A B S T R A C T S

of Papers

presented at

MathFest 2009

Portland, Oregon
August 6–8, 2009

Abstracts of Papers Presented at

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Contents

Invited Addresses

Earl Raymond Hedrick Lecture Series

Ravi Vakil Stanford University

Classical Structure in Modern Geometry, or Modern Structure in Classical Geometry

One of the beauties of mathematics is the fact that many themes run through the subject, over many centuries. Many classical ideas continue to bear fruit in modern contexts, and modern ideas can still shed new light on classical problems. The Hedrick series will explore this in geometry. This series is intended for a general mathematical audience, and the talks will be independent.

Lecture 1: The Mathematics of Doodling
Thursday, August 6, 10:30–11:20 AM

Doodling has many mathematical aspects: patterns, shapes, numbers, and more. Not surprisingly, there is often some sophisticated and fun mathematics buried inside common doodles. I'll begin by doodling, and see where it takes us.

Lecture 2: Murphy's Law in Geometry
Friday, August 7, 9:30–10:20 AM

When mathematicians consider their favorite kind of object, the set of such objects often has a richer structure than just a set often some sort of geometric structure. For example, it may make sense to say that one object is "close to" another. As another example, solutions to equations (or differential equations) may form manifolds. These "moduli spaces" often are hoped to behave well (for example be smooth). I'll explain how many ones algebraic geometers work with are unexpectedly as far from smooth as they possibly can be.

Lecture 3: Generalizing the Cross Ratio: The Space of n Points on the Projective Line
Saturday, August 8, 9:30–10:20 AM

Four ordered points on the projective line, up to projective equivalence, are classified by the cross ratio, a notion introduced by Cayley in the nineteenth century. This theory can be extended to more points, leading to one of the first important examples of an invariant theory problem, studied by Kempe, Hilbert, and others. Instead of the cross ratio (a point on the projective line), we get a point in a larger projective space, and the equations necessarily satisfied by such points exhibit classical combinatorial and geometric structure. For example, the case of six points is intimately connected to the outer automorphism of S_6. Much of the talk will be spent discussing the problem, and an elementary graphical means of understanding it. This is joint work with Ben Howard, John Millson, and Andrew Snowden.

MAA Invited Address

Thursday, August 6, 8:30–9:20 AM

Alan Taylor Union College

Predicting Values of Arbitrary Functions

To what extent is a function's value at a point x of a topological space determined by its values in an arbitrarily small (deleted) neighborhood of x? For continuous functions, the answer is typically "always" and the method of prediction of $f(x)$ is just the limit operator. Chris Hardin and I generalized this observation on limits to the case of an arbitrary function mapping a topological space to an arbitrary set, and showed that the best one can ever hope to do is to predict correctly except on a scattered set. Moreover, we produced a predictor whose error set is always scattered. In this talk, we outline the proofs of these two theorems and then derive some of the main results from our two earlier papers, "An introduction to infinite hat problems" (Mathematical Intelligencer, 2008) and "A peculiar connection between the axiom of choice and predicting the future" (American Mathematical Monthly, 2008). In particular, we show that given the values of a function on an interval $(-\infty, t)$, the strategy produces a guess for the value of the function at t and this guess is correct except for a countable set that is nowhere dense. In this sense, if time is modeled by the real line, then the present can almost always be correctly predicted from the past.

MAA Invited Address

Thursday, August 6, 9:30–10:20 AM

Steven Strogatz Cornell University

The Mathematics of Collective Synchronization

Every night along the tidal rivers of Malaysia, thousands of male fireflies congregate in the mangrove trees and flash on and off in silent, hypnotic unison. This display extends for miles along the river and occurs spontaneously; it does not require any leader or cue from the environment. Similar feats of synchronization occur throughout the natural world, whenever large groups of self-sustained oscillators interact. This lecture will provide an introduction to the Kuramoto model, the simplest mathematical model of collective synchronization. Its analysis has fascinated theorists for the past 35 years, and involves a beautiful interplay of ideas from nonlinear dynamics, statistical physics, and fluid mechanics. Classic results, recent breakthroughs, and open problems will be discussed, and a video of synchronous fireflies will be shown.

MAA Invited Address

Saturday, August 8, 8:30–9:20 AM

Greg Warrington University of Vermont

Statistics in Algebraic Combinatorics

A central tension in mathematics is knowing how much to forget. Retain too many properties and the conjecture is not true. Lose too much structure and there is nothing meaningful to say. A variation of this balance is especially evident in algebraic combinatorics; oftentimes the objects of study are shadows of deep algebraic and geometric constructs.

The association of statistics (i.e., weights) to simple combinatorial objects lets us recover some of the deeper structure. For example, permutations index a class of geometric objects known as Schubert varieties. By recording the number of inversions of a permutation we obtain the dimension of the corresponding variety.

In this talk I describe some statistics on familiar combinatorial objects such as permutations, lattice paths and partitions. These statistics can be appreciated for the beautiful identities they satisfy and the surprising relationships among them. I will illustrate both qualities with examples. However, such statistics can also serve to illuminate the theory of symmetric functions. I will describe several situations where the underlying algebra suggests we should be able to find statistics satisfying certain properties. In a few cases such statistics have been found/invented; in other cases we are still looking.

MAA Invited Address

Saturday, August 8, 10:30–11:20 AM

Alice Silverberg University of California at Irvine

Cryptography: How to Keep a Secret

When you send your credit card number over the Internet, cryptography helps to ensure that no one can steal the number in transit. Julius Caesar and Mary Queen of Scots used cryptography to send secret messages, in the latter case with ill-fated results. More recently, cryptography is used in electronic voting, and it is also used to "sign" documents electronically. While cryptography has been used for thousands of years, public-key cryptography dates only from the 1970's. Some recent exciting breakthroughs in public-key cryptography include elliptic curve cryptography, pairing-based cryptography, and identity-based cryptography, all of which are based on the number theory of elliptic curves. This talk with give an elementary introduction to cryptography, including elliptic curves and pairing-based cryptography.

MAA Lecture for Students

Thursday, August 6, 1:00–1:50 PM

Colm Mulcahy Spelman College

Mathemagic with a Deck of Cards on the Interval Between 5.700439718
and 80658175170943878571660636856403766975289505440883277824000000000000

Some unavoidable coincidences—as well as some truly surprising ones—will be explored as we survey 21^{st}-century mathemagical creations/entertainments with a deck of cards, touching on topics in combinatorics, algebra, and probability.

James R. Leitzel Lecture

Friday, August 7, 10:30–11:20 AM

Joan Ferrini-Mundy Michigan State University and the National Science Foundation

Communicating Among Communities and Calling for Change: Continuing the Improvement of Mathematics Education

Almost two decades ago Jim Leitzel's vision for the continued improvement of mathematics education called for communication among mathematicians, educational researchers, teacher educators, and others. Collaborations among stakeholders with diverse perspectives are central to many of today's major mathematics education initiatives. What shared commitments have emerged as most promising for improving mathematics learning? What is the role of undergraduate mathematics education, mathematics education research, and the mathematical education of teachers in addressing problems of national scope and urgency? A discussion of the challenges and opportunities in the current federal policy climate for continuing to call for change in mathematics education.

NAM David Blackwell Lecture

Friday, August 7, 1:00–1:50 PM

Edray Goins Purdue University

Why Should I Care About Elliptic Curves?

An elliptic curve E possessing a rational point is an arithmetic-algebraic object: It is simultaneously a nonsingular projective curve with an affine equation $y^2 = x^3 + A\,x + B$, which allows one to perform arithmetic on its points; and a finitely generated abelian group $E(\mathbb{Q}) \simeq E(\mathbb{Q})_{\mathrm{tors}} \times \mathbb{Z}^r$, which allows one to apply results from abstract algebra.

In this talk, we discuss some basic properties of elliptic curves, and give applications along the way.

Pi Mu Epsilon J. Sutherland Frame Lecture

Friday, August 7, 8:00–8:50 PM

Persi Diaconis Stanford University

The Mathematics of Perfect Shuffles

Magicians and gamblers can shuffle cards perfectly (demonstrations provided). Understanding what can (and cannot) be done with shuffles leads to math problems, some beyond modern mathematics. The math is also useful for describing all sorts of computer algorithms.

AWM-MAA Etta Z. Falconer Lecture

Friday, August 7, 8:30–9:20 AM

Kate Okikiolu University of California at San Diego

The Sum of Squares of Wavelengths of a Closed Surface

For the Laplacian on a closed manifold, we define a spectral invariant which is heuristically the sum of squares of the wavelengths which is a regularized trace of the inverse of the Laplacian. On a technical level, this is an analog for surfaces of the ADM mass from general relativity. We discuss a negative mass theorem for surfaces of positive genus, and give a probabilistic interpretation.

Invited Paper Sessions

History of Mathematics

Thursday, August 6, 1:00–3:20 PM

Fernando Gouvea (FQGOUVEA@COLBY.EDU) Colby College

Emilia's Arithmetic: A Brazilian Intellectual Tackles Mathematics Education

In the first half of the twentieth century Monteiro Lobato was one of Brazil's most prominent public intellectuals. Around 1930, he set out to write a sequence of children's books treating various subjects from the school curriculum. After doing one on language and grammar, he tackled arithmetic. We will take a look at the results and ponder what they tell us about mathematics education.

Charlotte Simmons (cksimmons@uco.edu) University of Central Oklahoma

Augustus De Morgan: The Man Behind the Scenes

Augustus De Morgan was a nineteenth century mathematician whose contributions to mathematics are not fully appreciated by historians. "[C]ertainly he made no lasting original contribution to mathematics," says Smith. This talk will explore the significant contributions made by De Morgan from "behind the scenes." De Morgan was a close friend and correspondent of Sir William Rowan Hamilton and George Boole, two of the greatest algebraists of the nineteenth century. He significantly impacted both of their careers, and it is doubtful that they would have attained the level of success that they ultimately achieved without his help. We also explore his contributions to the field of actuarial science, both direct (via his own papers) and indirect (via his friendship and support of actuary pioneer Benjamin Gompertz). If he had done nothing else noteworthy in mathematics besides supporting the efforts of these three men, we would still owe him a great debt.

Jim Tattersall (tat@providence.edu) Providence College

Mathematical Queries from Late Nineteenth Century American Publications

We focus on the mathematical columns that appeared in the *American Mathematical Monthly, Mathematical Visitor, Mathematical Magazine*, and *Yates County Chronicle* in the period from 1870 to 1900. We briefly discuss the history of the mathematical departments in each publication before illustrating a few of the problems that were contributed by readers.

Janet Beery (janet_beery@redlands.edu) University of Redlands

Sums of Powers of Integers

Fermat's and Pascal's work on sums of powers of integers is relatively well known, while that of Thomas Harriot (*c.* 1560-1621) and Johann Faulhaber (1580-1635) remains little known. In this presentation, we review the history of sums of integer powers from Pythagoras (6th century BCE) through Jakob Bernoulli (1654-1705), focusing on Harriot's and Faulhaber's contributions.

Stacy Langton (langton@sandiego.edu) University of San Diego

Newton's Theory of the Speed of Sound

As an application of his principles of mechanics, Newton attempted, in the second book of the *Principia,* to calculate the speed of sound. The value he arrived at was about 15% too low. Today we can understand this discrepancy by noting that Newton had assumed, in effect, that sound waves were isothermal. Laplace proposed a different assumption in 1816: that sound waves are not isothermal but, as we now say, adiabatic. The resulting theory agrees well with measured values of the speed of sound. So Newton made a wrong assumption about the thermodynamic nature of sound waves. But had he made Laplace's correction, he would have obtained the correct value for the speed of sound. It would seem, therefore, that the rest of his theory must be more or less correct. But what Newton writes is very difficult to understand (at least for me!). He does not seem to be applying the basic principles that he stated in the first book of the *Principia*—in particular, the famous "Second Law". So what principles is his treatment actually based on, and how does it work? In this talk I will try to disentangle these questions.

Gems of Combinatorics

Friday, August 7, 3:30 PM–5:30 PM

Naiomi Cameron (ncameron@lclark.edu) Lewis & Clark College

Combinatorial Enumeration via the Riordan Group

The Riordan group consists of infinite lower triangular matrices whose columns are determined by two generating functions. After providing some background on this interesting combinatorial structure, I will present some lattice path enumeration and determinant evaluation problems where the Riordan group has proven useful.

Joe Buhler (jpb@ccrwest.org) Center for Communications Research

Juggling Bijections

Bijections arising from the mathematical theory of juggling patterns will be described, proved, and performed, as appropriate.

Gregory Warrington (gwarring@gmail.com) University of Vermont

A Combinatorial Variation of Sylvester's Four-Point Problem?

Sylvester's Four-Point Problem asks for the probability that four points chosen at random from a region R have a quadrilateral as their convex hull. If R is convex, the answer has bee shown to range between 2/3 and 0.704, depending on the particular region R. We present a combinatorial version of this problem involving permutations for which the answer appears to be exactly 3/4.

Ravi Vakil (vakil@math.stanford.edu) Stanford University

A New Description of the Outer Automorphism of S_6

I'll give a (hopefully) new and (hopefully) fun and (literally) colorful description of the outer automorphism of S_6, and describe how it connects to other descriptions.

The Mathematics of Poker

Saturday, August 8, 1:00–3:30 PM

Brian Roger Alspach (brian.alspach@newcastle.edu.au) University of Newcastle

Three Mathematical Gems Arising from Poker

There are many mathematical questions that arise in poker. This can provide some motivation for students to study certain topics. We discuss three problems that involve group theory, recurrence relations, and inclusion-exclusion.

David Bachman (bachman@pitzer.edu) Pitzer College

Convergence of Discrete Poker Endgame Models

Poker endgames are classically analyzed by making one of two simplifying assumptions: either the deck is small (e.g. three cards or less), or the deck is continuous. It is not possible to analyze large finite decks by hand, but with the aid of computers we can investigate the convergence of the discrete models to the continuous ones. This justifies the use of the continuous models to inform decisions in "real poker," where the number of possible hands is large, but finite.

Thomas S. Ferguson (tom@math.ucla.edu) UCLA

Variations on Basic-End-Game in Poker

Basic-End-Game in poker is a game-theoretical model of a situation that sometimes occurs in poker in the last round of betting when there are just two players left. It is simple enough for a complete analysis, yet the analysis shows some surprising features. However, being a mathematical model, some simplifying assumptions are made that may not be satisfied is realistic situations. For example, in a bluffing situation, the model assumes that both players know the exact probability the bluffer has a winning hand. But the players may have personal information given by the cards that lead to different assessments of the probability of winning. Worse, one of the players may be giving away information inadvertently, through his nervousness for example, that his opponent may be able to use. One may analyze this as a game, but if a person does not know how he is giving away information, or even that he is giving away information at all, it should be analyzed as a pseudo-game, that is, as a game in which one of the players does not know all the rules of the game. Here the nervous player doesn't know he is giving away information. If the game is played repeatedly, how should his opponent best take advantage of the situation? And what should the nervous player do to minimize the effects of the information leak?

Bill Chen The Susquehanna International Group
Jarrod Ankenman Brown University

Bet Sizing for No Limit

We will explore the optimal bet sizes to make in NL games, and the solution methods that we use. We will start with the [0,1] full river game and see how the strategy changes for various other games. The drawback of making multiple bet sizes is that you bifurcate your distribution and thus you become more predictable, however it has been shown that in certain cases the gain in being able to make different bet sizes outweighs this.

Steven Bleiler (bleilers@pdx.edu) Portland State University

Quantized Poker

Poker has become a popular pastime all over the world. At any given moment one can find tens, if not hundreds, of thousands of players playing poker via their computers on the major on-line gaming sites. Indeed, according to the Vancouver, B.C. based pokerpulse.com estimates, more than 190 million US dollars daily is bet in on-line poker rooms. But communication and computation are changing as the relentless application of Moore's Law brings computation and information into the quantum realm. Classical games when played in this coming quantum computing environment or if played with quantum information become "quantized" and in almost all cases, the "quantized" versions of these games afford many new strategic options to the players. The study of these so-called quantum games is quite new, arising from a seminal paper of D. Meyer published in Physics Review Letters in 1999. The ensuing near decade has seen an explosion of contributions and controversy over what exactly a quantized game really is and if there is indeed anything new for game theory. With the settling of some of these controversies it is now possible to fully analyze some of the basic endgame models from the game theory of Poker and predict with confidence just how the optimal play of Poker will change when played in the coming quantum computation environment. The analysis presented here shows that for certain players, "entangled" poker will allow results that outperform those available to players "in real life".

