distinct components to represent the complete firn sample, focusing on preserving the image's most persistent features. Climate scientists can in turn apply these topological techniques in order to obtain a clearer representation of firn samples and thus more accurately interpret climate records. This is a joint work with Yu-Min Chung (UNCG), Sarah Day (W&M), and Kaitlin Keegan (UNR).

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