Open and Accessible Problems in Knot Theory

Session 1: Friday, August 7, 10:30–11:50 AM

Matthew E. DeLong (mtdelong@taylor.edu) Taylor University

Knot Colorability: Undergraduate Research using Crayons, Computers and Linear Algebra

An easily accessible invariant that can sometimes be used to distinguish knots is colorability. Colorability, as its name suggests, can be investigated with crayons by knot theorists of any age and experience. Colorability also has an algebraic basis, and it is closely related to another knot invariant, the determinant of knot. Determinants of knots can be investigated, by hand or with computers, by undergraduates who know a little linear algebra. It is known how the determinants of certain classes of knots behave, for example how the determinant of an (m, n) torus knot depends upon m and n. There are other, more complicated, classes for which it is not completely known how the determinant behaves. One such class consists of the (m, n, r, s) twisted torus knots. In this talk I will report on the work of two undergraduate researchers who investigated the determinants of certain families of twisted torus knots. I will explain how they used MATLAB to form conjectures and how they subsequently used Mathematica to assist them in working out the proofs of these conjectures. The proof techniques that the students developed were original but elementary, and could be used on other classes of knots for which the behavior of the determinant is still unknown.

Ramin Naimi (raynonym@yahoo.com) Occidental College

Lots of New Intrinsically Knotted Graphs via Computer Programs

A graph is said to be intrinsically knotted (IK) if every embedding of it in 3-space contains a nontrivial knot. I will describe recent work done jointly with students and colleagues in the field of IK graphs, as well as projects that would be great for undergraduate students to work on. We have written various computer programs that have helped us discover lots of new IK graphs, as well as counterexamples to some questions. As the graphs get larger, however, the programs slow down and reach their limits of usefulness, at which point we need to write faster programs, find new tricks, or prove new theorems.

Thomas Mattman (TMattman@CSUChico.edu) California State University, Chico

Knotted Graphs and Boundary Slopes

We say that a graph is intrinsically knotted if every embedding of the graph has a knotted cycle. By work of Robertson and Seymour, intrinsic knotting is determined by a finite list of minor minimal knotted graphs. However, determining that list remains difficult. This leaves many open problems including classification of graphs on 9 vertices, bipartite graphs on 11 or 12 vertices and, more generally, questions about multipartite or other types of graphs. Marc Culler has proposed A-polynomials for several 3-braids. This leads to a listing of the boundary slopes of those knots. It would be very useful to have a description of the surfaces in a 3-braid that give rise to these boundary slopes.

Joel Foisy (foisyjs@potsdam.edu) SUNY Potsdam

Linking Graph Theory and Knot Theory

We say a graph is intrinsically linked if it contains, in every embedding into 3-space, two disjoint cycles that form a non-split link. In the early 1980's, Conway-Gordon and Sachs proved that the complete graph on 6 vertices is intrinsically linked. Since then, there have been many new related results, many of which involved research done by undergraduates. We will discuss some of these related results, including some that generalize the idea of intrinsically linked graphs to embeddings into other spaces.

Lew Ludwig (ludwigl@denison.edu) Denison University

Link and Knots in Complete Graphs with Linear Edges

In 1983 Conway and Gordon and Sachs proved that every embedding of the complete graph on six vertices, K_6, is intrinsically linked. In the same article, Conway and Gordon showed that the complete graph on seven vertices, K_7, is intrinsically knotted. In this presentation we discuss my work with undergraduates students classifying the number of links and knots in straight-edge embeddings of K_7. We also consider whether straight-edge embeddings of K_9 are triple linked, extending the work of Flapan, Naimi, and Pommersheim. Straight-edge embeddings are of interest to molecular chemists who try to synthesis knotted and linked molecules. A number of open questions will be posed. No prior knowledge of knot theory or graph theory is assumed for this presentation.

Session 2: Friday, August 7, 1:00–3:30 PM

Kenneth C. Millett (millett@math.ucsb.edu) University of California

Knots, Ephmeral Knots and Slipknots in Random Walks and Equilateral Polygons

Diao, Pippenger, Sumners, and Whittington proved the Delbruck-Frisch-Wasserman conjecture that the probability that a self-avoiding random walk or equilateral polygon contains a knot goes to one as the number of edges goes to infinity. An ephemeral knot is defined to be a knotted segment of a walk or polygon that is contained in a larger unknotted segment, called the associated slipknot. Millett, Dobay, and Stasiak showed that the statistics of random closures, to the sphere at infinity, of a polygonal segment provide an effective definition of knotting of the segment. We prove that the probability that a self-avoiding random walk or equilateral polygon, in 3-space or the simple cubic lattice, contains a slipknot goes to one as the number of edges goes to infinity.

Sandy Ganzell (sganzell@smcm.edu) St. Mary's College of Maryland

Hot Jones!

In the 1980s, Vaughan Jones revolutionized Knot Theory with the discovery of a new method for determining knot equivalence. The Jones polynomial has been widely studied since then, yet many of its most basic properties remain mysterious. In this talk we introduce the Jones polynomial and explain why it continues to be such a hot research topic for undergraduates and professionals alike.

Colin Adams (cadams@williams.edu) Williams College

Three Knotty Tales: Complementary Regions, Spiral Index and Spanning Surfaces

In this talk, we will consider three questions related to projections of knots: 1. If we restrict the complementary regions of knot projections, say for instance allowing only triangles, quadrilaterals and pentagons, what knots can we create? 2. If we draw knot projections without any inflection points, so they always curve the same direction, are we just getting versions of braids? 3. Given a single alternating projection of a knot, what can we say about the collection of all spanning surfaces for the knot? All three questions have somewhat surprising answers, and all leave open a variety of further questions to pursue. No particular background assumed.

Research with Undergraduates

Saturday, August 8, 2:00–5:00 PM

Diana M. Thomas (thomasdia@mail.montclair.edu) Montclair State University

A Mathematical Model of Weight Change with Adaptation

As obesity and its related health problems grow around the world, efforts to control and manage weight is increasing in importance. It is well known that altering and maintaining weight is problematic and this has led to specific studies trying to determine the cause of the difficulty. Recent research has identified that the body reacts to forced weight change by adapting individual total energy expenditure. Key factors are an adaptation of resting metabolic rate, non-exercise activity thermogenesis and dietary induced thermogenesis. We develop a differential equation model based on the first law of thermodynamics that incorporates all three adjustments along with natural age related reduction of the resting metabolic rate. Forward time simulations of the model compare well with mean data in both overfeeding and calorie restriction studies.

Aparna Higgins (Aparna.Higgins@notes.udayton.edu) University of Dayton

Directing Undergraduate Research—Issues Beyond Solving the Mathematical Problem

Creating an authentic research experience for an undergraduate student involves more than merely solving the mathematical problem. We will address issues that begin well before the research starts (such as choosing an appropriate problem for this student), that occur during the time the research is conducted (such as support for both student and faculty member), and that arise after the research has concluded (such as choosing an outlet for communicating the results of the research).

Ali Nadim (ali.nadim@cgu.edu) Claremont Graduate University

Electrowetting and Digital Microfluidics

Electrowetting actuation of individual liquid droplets on a solidsurface, known as digital microfluidics, has a variety of interesting applications. These include liquid lenses without mechanical moving parts (www.varioptic.com), novel displays for consumer electronics (www.liquavista.com), and liquid handling without the need for channels, pumps or valves (www.liquid-logic.com). In this talk, we review our group's progress in this area and describe some of the mathematical models we have developed that help us estimate the magnitudes of forces and speeds that can be achieved by electrowetting. Our models focus on the problem of electrowetting actuation of individual sessile drops on a patterned array of electrodes with a thin dielectric coating. For both the case when the drop is electrically grounded from below and when it is floating, we compute the electric field in the vicinity of the drop over a range of frequencies and use the traction derived from the Maxwell stress tensor to calculate the effective electrowetting force on the drop. At low frequencies when the drop behaves like a perfect conductor, the results are compared with previously derived lumped parameter models for the electrowetting force.

Cynthia Wyels (cynthia.wyels@csuci.edu) California State University Channel Islands

Graph Labeling Problems Appropriate for Undergraduate Research

Distance labeling requires an assignment of labels (positive integers) to the vertices of the graph so as to satisfy one of several conditions involving the differences between the labels and the distances between the vertices. (The motivating context is that of assigning frequencies to transmitters so that interference is avoided.) Many labeling problems are immediately accessible to undergraduates and are valuable to their learning what research is and how it may be conducted. We'll discuss solved problems to which undergraduates contributed and some open problems ripe for undergraduate exploration.

Abdul-Aziz Yakubu (ayakubu@howard.edu) Howard University

Mathematical Models in Ecology and Epidemiology

More recently, mathematics has benefited from infusion of interests and techniques from biology. Simple mathematical models often provide useful insights that help our understanding of complex biological problems. Using three mathematical models, we will illustrate the role of mathematical models in understanding the interplay among human behavior, clinical diagnostic accuracy, and biosurveillance; monarch butterfly population dynamics; and implications of harvesting on the stability and resilience of exploited fisheries systems.

Ami Radunskaya (aer04747@pomona.edu) Pomona College

Mathematical Models of Tumor Growth: Lessons Learned from (and with) Undergraduates.

A research collaboration involving undergraduates is a qualitatively different undertaking from work with colleagues or graduate students. Successful outcomes can be encouraged by setting up realistic expectations, by matching students with topics of interest, by clearly articulating the project's focus and timetable, and by providing consistent, direct supervision. These are not easy tasks, especially when working with teams of students. In this talk, I will briefly describe a few lessons we have learned, sometimes the hard way, in the context of our ongoing research program modelling immunotherapy for cancer. In particular, I will discuss lessons learned on how to design projects, how to get funding, how to attract students, and how to complete the proposed research.

Mathematical and Computational Genomics

Thursday, August 6, 3:00–5:30 PM

Cedric Chauve (cedric.chauve@sfu.ca) Simon Fraser University

An Introduction to Some Mathematical Aspects of Genome Rearrangements

Genomes evolve through both small-scale mutations at the nucleotide level, but also through major mutations that concern large genome segments, called genome rearrangements. This talk will describe how these rearrangements are modelled as operations on permutations or sequences and introduce the very rich combinatorial theory that has been developped in the last 20 years to analyse such genome rearrangements.

Anne Bergeron (bergeron.anne@uqam.ca) Universite du Quebec a Montreal

Sorting Genomes with Parking Functions

One of the most rewarding aspects of working in computational biology is to stumble upon unusual connections between biology and computer science. In this talk we will describe such a connection: genome sorting is a way to trace back the genomes of two species to their common ancestor, while parking functions are related to the problem of assigning an index to a data record. We will show how parking functions code for specific ways of sorting genomes.

Binhai Zhu (bhz@cs.montana.edu) Montana State University

Approximability and Fixed-Parameter Tractability for the Exemplar Genomic Distance Problems

Handling errors and redundancies in genomic data is an important problem in comparative genomics. In this talk, I will survey the approximability and fixed-parameter tractability for some Exemplar Genomic Distance problems. I will mainly focus on three problems: the exemplar breakpoint distance (EBD) problem, the exemplar non-breaking similarity (ENbS) or the exemplar adjacency number problem (which is the complement of EBD), and the maximal strip recovery (MSR) problem. Basically, EBD does not admit any polynomial-time approximation or any FPT algorithm unless P=NP. ENbS is linearly reducible from Independent Set, so it does not admit an approximation of factor ¡math¿ n^{ϵ} ¡/math¿ unless P=NP, and it is W[1]-complete (loosely speaking, there is no FPT algorithm for ENbS unless FPT=W[1]). The status on MSR is more positive, it was shown to have a factor-4 approximation previously. Recently, we proved that it is NP-complete and we designed an FPT algorithm for it. Some interesting open problems will be covered in the talk as well.

Aida Ouangraoua (aida_ouangraoua@sfu.ca) SFU and LaCIM, UQAM

Ancestral Genome Architecture Reconstruction

We describe a general methodological framework for the reconstruction of ancestral genome architectures based on the detection of conserved groups of genomic markers and a classical graph-theoretical problem, the Consecutive Ones Problem and the associated PQ-tree.

Hamidreza Chitsaz (hrc4@sfu.ca) Simon Fraser University

Predicting RNA-RNA Interaction Probability and Structure

Background: Until recently, RNA has been viewed as a simple "working copy" of the genomic DNA, simply transporting information from the genome into the proteins. In the 1980s, this picture changed, to certain extent, with the discovery of ribozymes and the realization that the ribosome is essentially an "RNA machine". Since the turn of the millenium, however, RNA has moved from a fringe topic to a central research topic following the discovery of RNA interference (RNAi), the post transcriptional silencing of gene expression via interactions between mRNAs and their regulatory RNAs. Motivation: Recent interests, such as RNA interference and antisense RNA regulation, strongly motivate the problem of predicting whether two nucleic acid strands interact. The specificity of these interactions depends on the stability of intermolecular and intramolecular base pairing. While methods like deep sequencing allow to discover an ever increasing set of ncRNAs, there are no high-throughput methods available to detect their associated targets. Hence, there is an increasing need for precise computational target prediction. In order to predict base-pairing probability of any two bases in interacting nucleic acids, it is necessary to compute the interaction partition function over the whole Boltzmann ensemble. The partition function is a scalar value from which various thermodynamic quantities can also be derived. For example, the equilibrium concentration of each complex nucleic acid species and also the melting temperature of interacting nucleic acids can be calculated based on the partition function of the complex. Results: We present a model for analyzing the thermodynamics of two interacting nucleic acid strands considering the most general type of interactions studied in the literature. We also present a corresponding dynamic programming algorithm which computes the partition function over (almost) all physically possible joint secondary structures formed by two interacting nucleic acids in $O(n^6)$ time and $O(n^4)$ space. The algorithm involves a detailed case analysis which is also interesting from a purely combinatorial point of view. The same combinatorial recursion can be used to predict the joint secondary structure of two nucleic acids. We verify the predictive power of our algorithm by computing (1) the melting temperature for interacting RNA pairs studied in the literature and (2) the equilibrium concentration for several variants of the OxyS-fhlA complex. In both experiments our algorithm shows high accuracy and outperforms competitors.

Applications of Fluid Dynamics

Saturday, August 8, 8:30–10:30 AM

Benjamin Akers (akers@math.uic.edu) University of Illinois at Chicago

Stokes, Taylor, Wilton and the Power of Series

Applications of perturbation series will be presented, with historical perspective. Perturbation series are considered in the context of water waves and roots of polynomials. The role of resonances in perturbation series expansions are explored. Modern boundary perturbation methods for solving the water wave problem are motivated.

Kurt Ehlers (kehlers@tmcc.edu) Truckee Meadows Community College

Micro-Locomotion: Squirmers, Rowers, Spinners, and Singers

Microorganisms live in an Aristotelian world dominated by viscosity where inertia plays absolutely no role. After a brief review of known mechanisms for bacterial self-propulsion, I will introduce a new model for the self-propulsion of certain strains of Synecococcus (blue-green algae) based on acoustic streaming (AS). These one-cell organisms swim at 10-20 diameters per second without flagella or visible changes in shape. Biologists have discovered that some cells are able to generate high frequency small amplitude surface acoustic waves (SAW's) on their outer membrane using coupled molecular motors. Could traveling SAW's generating AS be the mechanism for Synechococcus self-propulsion? Do Synechococcus "sing" themselves along? I will present theoretical and experimental evidence.

Malgorzata Peszynska (mpesz@math.oregonstate.edu) Oregon State University

Flow and Transport in Subsurface with Multiple Scales

Modeling the remediation of contaminated aquifers, oil and gas recovery, and storage and management of greenhouse gases in subsurface requires good understanding and accurate computational simulations of underlying processes of flow and transport. These phenomena are described by coupled nonlinear partial differential equations with data at multiple spatial and temporal scales. Novel mathematical and computational models guide us how to battle the "tyranny of scales" and multi-physics phenomena in order to extract reasonably useful information in a practical time. In the talk we will show recent results of modeling of i) flow with inertia at porescale, ii) flow and transport at lab scale with non-separated scales, and iii) simulation of methane hydrates ("Ice that Burns") over thousands of years.

Roger Samelson (rsamelson@coas.oregonstate.edu) Oregon State University

Mathematical Modeling of Coastal Ocean Circulation

Mathematical modeling of coastal ocean circulation spans a wide range of approaches and techniques, including both numerical simulation and analytical solution of equations expressing basic physical principles. For example, the linear theory of coastal trapped wave dynamics has provided deep and lasting insights into the response of the coastal ocean to wind forcing, while high resolution three-dimensional, primitive-equation numerical models that include nonlinear parameterizations of the effects of stratified boundary layer turbulence are now being used to explore the complex dynamics of wind- and buoyancy-driven coastal ocean flow over realistic shelf and slope topography. Advanced data assimilation methods, similar to those used for numerical weather prediction, are now also being used with these high-resolution primitive-equation models.

Discrete Mathematics

Thursday, August 6, 3:30–6:30 PM

Colin Starr (cstarr@willamette.edu) Willamette University
Josh Laison (jlaison@willamette.edu) Willamette University

2-odd Graphs and Prime Distance Graphs

A graph G is a **prime-distance graph** if the vertices can be labeled with distinct integers in such a way that the differences between the labels on adjacent vertices are all prime. A graph is **2-odd** if the differences are either exactly 2 or odd. We present a characterization of 2-odd graphs and a family of 2-odd circulant graphs. We also offer a conjecture relating prime distance graphs and 2-odd graphs.

Rob Beezer (beezer@ups.edu) University of Puget Sound

Everything You Wanted to Know About the Hoffman-Singleton Graph, But Were Afraid to Draw

The Hoffman-Singleton Graph is an amazing combinatorial object. We'll use its 50 vertices and 175 edges to motivate a tour of a variety of combinatorial structures, with an emphasis on visual representations of the graph's properties.

John Caughman (caughman@pdx.edu) Portland State University

Higher Order Lattice Chains and Delannoy Numbers: The enumeration

Lattice chains and Delannoy paths represent two different ways to progress through a lattice. We use elementary combinatorial arguments to derive new expressions for the number of chains and the number of Delannoy paths in a lattice of arbitrary finite dimension. Specifically, fix nonnegative integers $n_1, \ldots, n_d$, and let L denote the lattice of points $(a_1, \ldots, a_d) \in \mathbb{Z}^d$ that satisfy $0 \le a_i \le n_i$ for $1 \le i \le d$. We prove that the number of chains in L is given by

$$2^{n_d+1} \sum_{k=1}^{k'_{\max}} \sum_{i=1}^{k} (-1)^{i+k} \binom{k-1}{i-1} \binom{n_d+k-1}{n_d} \prod_{j=1}^{d-1} \binom{n_j+i-1}{n_j},$$

where $k'_{\max} = n_1 + \cdots n_{d-1} + 1$. We also show that the number of Delannoy paths in L equals

$$\sum_{k=1}^{k'_{\max}} \sum_{i=1}^{k} (-1)^{i+k} \binom{k-1}{i-1} \binom{n_d+k-1}{n_d} \prod_{j=1}^{d-1} \binom{n_d+i-1}{n_j}.$$

Setting $n_i = n$ (for all i) in these expressions yields a new proof of a recent result of Duichi and Sulanke relating the total number of chains to the central Delannoy numbers.

Chuck Dunn (cdunn@linfield.edu) Linfield College

Higher Order Lattice Chains and Delannoy Numbers: The generating functions

The recurrence relations satisfied by the n-dimensional chain numbers and n-dimensional Delannoy numbers have similar characteristics. We define a concept which describes the recursive structure these two sequences share and use it to derive an explicit formula for the generating function for any such sequence.

Erin McNicholas (emcnicho@willamette.edu) Willamette University

Vertex Adjacencies in Random Planar Trees

A planar tree is a tree having a designated root vertex and a specified cyclic ordering of edges emanating from this vertex. We show how a Dyck path representation can be used to recast questions about the probabilistic structure of random planar trees into straightforward counting problems. Using this Dyck path approach, we reveal the binomial structure of vertex adjacencies in random planar trees as the number of edges goes to infinity.

Daniel Cranston (dcransto@rutgers.edu) DIMACS (Rutgers University)
I. Hal Sudborough (hal@utdallas.edu) University of Texas at Dallas
Doug West (west@math.uiuc.edu) University of Illinois, Urbana-Champaign

Bounds for Cut-and-Paste Sorting of Permutations

We consider the problem of finding the maximum number of moves required to sort a permutation of $1, 2, ..., n$ using cut-and-paste operations, in which a segment is cut out and then pasted into the remaining string, possibly reversed. Every permutation of $1, 2, ..., n$ can be transformed to the identity in at most $\lceil 2n/3 \rceil$ moves and some permutations require at least $\lfloor n/2 \rfloor$ moves.

Matroids You Have Known

Friday, August 7, 2:00–5:00 PM

David Neel (neeld@seattleu.edu) Seattle University

Matroids and the Third Deadly Sin

We will first whirlwind our way through various definitions of matroids to prepare the way for later talks. We will conclude with a discussion of greed and an associated algorithm, connections to matroids, and an example or two.

Gary Gordon (gordong@lafayette.edu) Lafayette College

Pretty pictures produce pretty matroids

Matroids that arise from highly symmetric objects are highly symmetric as geometries. We'll look at several examples where there are close connections between the symmetry of the original object (a polytope, a subset of a lattice or a root configuration) and the corresponding matroid.

Jakayla R. Robbins (jrobbins@ms.uky.edu) University of Kentucky

From Digraphs and Determinants to Oriented Matroids

Many of the results in Matroid Theory were motivated by results in Graph Theory and Linear Algebra. In fact, the edge set of every graph and the column vectors of every matrix over a field can be viewed as the element set of a matroid. Directed graphs, or digraphs, are one of the natural extensions of graphs. An oriented matroid is to a matroid what a digraph is to a graph. In this talk, we will see how digraphs and matrices over ordered fields motivate the study of oriented matroids. Oriented matroids that can be derived from a digraph or a matrix over an ordered field are a special class or oriented matroids. An oriented matroid that can be derived from a matrix over an ordered field is said to be realizable. We will conclude the talk with some results about realizable oriented matroids.

Carla Denise Cotwright (cdcotwright@gmail.com) Hampton University

"Clone Wars"—Clones in Matroids

We'll discuss the notion of clones in matroids. Results relate clones in a matroid to minors of that matroid. Characterizations of matroids that contain few clonal-classes will be discussed, as well.

Talmage James Reid (mmreid@gmail.com) The University of Mississippi

On Circuit Sizes in Matroids

Murty in 1971 characterized the connected binary matroids with all circuits having the same size. We characterize the connected binary matroids with circuits of two different sizes, where the largest size is odd. As a consequence of this result we obtain both Murty's result and other results on matroids with circuits of only two sizes. We also show that it will be difficult to complete the general case of this problem. This is joint work with Manoel Lemos and Haidong Wu.

Brett Stevens (brett@math.carleton.ca) Carleton University

The Solution to the Shannon Switching Game

Shannon's Switching Game is a problem about the connectivity of a network under attack. One player deletes edges and the other makes some invulnerable attempting to connect two vertices before the other player destroys all possibility of connection. In a 1964 article that is now regarded as a classic, A. Lehman generalized the game to matroids and completely solved it. A. Lehman was awarded the Fulkerson prize for his work in discrete mathematics. We present an outline of the solution and give examples of the application of Lehman's necessary and sufficient conditions for a player to win.

Graphs, Networks and Inverse Problems

Saturday, August 8, 2:30–5:00 PM

Chad Klumb (cklumb@gmail.com) University of Washington

Nonunique Solutions to the Inverse Problem for Electrical Networks

We discuss networks for which the discrete inverse problem has nonunique solutions. These networks have the distinctive property that, while there are multiple solutions to the inverse problem, there are still only finitely many solutions.

Tom Boothby (boothby@math.washington.edu) University of Washington

Embedding Permutation String Diagrams

Given any permutation in $\sigma \in S_n$, we can visualize that permutation by drawing n pairs of points, and connecting them according to the permutation. That is, a graph with $2n$ vertices placed on the boundary of a disk in circular order, $\{x_1, \ldots, x_n, y_n, \ldots, y_1, x_1, \ldots\}$, and edges $(x_j, x_{\phi(j)})$. We then compute the minimal genus surface that such a graph can be embedded on, while preserving the ordering of the vertices, and show that it is equal to the minimal number of terms in a factorization of σ into block interchanges.

Liz Stanhope (stanhope@lclark.edu) Lewis and Clark College

What is an Orbifold Graph

Orbifolds are objects in differential geometry that have recently attracted a lot of interest in the mathematics and mathematical physics communities. One line of inquiry is to look at what characteristics of an orbifold can be 'heard' through the spectrum of the orbifold's Laplace operator. One may ask if spectral questions about orbifolds can be transferred to the setting of graph theory. This possible extension of familiar spectral graph theory is the subject of this talk

Richard Froese (rfroese@math.ubc.ca) University of British Columbia

Absolutely Continuous Spectrum for Discrete Schrodinger Operators

If the Laplacian on an infinite graph has absolutely continuous spectrum, it is reasonable to expect that some absolutely continuous spectrum should persist if we add a small random potential. However, this is not always true, and in most cases where it is expected to be true the problem is completely open. In this talk I will explain how the existence of absolutely continuous spectrum for small random potentials can be established for some tree-like graphs.

Alberto Grunbaum (albertogrunbaum@yahoo.com) University of California, Berkeley

A Few Nonlinear Inverse Problems for Simple Networks

I will discuss some inverse problems for networks where a certain random walk takes place and one can only make probabilistic measurements involving pairs of ”external nodes”. This is motivated by medical imaging but may have applications in other areas.

Contributed Paper Sessions

Effective Use of Dynamic Mathematical Software in the Classroom

Session 1: Thursday, August 6, 8:30–10:30 AM

Marcia Weller Weinhold (mweinhold@calumet.purdue.edu) Purdue University Calumet

Designer Functions: Power Tools for Mathematics Instruction

This Geometer's Sketchpad lesson approaches polynomials as being built of linear factors. Students find that knowing the factors (or the zeros) of a polynomial tells them precisely where the polynomial will be positive and negative, and hence the shape of the polynomial. As they introduce new factors to the polynomial, students learn that they can predict its change in shape, so they see that polynomials are truly "designer functions." The lesson can be extended to include nonlinear irreducible factors, the fundamental theorem of algebra, and the constant factor.

Nora Strasser (strasser@friends.edu) Friends Universtiy

Drawing Cycloids with Geometer's Sketchpad

An activity using Geometer's Sketchpad to demonstrate a wheel rolling along the ground using a marked point to trace a cycloid will be presented. Students are given instructions to create a cycloid using geometer's sketchpad. The object will be animated and the students are asked to extend the exercise by investigating different types of cycloids. The types of cycloids investigated include hypocycloids, epicycloids, curtate cycloids, and prolate cycloids. This is an activity that is used in Calculus when the parametric equations of cycloids are studied. Students are better able to understand and visualize cycloids after participating in this activity. The step-by-step instructions for the students will be included as well as a Geometer's sketchpad file containing the completed sketches.

James Parson (parson@hood.edu) Hood College

Inversive Geometry in Geometer's Sketchpad

Plane inversive geometry is a type of geometry that includes simultaneously Euclidean geometry, hyperbolic geometry, and spherical geometry. I will present some Sketchpad activities that lead students to discover the basic properties of this geometry, to draw a striking picture, and to develop the tools later used for exploring non-Euclidean geometries.

Daniel J. Heath (heathdj@plu.edu) PLU

Rethinking Pythagorus with Geometry Playground

The Pythagorean Theorem, perhaps the most widely known result in mathematics, has been proven in countless ways, and remains a basic building block of Euclidean geometry. We provide a discovery activity utilizing Geometry Playground to see that Pythagorean Theorem analogs can provide a gateway into understanding non-Euclidean geometries as well, including our own (spherical) world.

Larry Ottman (ottmanl@hhsd.k12.nj.us) Haddon Heights Public Schools

Using Geometry Expressions to Explore Crop Circles

What is the best way for a farmer to maximize field space using center pivot irrigation? The landscape in parts of the arid west is a patchwork laboratory of circular fields that show numerous possible configurations. The need to work around existing terrain features and irregular shaped plots provides many interesting opportunities for geometrical investigations. Students can use the unique capabilities of Geometry Expressions to explore the possibilities both algebraically and geometrically in the same software package.

Bruce Torrence (btorrenc@rmc.edu) Randolph-Macon College

Lights Out for Linear Algebra

The popular game "Lights Out" (it's even available as an iPhone app) can be analyzed completely using elementary linear algebra. Adding a dynamic playing board and some other dynamic exploratory tools makes for an engaging classroom experience. A new twist on this puzzle will be discussed: which starting configurations can be solved by clicking on exactly those lights that are initially lit? This leads to an exploration of the eigenspace of the Lights Out transition matrix, which for an $n \times n$ board leads to an ¡math¿n dimensional subspace of $(\mathbb{Z}_2)^{n^2}$.

Session 2: Friday, August 7, 8:30–11:30 AM

David Robert Gurney (dgurney@selu.edu) Southeastern Louisiana University

Demonstrating the Central Limit Theorem with Minitab

Minitab statistical software provides a way for students to generate random samples of data from a variety of distributions - binomial, chi-square, normal, Poisson, uniform, etc. Checking whether a sample is normal or not can easily be done using a built-in Minitab procedure. Furthermore, many more random samples can be quickly generated, their means can be easily found and the Central Limit Theorem can be shown to apply to each of these distributions.

James Quinlan (quinlan@math.ohio-state.edu) Ohio State University

Dynamic Graphical User Interface in MATLAB using GUIDE

Building an interactive interface in MATLAB is easy, fun, and useful. Teachers and students can harness the power of MATLAB from an interface with standard interface elements such as text boxes, drop-down menus, and slider bars, then display numerical (or symbolic) results and graphs using a built-in integrated developing environment (GUIDE). The interface is an excellent way for teachers to demonstrate a process and for students to develop conceptual understanding while also acquiring knowledge of MATLAB, a useful skill in it's own right. The demonstration will include elements of proper interface design as well as syntax to handle user input and MATLAB output.

Leon Brin (brinl1@southernct.edu) Southern CT State University

Identifying Self-Similarity

A collection of self-similar fractals from classics like the Sierpinski Triangle and its variants to originals from the author will be presented. The task to the student is to collage the fractals and, for each part of each collage, propose an affine transformation that would map the whole to the part. IFS Freestyle, open source dynamic software for exploring fractals, is then used by the student to check their proposals. If a proposal is correct, great. They will see a replica of the collaged fractal on screen. If not, the real power of IFS Freestyle is unleashed. From the incorrect set of affine transformations, the student may click-and-drag the proposed transformations or add or subtract transformations as needed until they correctly reproduce the given fractal. The effect of changing the transformations is displayed in real time.

Sarah L. Mabrouk (smabrouk@framingham.edu) Framingham State College

Interactive MS Excel Workbooks: Learning Tools That Are More Than Spreadsheets

Since 2002, I have created a variety of tools that I have used in courses such as Introductory Statistics, College Algebra, Precalculus, and Calculus. These tools are useful in concept exploration as well as for examples and drill problems for students; many provide an infinite number of practice problems for students as well. In this presentation, I will give the audience a choice of topic to explore, for example, (standard) normal distribution and confidence intervals for Introductory Statistics, equations of lines and polynomial functions for College Algebra, trigonometric functions for Precalculus, and slope graphs and the derivative for Calculus. You are welcome to examine some of the available tools on my web site, http://www.framingham.edu/faculty/smabrouk/Interactive/.

Jason Grout (grout@iastate.edu) Iowa State University

Online Interactive Worksheets with Sage

I will describe how to use interactive features of Sage and its web notebook interface to teach calculus topics. Sage (http://www.sagemath.org) is a free open-source computer algebra system that includes several features that make it ideal for classroom use. The online notebook interface makes it trivial to share worksheets and collaborate. It is easy to have Sage automatically build an "interact," an online interactive form in which students can adjust or input parameters using sliders, buttons, text boxes, etc., and see results immediately. Sage's powerful computer algebra and visualization capabilities encourage exploration. A vibrant and helpful community quickly addresses questions and requests. In addition, the free and open-source nature of Sage allows students to use Sage in their own future workplaces or classrooms with no restrictions or fees.

Constance Edwards (constance.edwards@wku.edu) Western Kentucky University

Using E-activities with the ClassPad 300

We will show how to use the e-activity feature on the Casio ClassPad to engage students in a step-by-step learning process.

Diane Whitfield (dwhitfield@casio.com) CASIO MRD Center, Portland Community College

Using the ClassPad Manager Software to Assist in Teaching Mathematics

Can you improve your lecture by solving problems from both a Geometric and Algebraic view point? The answer is yes! Following a brief introduction and discussion, examples I have used to enhance lectures in Algebra to Pre-Calculus will be shared.

Paul Seeburger (pseeburger@monroecc.edu) Monroe Community College

Verifying Surface Intersection Curves Visually

In multivariable calculus we ask students to determine the parametric equations for the curve of intersection of two surfaces. This includes finding the intersection of two planes (a three-dimensional line), and the intersection of a pair of surfaces including spheres, paraboloids, hyperboloids, and planes. Using a freely available online multivariable calculus applet called CalcPlot3D, a pair of surfaces can be graphed along with the intersection curve (entered parametrically), and the intersection curve can be verified visually. CalcPlot3D is part of an NSF-funded grant project called Dynamic Visualization Tools for Multivariable Calculus (DUE-CCLI #0736968). See http://web.monroecc.edu/calcNSF/. This lesson includes an example to demonstrate in class and a worksheet for students to complete as homework. Students are asked to use the CalcPlot3D applet to graph the surfaces in their problems along with the space curves they obtain to represent the surface intersections. The applet allows the graphs to be printed and includes a date/time stamp and the name of the computer used for the exploration.

Jack Ryder (jack@jimryder.com) Kean University

Visualizing Cycles in Random Sequences

This presentation demonstrates a recursive method for generating pseudorandom numbers and then dynamically displays its related cycle sequence. Students can investigate different initial seeds and constants for the generator, and immediately visualize their related cycles. The material is applicable for use is discrete mathematics, statistics, numeric computing, simulation and abstract algebra courses. Generating random number sequences is essential in diverse areas of applied mathematics such as statistics, cryptography and for testing mathematical models on a computer via simulation. One of the best known pseudorandom number generator algorithms is the linear congruential generator given by the recurrence relation: $X_{n+1} = (aX_n + c)\text{mod}(m)$ where a, c and m are positive integers. The longest sequence before the generator starts to repeat is called the period. The best generators have periods at, or close to, m. The sequence of random numbers $r_1, r_2, \ldots, r_{n-1}, r_n$ can be plotted in two dimensions as a sequence of ordered pairs $(r_1, r_2), (r_2, r_3), \ldots, (r_{n-1}, r_n)$ which becomes a random walk. The corresponding graph is guaranteed to display a cycle in its random walk since the numbers will repeat after at most m numbers. There may be one or more cycles depending on the choice of a, c, m and the initial seed. Each cycle forms a ”cyclic group” with related mathematical properties. Students and teachers can explore random number generation and the concept of cyclic groups with this software. A web link is provided for running the software, and displaying overviews of cycles, groups, random number generation and student exercises.

Session 3: Saturday, August 8, 8:30–11:30 AM

Pallavi Jayawant (pjayawan@bates.edu) Bates College

Decision-Making and Iteration

I will present a lesson that I have used in two of my courses - Graph Algorithms and Enumerative Combinatorics. The lesson is a Maple worksheet which begins with a quick introduction to Maple and then moves on to writing simple procedures with decision-making using statements such as “if..then..” and procedures with repetitions using for loops or while loops. The mathematical content of the lesson is accessible to anyone with a calculus background. I will describe how I use the lesson in my classes to introduce the students to some of the basic programming skills. I will then talk about how the lesson can lead to other Maple lessons that enhance the understanding of the mathematical content of the course in which the lesson is used.

Michael Rogers (mroge02@emory.edu) Oxford College of Emory University

A Game to Illustrate the Definite integral

A game written in *Mathematica* by the speaker and freely available on the internet can be used to teach students the concept of a definite integral. In the game a student draws the graph of a function so that its integral from 0 to x, $0 \leq x \leq 4$, meets certain criteria. In this way, the student explores the relation of the integral to area, the integral as a function of its upper limit, and the corresponding fundamental theorem of calculus. The criteria in the game to be met can be randomly generated so that each student gets a unique exercise to complete. The speaker assigns this as a –quiz” in his classes.

Helmut Knaust (helmut@math.utep.edu) University of Texas at El Paso

Exploring Machin’s Approximation of π

In 1706 John Machin cleverly combined the tangent addition formula with Gregory’s power series representation of the arctangent function to approximate the circular constant π to 100 digits. Following in Machin’s footsteps today gives students in a second calculus course the opportunity to see a significant application of Taylor series techniques in action. With the help of a computer algebra system such as *Mathematica* they can also quite effortlessly explore possible alternative designs for such an approximation formula. In our talk we will present a hands-on lesson that is suitable both for an in-class laboratory activity or an out-of-class project assignment.

Therese Shelton (shelton@southwestern.edu) Southwestern University

Visualizing an Accumulation Function

Students often have difficulty with accumulation functions, such as the cumulative distribution functions in Probability. Dynamic graphing (using slider bars) in a computer algebra system (Mathematica) helps to clarify the relationship between the original function and the accumulation function.

Lyle E. Cochran (lcochran@whitworth.edu) Whitworth University

Interactive Demonstrations for Calculus

The key ideas of calculus are brought to life using high-quality, interactive Mathematica 7.0 graphics. These interactive, user-friendly demonstrations help students develop an intuitive, geometric understanding of the important ideas of calculus while providing professors with powerful new teaching tools. Two examples will be provided along with ideas of how to engage students in the learning process.

Chris Camfield (camfieldc@kenyon.edu) Kenyon College

Animating Multiple Maple Plots to Demonstrate Damped Harmonic Oscillation

Simultaneous animation of multiple Maple plots is a great way to demonstrate how damping effects a harmonic oscillator. By watching the characteristic polynomial, the displacement-velocity phase portrait, and the displacement versus time plot change with different levels of damping, students can see precisely what happens when a system transitions from underdamping to overdamping. Seeing a straight-line solution suddenly arrive with the appearance of a real eigenvalue, students realize why the system instantly changes from infinite oscillations to at most one oscillation. I have used this lesson in Differential Equations and also in a second semester of Linear Algebra as an application of eigenvalues. A physics student in class even used the file while giving his own presentation on the motion of spring-mass systems.

Philip B. Yasskin (yasskin@math.tamu.edu) Texas A&M University
Douglas B. Meade (meade@math.sc.edu) University of South Carolina

Maplets for Calculus - Developing Intuition in a Computer Lab

Maplets for Calculus is a collection of 94 Maple applets for teaching single-variable calculus, including precalculus, limits, derivatives, integrals, differential equations, sequences, series, and polar coordinates. They are highly pedagogical, using algorithmically-generated or user-entered problems. Most Maplets ask students one or more questions and guide them through the solution process. They build intuition, guide students through the steps of simple proofs and/or provide routine computational practice. The Maplets balance the development of understanding and technical skills. Students find the hints, immediate feedback, step-by-step checking of their responses and infinite patience as effective as a private tutor. Instructors also use the maplets as effective demonstrations making use of the 2D/3D graphics and animations and the ability to launch a Maplet with a specific example. Other instructors develop a project or a lab utilizing a Maplet in a guided-discovery exercise. The authors are in the process of incorporating a grading capability into the Maplets. The presentation will highlight two activities used in a lab setting to develop the intuition 1) between one and two sided limits and continuity and 2) on the approach of secant lines to tangent lines. This collection of Maplets are proving useful at both the college and high school level. A Table of Contents and sample videos may be seen at http://m4c.math.tamu.edu/. This project was awarded the 2008 ICTCM Award for Excellence and Innovation using Technology in Collegiate Mathematics and is supported by NSF DUE CCLI Grants 0737209 (Meade) and 0737248 (Yasskin).

Brian Hollenbeck (bhollenb@emporia.edu) Emporia State University

Using Maple to Apply Cubic Spline Techniques in Calculus and Mathematical Modeling

The technique inherent in finding a cubic spline can be adapted to fit many applications and to emphasize concepts in various mathematics classes. We will first introduce a Maple worksheet intended for Calculus I students to see how a cubic spline is fit to data points. Students are then asked to alter the cubic spline method to design an initial rise and drop in a roller coaster using their own parameters. This problem is intended to reinforce concepts associated with the derivative that students have recently learned. We will also discuss how students can use Maple and splines in other courses. For example, cubic splines can be used in Calculus II to approximate the shape of a vase in order to find its volume. In Math Modelling, students are asked to explore multiple techniques to design a high-speed railway route connecting several cities in an optimal way.

The History and Philosophy of Mathematics, and Their Uses in the Classroom

Session 1: Friday, August 7, 8:30–11:50 AM

Mike Krebs (mkrebs@calstatela.edu) California State University, Los Angeles

A Video Introduction to the Epsilon-Delta Definition of the Limit

We present a short video which can be used in a Calculus, Analysis, or History of Math class as an initial introduction to the definition of the limit. After first discussing the historical development of the epsilon-delta definition, the video then briefly takes students through the underlying visual logic. The video will be available online (for free) through the NSF-sponsored National Curve Bank. This presentation will end with a sing-a-long.

Ruth O'Keefe (okeefer@ferris.edu) Kendall College of Art and Design

History of Mathematics in Art and Design

Teaching mathematics in a college of art and design is a formidable challenge. However, making the subject relevant to the History of Art opens all possibilities to capture the interest of even the most skeptical student. Many artists and designers, through the ages, used mathematics as a problem solving tool. In most situations they were not fluent in the knowledge of mathematics, but rather used their natural instinct of intuition to accomplish their goals. By example, I propose to show examples of the "beauty of mathematics" exhibited through the eyes of the artist and designer.

Susan Kelly (kelly.susa@uwlax.edu) University of Wisconsin-La Crosse

Multiple Integrals Provide an Opportunity to Introduce a Women's History Lesson in a Mathematics Class

Winifred Edgerton Merrill was the first American woman to receive her Ph.D. in Mathematics. She earned her degree from Columbia University in 1886, a time when women were not allowed to be officially admitted to this male only university. In her thesis, Multiple Integrals, she uses an application of Green's Theorem which would work nicely in a Multidimensional Calculus course. Highlights of her thesis and her life, obtained from published works, archives, and a personal family journal, will detail some of her contributions to mathematics and to the advancement of women.

Vasos Pavlika (V.L.Pavlika@wmin.ac.uk) University of Westminster

Teaching Mathematics via a Detour into her History.

This paper discusses how one may introduce topics in mathematics by initially discussing the mathematicians responsible for the development of the topic. The topics discussed were algebraic inversion formulae leading to a discussion of groups and discourses on Lagrange, Abel and Galois, complex numbers including discussions on such giants as: Gauss, Cauchy and Riemann and on the calculus including discussions on Sir Isaac Newton and Gottfried Wilhelm Leibniz. The topics were discussed in courses delivered at the University of Oxford, the University of London and the University of Westminster. The methods were considered as a teaching tool due to the author's observation over a number of years that student appreciated and requested a knowledge of the history, development and background of the lives of the mathematicians responsible for the topics being taught.

Carol J. Browning (cbrowning@drury.edu) Drury University

A Student Reading Guide for Cayley's Paper on the Theory of Groups

Students can gain much from reading original sources. However, even in original sources written in English, the terminology can be different and the writing style unfamiliar. For students who are just learning new concepts using modern terminology, the wording of original sources for those concepts can be indecipherable. In 1854, Cayley published a paper on the theory of groups. He begins by describing why the set of permutations on n items forms a non-commutative group. He then determines all possible groups of order 2 and of order 6. For the past several years, students in my abstract algebra class have been reading this paper just after studying these concepts in our course. Over the years, I have developed a reading guide to lead students through the terminology. Students complete the reading guide as they read the paper and then submit the completed guide. In this talk I will present the reading guide, discuss the context for the assignment, describe the results, and consider how this model could be used for similar assignments in other areas of mathematics.

Steven Edwards (sedwards@spsu.edu) Southern Polytechnic State University

Aristarchus: The Man Who Introduced Mathematics to Astronomy

We examine the state of astronomical knowledge 300 BC and the contributions of Aristarchus of Samos, who was the first to use astronomical observations and geometry to estimate the relative sizes and distances of the earth, sun, and moon.

Colin Bryan Powell McKinney (cbmckinn@math.uiowa.edu) University of Iowa

Compound Ratio, Revisited

The notion of compound ratios is historically a sore subject for Greek mathematics. It is used once, but never defined, by Euclid in the *Elements*. However, more advanced works such as the *Conics* of Apollonius make extensive use of compound ratios despite the term being undefined and poorly elucidated in the primary Greek corpus. Later ancient authors, especially the commentators, make an attempt at correcting this flaw by providing explications of the concept. In this talk, I will concentrate on two different accounts provided by Eutocius of Ascalon in his commentaries on Archimedes' *On the Sphere and the Cylinder* and Apollonius' *Conics*. Eutocius' two descriptions of compound ratios are very similar, but I will demonstrate how the small differences indicate something about Eutocius' mathematical growth over the course of his life.

William Mercier (wmercier@ivytech.edu) Ivy Tech Community College - Lafayette

Why the Centroid is the Centroid: An Ode on a Theme by Archimedes

It is shown with modern rigor, using the modern notion of self-similarity, that, by adopting precisely the formulation of mechanics given by Archimedes in his *On the Equilibrium of Planes,* we can geometrically locate the point of mechanical equilibrium of an arbitrary triangle by means that would be immediately comprehended in the 3rd century BC. We then extrapolate to the tetrahedron. The arguments given appear to be new, although elementary. Though all steps can be phrased in a manner apprehensible to the ancient Greeks, several elementary illustrations of modern notions and methods from geometry and analysis fall out along the way. This is not to say that all steps employed would be approved by the ancients, with Archimedes a possible exception. This issue is discussed.

Mary Lynn Elizabeth Doan (doanm@vvc.edu) Victor Valley College

Euler's Lost Solution to Longitude at Sea

Leonhard Euler and Tobias Mayer corresponded from 1751 to 1755. This rather brief but interesting episode in Euler's life connects him as a mathematician and physicist to one of the most elusive problems of his day, finding longitude at sea. Coupling Mayer's fastidious measurements of the position and motion of the moon, planet and stars with Euler's skill in using infinite series to approximate logarithmic and trigonometric values allowed them to devise improved astronomical tables. These new tables allowed Euler and Mayer to solve the Longitude Problem to within 22", well within the required accuracy for the Longitude Prize money. Their tables, which were later refined by Euler, allowed for rather shorter and far more accurate calculations than previously, finally making an astronomical approach practical. The fact that it occurred simultaneously with the acceptance of Harrison's chronometer only muted our knowledge of their accomplishment.

George Heine (gheine@mathnmaps.com) US Bureau of Land Management

Leonhard Euler's Contributions to Mathematical Cartography

In 1777, Euler published a series of three articles about mathematical cartography E490: "De representatione superficiei sphaericae super plano"; E491: "De proiectione geographica superficiei sphaericae"; E492: "De proiectione geographicae de Lisliana in mappa generali imperii russici usitata". The speaker has recently translated these into English. We discuss these three articles, especially the first, which is the longest and most theoretical. In it, Euler gives a purely analytic proof that no mapping from sphere to plane can preserve both angles and distances. Three desirable conditions for a map are then proposed and analyzed as sets of solutions to a pair of differential equations in two variables. We examine Euler's interest in cartography in the context of the developing science of cartography, the developing Russian nation-state, and the internal politics of the St. Petersburg Academy.

Session 2: Friday, August 7, 1:00–6:20 PM

Martin E. Flashman (flashman@humboldt.edu) Humboldt State University

Which Came First? The Philosophy, the History, or the Mathematics?

The author will give examples from instruction where mathematics interacts with its history and philosophy in the context of a content based course illustrating how this interaction can enhance learning.

Daniel Sloughter (dan.sloughter@furman.edu) Furman University

Should My Philosophy of Mathematics Influence My Teaching of Mathematics?

My short answer to the title question is, of course, "Yes, no, and it depends." There are times when the answer should be "yes," such as when one is considering how much time to devote to the Bayesian approach, or that of Neyman and Pearson, or Fisher, when teaching a course in statistics. At other times, it seems to me the answer is clearly "no." For example, I would argue that if you do not, on philosophical grounds, accept arguments from contradiction or the law of the excluded middle, you nevertheless would be doing your students in an analysis course a serious disservice not to introduce them to results depending on these principles. Mathematics without the laws of noncontradiction and the excluded middle is very different from mathematics with them. Yet in other cases the choices may not be so radical. For example, the approaches of nonstandard analysis and standard analysis lead to the same results by different routes. Would students be harmed if you chose the route you found most philosophically attractive?

Bonnie Gold (bgold@monmouth.edu) Monmouth University

Philosophical Questions You DO Take a Stand on When You Teach First-year Mathematics Courses

Most mathematicians have no interest in the philosophy of mathematics, and, when asked about their philosophical views, reply that they leave that to philosophers. However, in fact, in the process of teaching undergraduate courses, we DO take a stand on a range of philosophical questions, in most cases unconsciously. I've become aware of more and more of these as I've gotten involved in the philosophy of mathematics. They range from the well-known - the Intermediate Value Theorem is not a theorem from an intuitionistic perspective - to the more subtle. Some of them are closely related to errors students persistently make or misunderstandings they have. I will discuss several that we all take stands on when we teach first-year mathematics courses such as calculus and introduction to proof, and how I have begun to alert students to the subtleties involved.

Jeff Buechner (buechner@rci.rutgers.edu) Rutgers University

Using the Philosophy of Intuitionistic Mathematics to Strengthen Proof Skills

There are several issues within the philosophy of intuitionistic mathematics that are useful for developing proof skills in undergraduate mathematics majors. This talk will examine the role of classical and intuitionistic logic in constructing proofs, the intuitionistic proscription of proof by contradiction, and the nature of constructive existence proofs and how attention to these issues can foster proof skills.

Robert Ely (ely@uidaho.edu) University of Idaho
Kimberly Vincent (vincent.kimberly@gmail.com) Washington State University

Context "vs." Notation: Lessons Learned From History and the Classroom

The history of mathematics is full of cases where a real-world context was crucial for sparking the creation of a powerful mathematical idea, but this same context later hindered the development of more powerful general methods. For example, Babylonian, Greek, and medieval Islamic mathematicians used the powerful context of cutting and pasting areas, which facilitated the creation and solution of quadratic and cubic equations but ultimately hindered the recognition of negative and complex solutions. Only by ignoring this context and pursuing the affordances of symbolic notation could later mathematicians successfully generalize solutions to quadratic and cubic equations.

Learners also experience a context as a springboard for understanding that later may become an obstacle to further understanding. In-service teachers participating in the mathematics professional development project we studied worked with many rich contexts, in which to develop models of and narratives about functions. But later these contexts became a hindrance to more generalized reasoning about functional relationships, ultimately needing to be abandoned in favor of pursuing abstractions afforded by functional notation.

This parallel between history and the classroom suggests that an important skill for teachers is knowing when and how to provide contexts for learners, and when these contexts are getting in the way of further learning. This lesson from history also indicates a way to move beyond the debate about traditional or reform mathematics curricula. We must study how to help students negotiate the transitions between context and notation, rather than thinking that we must choose between them.

Tommy Leavelle (leavelle@mc.edu) Mississippi College

Defining the Sum of an Infinite Series

In the calculus sequence the sum of an infinite series is usually defined as the limit of the sequence of partial sums, if it exists. But there are infinite series, such as Grandi's series, which are not given a sum by this definition and for which there are good arguments in favor of a sum. In this talk, we will consider a discussion which the presenter uses in his calculus classes to explore the concept of a "better" definition based on historical approaches to the sum of an infinite series proposed by Cesro and Euler.

J. Christopher Tweddle (ct55@evansville.edu) University of Evansville

The Development of the Irrational Numbers.

The 19th century saw a push to place mathematics on a rigorously sound logical base. Prior to the late 1800s, the existence of the irrational numbers was based upon our intuitive concept of the completeness (or continuity) of a line. During this period, Richard Dedekind and Georg Cantor introduced the now familiar constructions of the real numbers using a geometric notion of "cuts" of the number line, and Cauchy sequence of rational numbers, respectively. In this talk, we'll look at some lesser known contributions to the study of irrational numbers made by Heinrich Eduard Heine and Charles Meray. We will also see an alternative characterization given by Karl Weierstrass. In the classroom, the subtlety of these constructions provides an example of the difficulty of pinning down an intuitively "obvious" mathematical concept. Additionally, it illustrates the great lengths to which we must go to avoid the pitfall of assuming what we want to prove.

Meighan Dillon (mdillon@spsu.edu) Southern Polytechnic State University

Teaching Geometry from a Historical Point of View

An elective geometry course presents an opportunity to bring students into the history of mathematics for a well-defined purpose, that is, to understand the development of non-Euclidean geometries as arising from attempts to prove the parallel postulate. This is a natural approach and yields a satisfying course. More generally, though, the history of the parallel postulate serves as a window into the history of *The Elements* of Euclid itself. Over its 2400 year history, *The Elements* has played a critical role in Western thought and pedagogy. The earliest record of the axiomatic approach to mathematics, *The Elements* can be considered the first step in a line that leads to Hilbert and to Bourbaki. This is a powerful story that can and should be told to our students, because it is, in part, the story of why we teach what we teach.

Julius Barbanel (barbanej@union.edu) Union College

A Course on Ancient Greek Mathematics

We will discuss our course "Ancient Greek Mathematics," which we developed and have offered twice at Union College. It is part of a collection of General Education courses offered at Union and intended for students who will take no other college mathematics courses. In the course, we explore both purely mathematical issues and contributions (such as: Pythagorean number theory, the problem of incommensurables, Euclid's Elements, the method of exhaustion, the three special construction problems, and Archimedes' law of the lever and mechanical method) and meta-mathematical ideas (such as: what is mathematics, how and why was Greek mathematics different, consequences of the discovery of incommensurables, and the influence of Plato on the development of Greek mathematics). We will discuss the course structure and shall present samples of exams and homework assignments.

Gary W. Hagerty (garyhagerty@bhsu.edu) Black Hills State University

Issues of Implementing History Modules into College Algebra

In 2002, we began redesigning our College Algebra course with an added emphasis on improving the students understanding of the purpose and usefulness of mathematics. This included adding a brief weekly history discussion to the course. The original results were made available in a paper titled, "The Unique Effects of Including History in College Algebra", published in the MAA's online journal *Loci: Convergence*. This discussion will look at the overall use of history in College Algebra, implementation issues, pitfalls and results since the original paper.

Randy Ken Schwartz (rschwart@schoolcraft.edu) Schoolcraft College

Silk Threads of Many Colors: Teaching Ibn Mun'im's Combinatorics

This talk summarizes a course activity that the presenter has developed and used for several years in classes in Finite Mathematics and Statistics. The activity takes the form of a pair of written, self-paced lessons that guide students to discover important combinatorial relationships through a mixture of exposition and exercises. The work is based on a triangle of binomial coefficients that was developed by Ahmad al-Ab'dari ibn Mun'im (d. 1228 CE), a physician and mathematician born in Andalusia and who lived much of his life in Marrakech. The starting point is ibn Mun'im's explanation of his triangle in terms of counting the number of ways to combine silk threads of various colors into tassels. Going through the tassel problem allows students to understand two different patterns among the entries in the triangle, involving row and column sums, respectively. They go on to exploit these patterns to solve a variety of practical problems. Each of the two lessons requires approximately one hour, whether completed individually or in groups.

Sarah L. Mabrouk (smabrouk@framingham.edu) Framingham State College

Sharing the History of Mathematics Online

The history of mathematics is a regular component in my mathematics and statistics courses. During the past several years, I have taken advantage of students' listening to one another differently than they listen to an instructor as well as modern course technology in the manner in which this history is presented and shared: students research the lives and work of mathematicians/statisticians and share this research as they discuss and compare mathematicians/statisticians online. This sharing of the lives and work of famous mathematicians/statisticians can inspire students to learn more mathematics, to explore new ideas, and foster curiosity. In addition, through this research, students come to see these mathematicians/statisticians as real people with personal lives and interests just like them as well as see these "heroes" of mathematics/statistics as having diverse interests that developed and changed over time. In this presentation, I will share student reaction to these discussions of the history of mathematics/statistics as well as some of the aspects of the lives and work of these mathematicians/statisticians that have fascinated, inspired, and delighted students.

Janet Heine Barnett (janet.barnett@colostate-pueblo.edu) Colorado State University - Pueblo

Straight from the Source's Mouth: A Two-Valued Approach to Learning Boolean Algebra and More

In 1847, George Boole (1832–1916) launched the study of boolean algebra with a bold new algebraic approach to logic. Axiomatized as an abstract structure by Edward V. Huntington in 1904, boolean algebra was later recognized by Claude E. Shannon as an important tool in circuit design in 1938. Today, boolean algebra remains a subject of interest, not only as an interesting mathematical object in its own right, but as a powerful tool for applied practitioners as well. This talks highlights student projects which bring the values of the past and the present together as students build their own understanding of boolean algebra by exploring the mathematics of the initial discovery and its subsequent lines of inquiry through guided reading and activities based entirely on original source readings. Intended in part to restore context and motivation to the study of discrete mathematics, the projects are part of a larger collection under development by an interdisciplinary team of faculty at CSU-Pueblo and New Mexico State University through the NSF-funded project Learning Discrete Mathematics and Computer Science via Primary Historical Sources. Original source authors represented in the collection (available at www.cs.nmsu.edu/historical-projects) include Archimedes, Cantor, Euler, Leibniz, Pascal, Turing, Veblen, and von Neumann, writing on topics such as mathematical induction, finite sums of powers, graph theory, transfinite arithmetic, binary arithmetic, combinatorics, algorithm design, computability, and decidability. Our pedagogical approach and its rationale will be illustrated through excerpts from Boole, Huntington, Shannon and others employed in the boolean algebra projects of the collection.

Eugene Clayton Boman (ecb5@psu.edu) Penn State-Harrisburg

False Position, Double False Position, and Cramer's Rule

The methods of False Position and Double False Position appear in the *Nine Chapters of the Mathematical Arts* of ancient China, in the Ahmes papyrus of ancient Egypt, and in Fibonacci's *Liber Abaci* in the thirteenth century. I will discuss the two schemes and the connections between them, and show how Double False Position is connected to Cramer's Rule.

Joyati Debnath (jdebnath@winona.edu) Winona State University

π, e, and Prime Numbers

The concept of irrationality was clearly expressed in the *Sulba Satras* (literally, "Aphorisms of the Chords" in Vedic Sanskrit) in India in the 7^{th} century BC. Ancient and medieval mathematical works arose in these sutras as a part of religious ritual and astronomy. Manava (750 BC–690 BC), a Vedic priest and a skilled craftsman, wrote *Sulba Satras* to provide rules for religious rites and detailed accurate construction of altars needed for sacrifices. It contained constructions of circles from rectangles and squares from circles giving approximate values of π. Baudhayana *Sulba Satras* contained examples of Pythagorean triples and an accurate formula of $\sqrt{2}$ up to five decimal places. This presentation will elaborate on the early history of the existence of π, e and their connection with the prime numbers, which is both fascinating and intriguing. It causes students to appreciate the number system and enhances mathematical thinking.

Una Bray (ubray@skidmore.edu) Skidmore College

The Enigmatic Gerolamo Cardano - Mathematician, Medical Doctor, Inventor, Food Critic, Gambler

Cardano lived during an important transition period in the history of mathematics. I will summarize his contributions to mathematics, but will also talk about his work in other areas.

Graph Theory and Applications

Saturday, August 8, 1:00–6:35 PM

Ralucca M. Gera (rgera@nps.edu) Naval Postgraduate School

Set Colorings in Graphs.

Let $G = (V(G), E(G))$ be a connected graph, and let $c : V(G) \longrightarrow \{1, 2, \ldots, n\}$ be a vertex coloring (not necessarily a proper one). The neighborhood color set $NC(u)$ is the set of colors of the neighbors of u. We then define a set coloring to be a coloring in which $N(u) \neq N(v)$, for every pair $u, v \in V(G)$ for which $uv \in E(G)$. We present results on the set coloring.

Mark Anderson (manderson@rollins.edu) Rollins College
Richard Vitray (RVitray@rollins.edu) Rollins College
Jay Yellen (JYellen@Rollins.edu) Rollins College

Maximal Irregular Colorings of Regular Graphs

An irregular coloring of a graph is a proper vertex coloring that distinguishes vertices in the graph either by their own colors or by the colors of their neighbors. In algebraic graph theory, groups with a certain amount of symmetry are specified in terms of a group and a smaller graph (e.g. voltage graphs). Radcliffe and Zhang found a bound for the irregular chromatic number of a graph on n vertices. In this paper we create voltage graphs achieving that bound.

Stephen Devlin (smdevlin@usfca.edu) University of San Francisco

Evolutionary Games on Graphs

Evolving biological systems are often modeled in the context of the social structure of the population, which is represented by a (usually big) graph. We will give examples of such models that are based on game theory, and show how various graph parameters can be used to effectively predict the dynamics of the system.

Abdul Jalil M. Khalaf (am_maths@yahoo.com) PhD student (UPM)
Yee-Hock Peng (yhpeng@math.upm.edu.my) Professor (UPM)

A Counter Example for a Chromatic Uniqueness Theorem

Let $P(G, \lambda)$ denote the chromatic polynomial of a graph G. Two graphs G and H are chromatically equivalent, written $G \sim H$, if $P(G, \lambda) = P(H, \lambda)$. A graph G is chromatically unique written $\chi-$unique, if for any graph H, $G \sim H$ implies that G is isomorphic with H. In this paper we introduce a counter example against one chromatic uniqueness theorem of 5-bridge graphs.

Bill Linderman (wclinder@king.edu) King College

A Closure for Claw-Free Graphs

A graph is said to be *claw-free* if it does not contain a copy of $K_{1,3}$ as an induced subgraph. Ryjácek has described a closure for claw-free graphs which preserves the length of the longest cycle. Thus, a claw-free graph with a complete closure of this type is hamiltonian. The closure is also a line graph for some graph. We examine this closure for claw-free graphs and some results related to hamiltonicity and line graphs, including a result on the minimum number of edges a graph can have if it has a complete closure of this type.

Rochelleo Esios Mariano (romeo_4747@yahoo.com) Ateneo de Zamboanga University

Edge Geodetic Covers in the Cartesian Product of Graphs

For any two vertices u and v of a graph $G = (V(G), E(G))$, the set $I^e_G[u, v]$ consists of all edges of G lying in any $u - v$ geodesic in G. If $S \subseteq V(G)$, then the set $I^e_G[S]$ denotes the union of all $I^e_G[u, v]$, where $u, v \in S$. A subset S of $V(G)$ is an *edge geodetic set* or an *edge geodetic cover* of G if every edge of G is contained in $I^e_G[S]$, that is, $I^e_G[S] = E(G)$. The *edge geodetic number* of G, denoted by $g_e(G)$, is the minimum cardinality of an edge geodetic cover of G. Any edge geodetic cover of G of cardinality $g_e(G)$ is called an *edge geodetic basis* of G. If G is a graph and $S = \{x_1, \ldots, x_k\}$ an edge geodetic set of G, then S is a $linear$ $edge\ geodetic\ set$ of G if for any edge $e \in E(G)$, there exists an i, $1 \leq i < k$, such that $e \in I^e_G[x_i, x_{i+1}]$. Moreover, a set of vertices S in a graph G is a $double\ edge\ geodetic\ set$ of G if for any pair of edges, $ab, xy \in E(G)$, there exist $p, q \in S$ such that both $ab, xy \in I^e_G[p, q]$. In this paper, we established the lower and upper bounds of the edge geodetic covers of the cartesian product of two connected graphs G and H where these bounds are sharp. In particular, if G and H are graphs with $g_e(G) = p \geq g_e(H) = q \geq 2$, then $g_e(G \times H) \leq pq - q$ while if G and H are connected graphs, then $max\{g_e(G), g_e(H)\} \leq g_e(G \times H)$. If we let G and H be connected graphs with $g_e(G) = p$ and $g_e(H) = q$ and if both G and H contain linear minimum edge geodetic covers, then $g_e(G \times H) \leq \lfloor \frac{pq}{2} \rfloor$. For any trees T_1 and T_2, $g_e(T_1 \times T_2) = max\{g_e(T_1), g_e(T_2)\}$. Lastly, if a graph G has a double minimum edge geodetic cover, then $g_e(G \times G) = g_e(G)$.

Crista Arangala (ccoles@elon.edu) Elon University

Turning Lights Out

The Tiger Electronics hand-held Lights Out toy is traditionally played on a board of 25 buttons set up in a 5×5 grid. Each button can start in an on or off state and the object of the game is to get all of the lights off by pressing the different buttons. However each time a button is pressed, it not only changes it's state, but it also changes the state of all adjacent buttons. This problem consumes hours of the average players time but with this module, one can determine solutions of any binary Lights Out game. In this talk the history behind the mathematics of Lights Out and a unique graph theory approach, to show that every Lights Out toy has a solution when all buttons start in the on state will be presented.

Vladimir Riabov (vriabov@rivier.edu) Rivier College

Graph Theory Applications in Code Analyses and Developing Software Test Strategies

The graph theory is effectively used in analyses of logical complexity of various computer-programming codes. It is shown that three branches of the graph theory (algebraic, predicative-logical, and topological approaches) provide simple algorithms for calculating the cyclomatic complexity of flowgraphs that represent the code structures. The main graph-based metrics (cyclomatic complexity, essential purely-structured-logic complexity, module design complexity, system design complexity, and system integration complexity parameters) are reviewed and applied for studying the C-programming code complexity and estimating the number of system integration and unit tests for two networking systems: Carrier Networks Support system with switches and routers, and Aggregation System for networking services. Comparing different code releases, it is found that the reduction of the code complexity leads to significant reduction of errors and maintainability efforts. The test and code coverage issues for these systems are also discussed.

Nancy E. Clarke (nancy.clarke@acadiau.ca) Acadia University
Gary MacGillivray (gmacgill@uvic.ca) University of Victoria

Characterizations of Cop-Win Graphs

The perfect information graph game –Cops and Robber" was introduced around 1980 by Quilliot, and Nowakowski and Winkler, independently. There are two sides: a collection of $k > 0$ cops and a single robber. The cops begin the game by each choosing a vertex to occupy. The robber then chooses a vertex and the two sides move alternately, with the cops moving first. A move for the cops consists of each cop traversing an edge to a neighbouring vertex, or remaining at his current vertex. A move for the robber is defined analogously. The cops win if any cop occupies the same vertex as the robber after a finite number of moves (some cop catches the robber), and otherwise the robber wins. Both Quilliot, and Nowakowski and Winkler gave a characterization, in terms of a certain vertex ordering, of the finite graphs in which one cop has a winning strategy. Nowakowski and Winkler also gave a relational characterization of these graphs that extends to the infinite case. We describe an extension of these characterizations to the case of k cops.

Dov Zazkis (zazkis@unbc.ca) University of California San Diego

Considering Symmetries of the Middle Levels (An Interesting Approach to a Special Case)

Let k be a positive integer. We define M_k to be the graph with a vertex set consisting of all binary strings of length $2k + 1$ which have either k or $k + 1$ ones and edge set consisting of all pairs of these binary strings which differ in exactly one bit. Showing that the graph M_k is Hamiltonian for all k is known as the Middle Levels problem. This problem was first posed in the early 1980's and to this day remains unsolved. In this paper we explore the symmetries of M_k and graphs related to it. We then use these symmetries to propose a method for finding Hamiltonian cycles in M_k when $2k + 1$ and k are prime. We believe that our method is more efficient than methods proposed by previous authors.

Ben Small (bentsm@gmail.com) Seattle University
Leanne Robertson (robertle@seattleu.edu) Seattle University

Extensions of Newman's Conjecture and Applications to Prime Trees

In 1980, Carl Pomerance and J. L. Selfridge proved D. J. Newman's coprime mapping conjecture: *If nis a positive integer and I is a set of n consecutive integers, then there is a bijection $f : \{1, 2, \ldots, n\} \to I$ such that* $\gcd(i, f(i)) = 1$ *for* $1 \leq i \leq n$. Around the same time, Roger Entringer conjectured that all trees are *prime*, that is, that if T is a tree with vertex set V, then there is a bijection $L : V \to \{1, 2, \ldots, |V|\}$ such that $\gcd(L(x), L(y)) = 1$ for all adjacent vertices x and y in V. There has been little progress so far towards a proof of this conjecture. In this talk, I will discuss extensions of Newman's conjecture and how they can be used to prove that various families of trees are prime, including palm trees, banana trees, binomial trees, and certain families of spider colonies.

Jonathan Adler (jonadler@wpi.edu) Worcester Polytechnic Institute
Nicholas LeCompte (nl@wpi.edu) Worcester Polytechnic Institute
Peter Christopher (peterrc@wpi.edu) Worcester Polytechnic Institute

Prism Complement Hamiltonian Graphs

Let G be a graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and let $\bar{G}$ be its complement with vertex set $\bar{V} = \{\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_n\}$. The *complementary prism* is defined to be the graph $G\bar{G}$ with vertices $V \cup \bar{V}$ and with edges consisting of the edges in G and $\bar{G}$ together with edges $\{v_i, \bar{v}_i\}$ for $i = 1, 2, \ldots, n$. G is said to be *prism complement Hamiltonian* if $G\bar{G}$ is Hamiltonian. We examine the classification of such graphs and consider how prism complement Hamiltonian relates to other traversibility problems.

Cameron Wickham (cwickham@missouristate.edu) Missouri State University

The Genus of a Zero-Divisor Graph.

To each commutative ring, one can associate a graph, called the zero-divisor graph, whose vertices are the (nonzero) zero divisors of the ring, and an edge is drawn between two vertices if the product of the vertices is zero. This graph was essentially introduced in 1988 by Beck, and has been studied extensively in the last ten years. We will present examples and briefly discuss some basic properties and structure of these graphs. In particular, we will discuss recent results about the genera of zero-divisor graphs of finite commutative rings and possible links to ring theoretic properties. Some of these results are derived from work done at the Research Experiences for Undergraduates at Missouri State University.

Josefina Flores (josefina.flores631@gmail.com) MAA

The Radio Number of Ladder Graphs

The radio labeling of graphs originated from the real world problem of radio transmitter frequency assignment, which depends on distance between transmitters. For a connected graph G, let $d(u, v)$ denote the distance between any two vertices u and v. The diameter, $diam(G)$, is the longest distance in G. A radio labeling c of G is an assignment of positive integer values to the vertices of G that satisfies $d(u, v) + |c(u) - c(v)| \geq diam(G) + 1$, for all vertex pairs u and v. The maximum integer produced by the labeling is the span of the labeling. The radio number of G, $rn(G)$, is the minimum achievable span. Let L_n be a ladder graph with n rungs and $2n$ vertices. As stated by Liu and Zhu, "…determining the radio number seems a difficult problem even for some basic families of graphs." We determine the radio number of ladder graphs.

Jennifer Gorman (gorman005@gannon.edu) Gannon University

Theshold Graphs, Linear Forests, and Hamiltonian Paths

There are well known necessary and sufficient conditions for a Threshold graph to have a Hamiltonian Path. In this talk we will investigate Hamiltonian paths that contain a given linear forest. We will give necessary and sufficient conditions for determining if a threshold graph G contains a specified linear forest P. We will also discuss an efficient algorithm to construct such a Hamiltonian path if one exists.

Sin-Chye Lew (sclew@unisim.edu.sg) SIM University

A Methodology for Converting a Two-Way Road Network to a One-Way Road Network

It is not uncommon in the field of traffic engineering for two-way roads to be converted to one-way roads and vice versa. In these situations, the traffic engineer is faced with the problem of selecting directions for all the one-way roads so that a strongly-connected network can be achieved. This problem is named the one-way road network problem. Graph theory and minimum traverse-time algorithms provide a model of the network. This coupled with minimum cut graph model form a model referred to as the one-way road network model for solving the above-mentioned problem. A real-life case study is used for testing and validating the model. Two-way road networks are represented by undirected graphs and one-way road networks by digraphs. The problem then becomes one of converting an undirected graph to a strongly-connected digraph. A brute-force algorithm was initially used for obtaining feasible digraphs. In order to obtain a true-to-live, efficient one-way road network, the topological aspects of network efficiency incorporating realistic delay times at junctions is explored. Dijkstra's algorithm is modified to calculate the network shortest total travel time in the search for a suitable initial road network layout. With the selected road network layouts, flipping of the various one-way roads at identified critical junctions on the network is tested using a minimum-cut graph model. This allows efficient consideration of not only each direction but combinations of directions from different roads as well. A theoretically optimal one-way road network for the given topology and traffic loading is obtained for the selected case study. The one-way road network is tested against the original two-way road network by means of simulation. Results of the simulation runs showed that the one-way road network performed better than the original two-way road network under given traffic load of the case study. This thus validates the one-way road network model.

Kathryn Weld (kathryn.weld@manhattan.edu) Manhattan College

On Groups Admitting a Cayley Prism Mapping

A group G is said to admit a Cayley prism mapping σ provided that σ is a bijection on G and moreover, for each Cayley subset $S \subset G$, σ induces an automorphism of the associated Cayley graph $Cay(G, S)$. The identity map on G is a trivial example. In the case that G is isomorphic to direct product of the quaternion group and an elementary abelian two group, the inversion map is an example of a prism map on G. In this paper we classify the groups that admitting a Cayley prism mapping which is neither the identity mapping nor the inversion mapping. In this paper we classify all groups that admit a Cayley prism mapping. (Preliminary report.)

Advances in Recreational Mathematics

Thursday, August 6, 2:00–5:15 PM

John Beam (beam@uwosh.edu) University of Wisconsin Oshkosh

Moviemaking Magic and Mishaps

Creating scale models for motion pictures requires much more mathematics than just proportional reasoning. Shutter speed and strength of materials, for instance, are crucially affected by the scaling factor. We will see why, by exploring some good and bad examples on film. The presentation is designed to serve as a motivating application of basic concepts from a first-semester calculus course.

Mike Spivey (mspivey@ups.edu) University of Puget Sound

Can War Last Forever?

The card game War usually takes so long to play that it often seems as if it will last forever. Is this actually possible, though? Can a War deck enter a cycle and thus truly last forever? In this talk we discuss a simplified version of War in which the fifty-two cards in the deck have a strict ranking from 1 to 52 and in which the winning card and losing card are immediately placed, in that order, at the bottom of the winning player's deck. Under this variation of War we show that it is possible for a fifty-two-card deck to cycle, and we exhibit such a cycle. This result is a special case of a more general result that exhibits a cycle construction for an ¡math¿n¡/math¿-card deck, for any value of ¡math¿n¡/math¿ that is not a power of 2 or 3 times a power of 2. We also discuss results that show that under some assumptions the types of cycles we exhibit are the only types of cycles that can occur. Time permitting, we may briefly discuss variations in which the losing card goes into a player's deck ahead of the winning card, as well as War with more than two players. This talk will be accessible to undergraduates.

Paul Cull (pc@cs.orst.edu) Oregon State University
Beth Skubak (elizabeth.skubak@bucknell.edu) Oregon State University
Nick Stevenson (stevnich.@onid.orst.edu) Oregon State University

Towers of Hanoi, Spin-Out, Graphs, and New Puzzles

Puzzles correspond to graphs in which the vertices are the puzzle's configurations and the edges specify the allowed moves. We show that the familiar puzzles, Towers of Hanoi and Spin-Out, correspond to highly structured graphs which we call "iterated complete graphs". These familiar puzzles are based on iterating triangles and lines. We define puzzles based on iterating any complete graph. For odd vertex graphs, our puzzles are Towers of Hanoi puzzles with multiple (more than 3) towers, but with restrictions on the allowed moves. For powers of 2, our puzzles are generalizations of Spin-Out that have multi-level spinners. For other integers, our puzzles are "products" of a generalized Towers of Hanoi puzzle and a generalized Spin-Out puzzle. We show how to solve these puzzles by using recursive, iterative, and "counting the moves" algorithms. We also show some remarkable properties of the "iterated complete graphs": (1) They support (essentially unique) perfect error-correcting codes; (2) they have Hamiltonian paths or circuits; and (3) the distance between vertices is easy to compute.

Gary R. Lawlor (lawlor@mathed.byu.edu) Brigham Young University

Abstract Algebra Sheds Light on Peg Solitaire, and Vice Versa

By looking to the mathematical heart of peg solitaire we find fresh ideas for variations on the game, generating a whole new array of puzzles and games. By delving into these ideas, beginning students of abstract algebra can get an idea of what the subject is about and what it is good for, and can have a concrete example of a group, subgroup, generators, and cosets.

Joel Haack (Joel.Haack@uni.edu) University of Northern Iowa

Slitherlink as a Model of Levels of Abstraction

A characteristic of mathematics is the opportunity to discover, then revisit and refine, properties and propositions in distinct or increasingly abstract settings; a favorite example is provided by the ring of integers, rings of polynomials over fields, and principal ideal domains. It can be difficult to help those without significant formal mathematical background to experience this process. The logic puzzle Slitherlink and its variant forms can provide such an opportunity.

Eric Gottlieb (gottlieb@rhodes.edu) Rhodes College
Whitney DuVal (duvwr@rhodes.edu) Rhodes College

Short Impartial Combinatorial Games Played on Partitions

We offer a variety of Nim-like games played on partitions or sets of partitions. We provide conditions on the objects of play, in some cases necessary, in some cases sufficient, in some cases both, for the second player to have a winning strategy.

Matthew Hudelson (mhudelson@wsu.edu) Washington State University

A Hadamard Matrix Coin-Sifting Algorithm

The most widely known coin weighing problems typically involve using a balance scale a small number of times to locate a very small number of counterfeit coins among a much larger number of coins. In this discussion, we consider the problem of efficiently using a digital scale to sift, i.e. separate into piles of counterfeits and non-counterfeits, a collection of coins that could have any number of counterfeits among them. We begin by demonstrating small examples, first by using the scale three times to sift four coins and then using the scale seven times to sift twelve coins. These examples are then generalized to a larger-scale sifting algorithm constructed using Hadamard matrices, with the result that the ratio of weighings to coins vanishes as the number of coins is made very large.

Brian Kelly (bkelly@bryant.edu) Bryant University

Valuation of Discount Points

In mortgage lending, mortgage discount points (if used) are collected up front in return for a reduction of the interest rate. A borrower would consider buying them because then the total paid over the life of the loan offsets the up front cost. We examine how the lender benefits, and consider how refinancing or foreclosure concerns can influence the lender's position. The principle tools are geometric sequences and the exponential series

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}.$$

One of the goals is to develop a rule of thumb similar to the rule of 70 for doubling time of an investment.

Cindy Traub (cytraub@siue.edu) Southern Illinois University Edwardsville

Geogebra, Geodesics, and Gift Wrapping

When tying a ribbon around a wrapped box (or other polyhedral gift), it is preferable that the ribbon bend neatly around the edges of the box without twisting or buckling. A ribbon with this property represents a family of parallel geodesics on the surface of the object being wrapped. In this talk we will discuss how the dynamic geometry software Geogebra can be used to visualize and study polyhedral geodesics. We will present some results towards a characterization of dodecahedral geodesics.

Christopher J. Dugaw (dugaw@humboldt.edu) Humboldt State University

Radioactive π?

Similar to Buffon's famous needle, I will describe an experimental method to calculate π using radioactive material. Surprisingly this method has nothing to do with circles or angles and the square root is the most complicated function needed. Calculus and undergraduate probability are only mathematical tools needed to understand the method.

Fascinating Examples from Combinatorics, Number Theory, and Discrete Mathematics

Session 1: Thursday, August 6, 8:30–10:30 AM

Aklilu Zeleke (zeleke@msu.edu) Michigan State University

New and Old Combinatorial Identities

Binomial coefficients $\binom{n}{k}$ appear in different areas of mathematics (for example, in Pascal's triangle, counting problems and computing probabilities). There are also many identities that involve binomial coefficients. In this talk we discuss new and old identities that represent real numbers of the form $xu^m \pm yv^n$, where x, y, u, v are real numbers and m, n are nonnegative integers. One such identity is

$$\begin{aligned}\frac{1}{2^{n+2}} \sum_{k=0}^{n} \frac{1}{2^k} \binom{n+k}{2k} &= \frac{1}{4^{2n+1}} \sum_{k=0}^{n} 9^k \binom{2n+1}{2k+1} \\ &= \frac{1}{6}\left(1 + \frac{2}{4^{n+1}}\right),\end{aligned}$$

These identities are derived from studying the asymptotic behavior of the roots of a generalized Fibonacci polynomial sequence given by $F_j(x) = x^{j-1} - \cdots - x - 1$.

Alexander Soifer (asoifer@uccs.edu) University of Colorado

Building a Bridge Between Problems of Mathematical Olympiads and "Real" Problems of Mathematics

New Olympiad problems occur to us in mysterious ways. This problem came to me one summer morning of 2003 as I was reading a never published 1980s manuscript of a Ramsey Theory monograph, while sitting by a mountain lake in Bavarian Alps. It all started with my finding a hole in a lemma, which prompted a construction of a counterexample (part b of the present problem). Problem (a) is a corrected particular case of that lemma, translated, of course, into a language of a nice "real" story of a chess tournament. I found three distinct striking solutions of (a) and an even more special solution of (b). As a result, this problem became the most beautiful Olympiad problem I have ever created. What is more, the journey that lead me from Ramsey Theory to problems of mathematical olympiads, continued to finite projective planes!

Chess 7 7 (21st Colorado Mathematical Olympiad, April 16, 2004, A.Soifer).

(a) Each member of two 7-member chess teams is to play once against each member of the opposing team. Prove that as soon as 22 games have been played, we can choose 4 players and seat them at a round table so that each pair of neighbors has already played.

(b) Prove that 22 is the best possible; i.e., after 21 games the result of (a) cannot be guaranteed.

I will present beautiful solutions of this problem, and show its link with "real" mathematics.

Valerio De Angelis (vdeangel@xula.edu) Xavier University of Louisiana
Victor Moll (vhm@tulane.edu) Tulane University
Tewodros Amdeberhan (tewodros@math.mit.edu) Tulane University

The 2-adic Valuation of the Complementary Bell Numbers

The n-th Bell number is the number of partitions of a set with n elements. The n-th complementary Bell number is the difference between the number of such partitions with an even number of sets, and those with an odd number of sets. While the 2-adic valuation (or largest power of 2 factor) of the Bell numbers is easily calculated, the 2-adic valuation of the complementary Bell numbers is much harder to find. In this talk we will present explicit values of the valuation when $n \not\equiv 14 \pmod{24}$, and discuss the more difficult case $n \equiv 14 \pmod{24}$.

Jeff Johannes (johannes@member.ams.org) SUNY Geneseo

Counting Student Groupings

A student teacher came back to campus one day and told me a Spanish teacher at her school had asked her (and the other math teachers) how many ways she could pair her students without including the same pair twice. I posed the question to my combinatorics class and one student has extensively worked on the problem and its extensions. I will present the problem and discuss related progress.

Joshua Holden (holden@rose-hulman.edu) Rose-Hulman Institute of Technology

"How Do You Say 'Cryptography' in Romanian?" Learning About Integers from Ciphers in Different Languages

Using simple ciphers based on modular arithmetic to illustrate facts about the integers is now a relatively common part of the discrete mathematics and number theory curricula. However, most of the time these illustrations stick to ciphers based on the English version of the Roman alphabet, which has 26 letters. Other languages have different numbers of letters — from as few as eleven or twelve (depending on how you count) for some Pacific languages to more than 50 for some languages of India and of the Caucasus region. But from the point of view of number theory, the important question is not how large the alphabet is, but the prime factorization of the number of letters! We will explore some effects that this prime factorization has on common arithmetic ciphers.

Annela Kelly (akelly@rwu.edu) Roger Williams University

Fixed Points and Modular Calculations in Cryptology

Our talk will discuss fixed points in some private-key substitution ciphers that incorporate modular calculations. The equations determining fixed points motivate interesting methods for their solutions. For example, I advised an undergraduate research project which categorized fixed points for the affine cipher and modified the cipher to eliminate the fixed points. In addition to this, we will discuss the Hill cipher and the properties of its fixed points.

Session 2: Friday, August 7, 8:30 AM–12:05 PM

Tyler J. Evans (evans@humboldt.edu) Humboldt State University

Generalizing Some Well Known Congruences Using Burnside's Theorem

Number theory and abstract algebra students are exposed to famous congruence theorems such as Fermat's little theorem and Wilson's theorem. Each of these theorems state that a certain integer valued sum is always divisible by a prime integer. In this talk, we will use some elementary actions by finite cyclic groups and Burnside's theorem to derive generalizations of the aforementioned theorems to the case of an arbitrary positive integer divisor.

Len Smiley (smiley@uaa.alaska.edu) University of Alaska Anchorage
Brian Wick (afbdw@uaa.alaska.edu) University of Alaska Anchorage

12 Coins Problem, Yet Again, a New Viewing

The 12 Coins Problem is famous, well-understood, and often presented to undergraduates. We produce, with computer assistance, a small set of essentially different solutions which we claim to be exhaustive. We model these solutions with displayed bouquets of 3-dimensional vectors.

Shanzhen Gao (sgao2@fau.edu) Florida Atlantic University

Pattern-Avoiding of Self-Avoiding Walks

A self-avoiding walk (SAW) is a sequence of moves on a lattice which does not visit the same point more than once. It was given as one of the two classical combinatorial problems in the Encylopaedia Britannica. A SAW is interesting for simulations because its properties cannot be calculated analytically. Calculating the number of self-avoiding walks in any given lattice is a common computational problem. We will present some interesting problems on pattern-avoiding of self-avoiding walks and show you how to solve a few of them.

Suzanne Ingrid Doree (doree@augsburg.edu) Augsburg College

Counting on the Hanoi Graphs

The Hanoi graphs are the state graphs for the multi-peg version of the Tower of Hanoi puzzle. In this talk we introduce this intricate family of graphs and derive formulas for the size, order, and degree of vertices. Along the way we provide an alternate combinatorial proof of Stirling's Identity.

Brian Hopkins (bhopkins@spc.edu) Saint Peter's College

Pell Numbers and Related Identities via Charged Tilings

Fibonacci numbers can be represented by tilings consisting of squares and dominoes, as popularized by Benjamin and Quinn, allowing insightful combinatorial proofs of many identities. By allowing two types of squares, with positive or negative charge, we present a related representation of Pell numbers, 1, 2, 5, 12, 29, 70... This allows for combinatorial proofs of identities relating to the Delannoy triangle, tribonacci numbers, and a result attributed to Moriarty.

Brant Jones (brant@math.ucdavis.edu) University of California at Davis

Permutation Patterns in Enumerative Combinatorics

Generating functions encode the solution to counting problems in enumerative combinatorics. Among the nicest generating functions are those which can be expressed as a ratio of two polynomials. The coefficients of these "rational generating functions" satisfy a linear constant coefficient recurrence. Permutation patterns are a source of many diverse generating functions in modern enumerative combinatorics. However, it remains a difficult open question to classify which permutation pattern classes lead to rational generating functions. Recent work of the speaker (with H. Denoncourt) has given one construction that transforms permutation pattern classes in a way that preserves rationality of the corresponding generating functions. In this talk, we present a methodology for developing undergraduate research projects around these ideas based on an REU at UC Davis led by the speaker during Summer 2008. One aspect of the project involves generating sequence data using computer programs. The goal of the project is to seek out new collections of examples of rational permutation pattern classes. Along the way, students learn how to use linear algebra to detect recurrences, and use available tools such as Sloane's Online Encyclopedia of Integer Sequences to compare their enumerative results with those of other researchers.

Silvia Heubach (sheubac@calstatela.edu) California State University Los Angeles
Matthieu Dufour (dufour.matthieu@uqam.ca) Universite de Quebeq a Montreal

Circular (n,k)-games

A circular (n,k)-game is a two player impartial combinatorial game consisting of n piles of tokens placed in a circle. A move consists of choosing k consecutive piles, and taking at least one token from the k piles. More precisely, if t_j is the number of tokens in pile j, and a_j is the number of tokens the player selects from pile j, then the rules are as follows: Pick piles $i, i+1, \dots, i+k-1$ (modulo n) and select $0 \le a_j \le t_j$ tokens from pile $j = i, i+1, \dots, i+k-1$ with $\sum_{j=i}^{i+k-1} a_j \ge 1$. Typical questions are whether there is a strategy for one of the two players that allows this player to win no matter how the other player plays. This question is answered by determining the set of losing states. We will present results on the structure of the losing states for small n, some conjectures, and variations on the game.

Scott Hochwald (shochwal@unf.edu) University of North florida

The Amazing, Astounding, Phantasmagorical Central Binomial Coefficient

What do Catalan Numbers and Bertrand's Postulate have in common? Other than the fact that they are misnamed and should really be known as Euler Numbers and Chebyshev's Theorem, respectively, they are both tied in some way to the Central Binomial Coefficient, C(2n,n). Other things that can be linked to the Central Binomial Coefficient include the Harmonic Series and related series, truncated, of course; Bernoulli Numbers, and some conjectures due to Erd̋os. This talk will expose all connections and present the ideas behind the proofs of most results that are discussed.

Adam G. Weyhaupt (aweyhau@siue.edu) Southern Illinois University Edwardsville

Students, Sudoku, Permanents, and Combinatorial Proof: An Upper Bound for Permanents of $(0,1)$-matrices

The permanent of a matrix is computed similarly to the determinant of a matrix, except that instead of summing over the permutations and alternating signs we make no sign change when computing the permanent. Permanents have myriad applications in combinatorics and graph theory; one particular combinatorial application involves the number of ways to choose a system of distinct representatives from a collection of sets. When working recently with a student on her senior project (related to Sudoku), we searched the literature to find a good bound for the permanent of a $(0,1)$-matrix. One such bound had a published proof that was complicated and algebraic; we present here a short combinatorial proof and a slight improvement of the bound. We will also look at some numerical results suggesting that for certain matrices this is a very good bound relative to other more general bounds on the permanent. Along the way, we tell a story about the joys of working with students and the delightful understanding possible in combinatorial proofs. We also mention some open questions and some possibilities for undergraduate research.

Gara Pruesse (Gara.Pruesse@viu.ca) Vancouver Island University

The Best Proof is Combinatorial—and now Reed-Dawson has one

A combinatorial proof uses counting to prove an identity. Given an identity such as

$$\sum_k \binom{n}{2k}\binom{2k}{k}2^{n-2k} = \binom{2n}{n},$$

we find a combinatorial object that is counted by the expression on the left of the equals sign using one method, and the expression on the right using another method, thus proving the equality. (In the case of the identity given, try proving that both sides count the number of words of length n over the alphabet $\Gamma = \{a, b, c, d\}$ where the number of a's is equal to the number of b's.) Such proofs are often the most elegant, satisfying, comprehensible, and blessedly short proofs that an identity can have, and are much sought-after for this reason. The Reed-Dawson Identity has admitted many long and arduous proofs since it appeared in Riordan's book, Combinatorial Identities, in 1968. In this talk, we present the first combinatorial proof of this identity, also known as Knuth's Old Sum:

$$\sum_k \binom{n}{k}\binom{2k}{k}(-2)^{n-k} = \binom{n}{n/2}$$

if n is even. If n is odd, the sum is 0.

Keith Brandt (keith.brandt@rockhurst.edu) Rockhurst University

The Pill Problem and Catalan Numbers

Suppose you have a bottle of pills and you would like a daily dose of half a pill. Each day, remove a pill from the bottle. If it is whole, break it in half, swallow one half, and return the other half to the bottle. If you remove a half pill, simply swallow it. Over time, you will have a mix of whole and half pills in the bottle. If you know the current number of whole and half pills, what is the probability that the pill you remove n days from now is whole? In my talk I will discuss this problem and its connections with the Catalan numbers.

Session 3: Saturday, August 8, 10:30 AM–12:25 PM

Eric Gottlieb (gottlieb@rhodes.edu) Rhodes College
Jonathan Fitz Gerald (fitzgeraldj@rhodes.edu) Rhodes College

Optimal Selection of Recombinant Inbred Lines for Locating Genes: A Preliminary Report

Biologists sometimes wish to find the chromosomal location of the gene that controls a given trait. One approach is to use a subset of commercially available recombinant inbred (RI) lines. RI lines are homozygous; their (fully mapped) chromosomes are mixtures of the chromosomes of a parent having the trait under consideration and of a parent lacking the trait. By comparing the chromosomes of the RI lines that carry the trait with those that do not, the biologist hopes to determine the location of the controlling gene. Determining which RI lines carry the trait can be costly with respect to time, laboratory space, money, and materials. Thus, it is desirable to use as few RI lines as possible, and also to ensure that the RI lines selected are collectively capable of resolving the location of the controlling gene. We propose an approach towards attaining these conflicting objectives that uses an algorithm that operates on the power set of the RI lines, partially ordered by inclusion.

James Mihalisin (mihalisi@meredith.edu) Meredith College

Clique Swaps, Polytopes and "Near Cubes"

Polytopes are the higher dimensional analogues of two-dimensional polygons or three-dimensional solids such as cubes and pyramids. "Clique Swaps" are a process of transforming one class of polytope into another. "Near Cubes" are polytopes that are only a few clique swaps away from the d-dimensional cube. This talk will describe some results from an REU in 2008 and outline some future directions for undergraduate-accessible research involving clique swaps.

Emil Daniel Schwab (eschwab@utep.edu) University of Texas at El Paso

Characterizations of Prime Numbers via Arithmetic Functions

Chapter 16 of the book "Mathematical Morsels" by Ross Honsberger is devoted to a nice characterization of prime numbers, due to C.A.Nicol and A.Makowski, using three arithmetic functions: *n is prime iff* $\sigma(n) + \Phi(n) = n\tau(n)$. Our talk will present a generalization of this characterization involving the Dirichlet convolution of arithmetic functions. This approach will also yield other interesting characterizations of prime numbers.

Kent E. Morrison (kmorriso@calpoly.edu) California Polytechnic State Univ. and American Institute of Mathematics

Groups of Perfect Shuffles

Although the structure of the two-handed perfect shuffle groups has been known for nearly thirty years, many open problems remain in the understanding of k-handed perfect shuffle groups for $k > 2$. Requiring some background in group theory and linear algebra these problems are suitable for undergraduate research projects.

Kenneth A. Ross (rossmath@pacinfo.com) University of Oregon

Repeating Decimals

The first interesting repeating decimal is the decimal expansion for $\frac{1}{7} = 0.\overline{142857}$. I've known forever that the repeating portions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$ are all cyclic permutations of 142857, but I only recently stumbled upon the well-known fact that the number 142857 has the property that if you break its set of digits into 2 strings of equal size, then the numbers add to 999: $142+857 = 999$. I will call this the 2-block property. The number 142857 also has the 3-block property: $14+28+57 = 99$. If we look at $\frac{3}{7} = 0.428571$, then again $428 + 571 = 999$ but $42 + 85 + 71 = 198$. That is not quite as nice as 99, but $198 = 2 \cdot 99$ and we regard this as close enough and say that 3/7 has the 3-block property. The m-block property always holds for t/p (provided m divides the period of t/p), where p is prime, and for $m = 2$ this is known as Midy's Theorem (1836). These nice block properties hold much more generally, but not always. Consider, for example, the fractions $\frac{13}{77} = 0.\overline{168831}$ and $\frac{150}{259} = 0.\overline{579150}$. I will give several more examples and two simple theorems that explain nearly all appearances of this phenomenon.

David Penniston (pennistd@uwosh.edu) University of Wisconsin Oshkosh

Some Odd Things About Partitions

A partition of a positive integer n is a nonincreasing sequence of positive integers whose sum is n. For example, the partitions of 4 are $4, 3+1, 2+2, 2+1+1$ and $1+1+1+1$. One of the best known open questions in this area is "which integers have an even number of partitions?" In this talk we will present some results on the divisibility of certain restricted partition functions, with special attention given to divisibility by 2.

Resources for Teaching Math and the Arts

Session 1: Thursday, August 6, 9:00–10:15 AM

William Fenton (wfenton@bellarmine.edu) Bellarmine University

An Honors Seminar in Music & Mathematics

In the spring 2008 semester, I taught a freshman seminar in our honors program that focused on music and mathematics. The course examined five broad themes: sound, pitch, rhythm, melody, and theory. The students read short articles, which were then discussed in class. My role was to guide the discussions and fill in the mathematical background. I also found musical examples to play in class. In this talk, I will present the course design, the readings, the student projects, and sample problems from the homework assignments and exams. I will also discuss things I would change for next time.

David Alan Becker (beckerd@fiu.edu) Florida International University

A Truth So Joyously Blueniversal: An Inherently Uniform Succinct 3-D Decoding of Archetypal Jazz/Blues Harmony

This presentation will feature a newly discovered code with demonstrable utility in the analysis and composition of music in the Jazz/Blues genre.

Ioana Mihaila (imihaila@csupomona.edu) Cal Poly Pomona

Fret Not!

This article will discuss applications of modular arithmetic in music, in particular in tuning and playing the guitar.

R. Daniel Hurwitz (dhurwitz@skidmore.edu) Skidmore College
Mark Sicilian (dhurwit1@nycap.rr.com) Skidmore College

Frieze Music

The concepts of transformation geometry are important tools in studying symmetry and patterns in visual art. In art, architecture, pottery, and elsewhere, such ideas as frieze groups and crystallography (wallpaper) groups allow analysis and classification of geometric objects from an algebraic standpoint. But these tools can also be applied to music, with interesting consequences. We will look at the seven frieze groups (as well as the twenty-four two-colored friezes) and apply them to the musical staff to see (hear) where a simple leitmotif can lead us.

Session 2: Thursday, August 6, 1:00–5:35 PM

Arthur Thomas White (arthur.white@wmich.edu) Western Michigan University

"Math and the Other Arts", a course at Western Michigan University

For 35 years I have been teaching a general education honors college course at Western Michigan University that focuses on math as a creative endeavor with aesthetic values, and the interaction of math with the "other arts": the visual arts (painting sculpture, dance, architecture, photography, film), literature (poetry, short stories, novels, drama), and music (classical, 12-tone, English church-bell ringing). Now I am teaching each year an upper level math-major course with the same focus, at a more advanced level. We adopt G. H. Hardy's criteria for creativity (unexpectedness, inevitability, economy) and apply them to math as an art form in its own right, and to various interactions of math with the more traditional arts. I will discuss some of these, such as magic squares in the work of Durer and Gaudi leading to elementary topics in group theory and surface topology, the importance of prime numbers for haiku and tanka, and the relevance of permutations, Fibonacci numbers, and graph theory to change ringing. I will also discuss David Auburn's play "Proof" (and the movie of the same title), including its themes of gender and age in math, and the theorem—not specified by Auburn—which was proved. (Auburn read Hardy's "A Mathematician's Apology" in preparation for writing his Pulitzer-Prize-winning play.) I have written a book to support the course.

Ann Robertson (arob@conncoll.edu) Connecticut College

Beyond Math and the Arts

The talk will highlight how Math and the Arts' material can be effectively incorporated into other liberal arts math courses with minimal added work. For example, Connecticut College's Introduction to Mathematical Thought course with its theme, "To describe, adapt and succeed in our world" is assisted by the novelette, *Flatland*, Salvador Dali's, "The Crucifixion (Corpus Hypercubicus)", M.C. Escher's mathematical tilings, Max Ernst's "Swamp Angel," African architecture, and Jackson Pollock's poured paintings. Also to be discussed, with math and the arts content are the Geometric Issues, Mathematics from a Cultural Perspective, Calculus I courses and a first year seminar in Fractals, Chaos, and Culture.

Cheryl McAllister (cjmcallister@semo.edu) Southeast Missouri State University

Using the Mathematics of Art as a Freshman Seminar Theme

'The Mathematics of Art' is the theme for a freshman seminar whose major goals are to introduce students to the purposes and goals of a liberal education. The session will present a brief outline of the course, activity and project ideas, and a discussion of how to grade such projects.

Steven B Zides (zidessb@wofford.edu) Wofford College

Liberal Arts Mathematics: "A Conceptual Art Form"

The Conceptual Art movement of the 1960's/1970's characterized itself as elevating "Platonic Ideas" to paramount importance, while downplaying material manifestation as nothing more than ephemeral modes of sensory stimulation. A classic example of this movement being Joseph Kosuth's "One and Three Chairs", which celebrates "chairness" through semantic, photographic and physical representations. Thought about in just the right way, the goals of the typical Liberal Arts Mathematics course are not far removed from this artistic manifesto. Such math courses elevate ideas such as infinity, symmetry, iteration, etc., knowing that the material resources (books, notes and handouts) will be long gone by the start of the next semester. With this epiphany in mind, I have designed a new math course which emphasizes is Conceptual Artiness. Much like the "One and Three Chairs" example, instead of introducing a concept like infinity through a single monolithic resource (such as the traditional $100 textbook), we will celebrate the concept of infinity through philosophical, historical, literary and artistic representations. Although students will still be expected to work through some traditional number crunching exercises, the real focus of the course is in the paintings, short stories, poems and plays that accentuate the mathematical ideas. In this talk I will discuss the structure of the class, the necessary resources involved and the assessment results obtained. Emphasis will be given to the specific course topics which are highly correlated to the visual arts.

Ruth G. Favro (favro@ltu.edu) Lawrence Technological University

Student Projects in a Geometry and Art Course

Discussion of projects that worked (or didn't) in a required math course designed for students with strength in the visual arts. The course moves from the visual to the symbolic, covering several areas, from symmetry and tiling and classification, to regular and semi-regular solids, to digging for gold (the ratio, the rectangle, and related topics), and perspective drawing.

Annalisa Crannell (annalisa.crannell@fandm.edu) Franklin & Marshall College

Using Fractals to Motivate Linear Algebra

Fractals are a catchy hook for luring students into Linear Algebra. In this talk, we describe a module that introduces fractals and Iterated Function Systems in a standard Linear Algebra course setting. We begin this module by introducing a project that encourages the student to adopt a fractal and then find out "everything" about this fractal. The rest of the module leads students through the concepts needed for the completion of the project: affine transformations, matrix notation and multiplication, determinants, eigenvalues, eigenvectors, and Hausdorff dimension.

Cynthia Lee McGinnis (mcginnic@nwfstatecollege.edu) Northwest Florida State College

Dance Choreography and Dance

Mathematics and Dance If mathematics is a study of pattern, then dance choreography can be described using mathematics. Dance choreography often displays pattern symmetry types such as reflection, rotation, translation and glide. Students explore mathematical patterns and use them to create dance choreography. In the past year, I worked in collaboration with the dance department to choreograph a piece for our spring dance program. We used Pascal's triangle as a choreographic tool, as well as, algebra and geometry. The pattern of Pascal's triangle, 1, 1,1, 121, 1331 gave us the number of dancers on stage. The formula 'length of segment' = t +1.5, where t was the length of the first stage of the dance. The patterns formed onstage were geometric, triangles; two triangles - or square. What resulted was a lively dance enjoyed by the dancers, as well as, the audience. Dance choreography can, also, be used as a teaching tool in a mathematics class. To choreograph a dance the choreographer must count the number of phrases in the musical score and note what time signature the music is in. From there, the choreographer can make a rule, use a math pattern or a combination of both to build the steps of the dance. Why use dance to teach math? Students who are art majors find an outlet for their creativity. Non-art majors enjoy the team work and the kinesthetic approach to learning. All students like being able to build something that is all their own.

Henry Segerman (henrys@math.utexas.edu) The University of Texas at Austin

The Mathfest 2009 Poster Image, Mathematical Art, Design and Education in Second Life

The image on the poster for Mathfest 2009 is a colourful numbered pinecone, which I designed with input from Ravi Vakil for the cover of the second edition of his book *A Mathematical Mosaic.* I will talk about the design process, which took place within the online 3D virtual world "Second Life" using a combination of algorithmic processes. I will also talk about mathematical artwork I and others have created within Second Life and some ideas on using it to teach mathematics.

Susan Kelly (kelly.susa@uwlax.edu) University of Wisconsin-La Crosse

Mathematical Methods for Determining Art Forgeries

Possibly the most famous art forgery story of the twentieth century involved a possible 17th century Jan Vermeer masterpiece, the Nazi Hermann Goring, and the 20th century painter Han Van Meegeren. Differential equations and radioactive decay were used to date the painting and solve the authenticity mystery, but what can be done if the master artist and possible forger worked during the same or similar time period? Such examples include Otto Wacker's forged 19th century Vincent Van Gogh works and works by 16th century artist Pieter Bruegel and his many imitators. While painting style and products used can aid in the determination in some cases, new research in harmonic analysis is providing a way to find an artist's "fingerprint" in the makeup of the brushstrokes. This talk will present both past and current mathematical methods as well as some fascinating art and history of the forgery mysteries.

Carolyn Yackel (yackel_ca@mercer.edu) Mercer University

Polyhedral Thread Balls

We discuss having students make temari balls (Japanese embroidered thread balls) to reify their understanding of Platonic solids and Euler's formula in a mathematics for liberal arts class. The presenter has used this activity successfully in three different semesters.

Ivona Grzegorczyk (ivona.grze@csuci.edu) California State University Channel Islands

Beauty of Surfaces

As a motivation we will show examples of interesting mathematical surfaces used in arts and architecture (including Gaudi's catenary constructions) and present interactive software-based activities involving students in meaningful artistic creations.

Douglas James Dunham (ddunham@d.umn.edu) University of Minnesota Duluth

The Use of Repeating Patterns to Teach Hyperbolic Geometry Concepts

Hyperbolic geometry is the least well-known and hardest to comprehend of the three classical geometries: Euclidean, spherical, and hyperbolic. Probably the main reason is that there is no smooth, distance-preserving embedding of it in the Euclidean 3-space - unlike spherical geometry (or the Euclidean plane). Thus we must rely on models of hyperbolic geometry: Euclidean constructs that can be interpreted to have meaning in hyperbolic geometry. One such model is the Poincare disk model, in which the hyperbolic points are represented by points inside a bounding circle. The hyperbolic lines in this model are represented by interior circular arcs that are orthogonal to the bounding circle. The Dutch artist M.C. Escher made use of this model to create his four appealing Circle Limit patterns. These patterns can be considered to be repeating patterns in the hyperbolic plane. By carefully examining these patterns, students can learn concepts of hyperbolic geometry. In fact repeating patterns are necessary to show the true hyperbolic nature of what is presented within the bounding circle. While Escher's Circle Limit patterns are a start, it is useful to have quite a few more repeating hyperbolic patterns to broaden the students' experience with them. We will show a number of new repeating hyperbolic patterns.

Fumiko Futamura (futamurf@southwestern.edu) Southwestern University

Drawing Humpty Dumpty In Perspective: Side Topics In Projective Geometry

Projective geometry provides a framework for introducing a host of fascinating topics, including mathematical philosophy, axiomatics, perspective drawing and the controversial development of non-Euclidean geometry. I will share some of the interesting side topics we explored in this course.

Zdenka Guadarrama (guadarrama@rockhurst.edu) Rockhurst University

"The Art of Balance"—Mobiles in a calculus class

This presentation will share the results of the implementation of a mobile construction project introduced at different stages in a second semester calculus class. Throughout the semester mobiles were used to motivate and bring together important concepts like areas, centers of mass and convergence. The project consisted of group activities and an individual component that required each student to build his/her own mobile documenting carefully its mathematical details. The process of constructing their own mobile helped students assimilate concepts, and encouraged them to be creative and investigate their own questions independently. I will discuss my reflections on the project, examples of student work, class discussions, and other extensions.

Active and Innovative Learning Approaches for Pre-service Mathematics Teachers at the K--12 and University Levels

Friday, August 7, 2:30 –6:05 PM

Joseph R. Ediger (ediger@pdx.edu) Portland State University
Eva Thanheiser (evat@pdx.edu) Portland State University

Mathematics for Middle School Teachers: A Program of Activity-Based Courses

For more than 20 years, the Department of Mathematics and Statistics of Portland State University has offered a series of courses designed specifically for the preparation of middle school math teachers. Now offered both as an undergraduate minor and as a graduate certificate, the mathematics for middle school teachers program consists of these eight courses: Computing in Mathematics for Middle School Teachers, Experimental Probability and Statistics for Middle School Teachers, Problem Solving for Middle School Teachers, Geometry for Middle School Teachers, Arithmetic and Algebraic Structures for Middle School Teachers, Historical Topics in Mathematics for Middle School Teachers, Concepts of Calculus for Middle School Teachers, and Teaching and Learning in Mathematics for Middle School Teachers. The courses emphasize problem-solving activities that promote exploration and experimentation, allowing students to construct and reconstruct mathematical understanding. Use of visual models, sharing of multiple approaches to problems, and development of oral and written mathematical communication skills are integral components of each course. The college level mathematics content can often be connected to content appropriate for middle school students, so that activities used in the class are frequently adaptable for use by middle school teachers. In some classes students view video of and/or work in individual interviews with middle school students. These experiences inform the discussion of the mathematical content in those classes. We will share a brief history of the program, its philosophy and focus, and will outline a typical syllabus for one of the courses. A sample activity from the course will be described.

Saad El-Zanati (saad@ilstu.edu) Illinois State University
David Barker (dbarker@ilstu.edu) Illinois State University

Research Experiences for Secondary Mathematics Teachers

We will report on two NSF funded programs that involve preservice (and inservice) secondary mathematics teachers in undergraduate mathematics research. One of the programs, the Teacher Scholar Program (TSP), is a yearlong capstone research experience. The second is a Research Experiences for Undergraduates Site (REU) for both preservice and inservice teachers. Both programs are collaborative efforts between mathematicians and mathematics educators to provide teachers authentic mathematical experiences. The experience of doing mathematics has caused a change in our teachers' views of the nature of mathematics and subsequently their beliefs about teaching and learning. The session will present problems and activities that can be used to prompt mathematics research by secondary mathematics teachers. These problems are accessible to high school students, but can be extended into interesting, challenging, and original mathematics. The development of deep and connected knowledge of high school mathematics allows teachers to provide tasks that challenge and instill curiosity into future generations. In addition to the mathematical component of REU and TSP, we will discuss the educational topics that were connected to these mathematical experiences. For instance, we conducted an investigation of how high school students develop mathematical generalizations, and the role that representations played in this process. In conclusion, we will present data that characterize the changes in our students beliefs and then provide an opportunity for the audience to discuss the role of authentic mathematical experiences in the preparation of mathematics teachers.

Elizabeth Burroughs (burrough@math.montana.edu) Montana State University
Jennifer Luebeck (luebeck@math.montana.edu) Montana State University

Including Pre-Service Teachers in Mathematics Lesson Study

This session will describe a project which partnered a middle school mathematics professional learning community with pre-service secondary mathematics methods students. Together they formed a lesson study team. By examining video and data gathered during the team's meetings, participants in this session will analyze how the pre-service teachers engaged in the lesson study cycle and contributed to the in-service teachers' professional growth. The research lesson's focus on the teaching and learning of slope enabled the pre-service teachers to investigate their own knowledge of algebra and functions in the way called for in *The Mathematical Education of Teachers* (CBMS, 2001). The session will highlight what pre-service teachers learned about worthwhile mathematical experiences.

Velma Venetta Tyson (vtyson@uvi.edu) University of the Virgn Islands
Vanere Goodwin (vgoodwi@uvi.edu) University of the Virgin Islands

Professionalization and the Exemplary Mathematics Teacher: Where's the Connection?

The 2001 Conference Board of the Mathematical Sciences (CBMS) report, *The Mathematical Education of Teachers,* and NCTM's Principles and Standards for Teaching Mathematics share many concerns and recommendations regarding the preparation of mathematics teachers. The 1986 Holmes Group report, Tomorrow's Teachers, strongly advocates improvement in the preparation of teachers. The 1995 report of the Holmes Group, Tomorrow's Schools of Education, makes a case for professionalization of teachers, supported by the Carnegie Task Force, NCTM and other organizations. NCTM strongly proposes the coming together of mathematicians and education faculty. The challenges of the twenty-first century intensify the need for stronger collaboration between these two professions that are key influences in the preparation and professionalization of mathematics teachers. This presentation proposes the importance of incorporating quality professionalization in every mathematics teacher preparation program, fostering motivation and awareness of desirable characteristics to a level that promotes strength, flexibility, the quest for lifelong learning and ability to provide outstanding service as mathematics educators. Of particular interest are recommendations of the CBMS report and implications surrounding research in mathematics education as provided by NCTM. A brief overview of issues such as collaboration between mathematicians and education faculty, types of teacher knowledge and their impact on mathematics teaching, pedagogy, interactive inquiry, technology integration, content and curriculum, performance assessment, lifelong learning and the power of action research will be explored. This may be particularly helpful for professionals involved in the preparation of mathematics teachers.

Jerome Trouba (troubaj@ferris.edu) Ferris State University

Designing, Implementing, and Evaluating a Teacher Training Workshop for New Graduate Mathematics Teaching Assistants

To help train new graduate mathematics teaching assistants (GMTAs), a teacher training workshop was designed based on K-12 literature on the components of effective professional development (Garet, et al., 2001). The workshop consisted of six two-hour seminars over a six week period followed by a classroom component of either peer coaching or observation, for a total time commitment of 18 hours. The content of the workshop addressed teaching techniques specifically targeted to GMTAs. These ideas included elements of reflection, techniques for engaging students with active learning, asking good questions and utilizing wait-time, and using formative assessment techniques. The assessment of the workshop was based on Guskey's (1999) work on the five levels of effective professional development evaluation. Results indicated that a well-designed workshop can impact teaching practices. Through interviews, GMTAs indicated they were more reflective of their teaching, thought more about the questions they asked, and tried to engage their students more during lectures. This project also contributes to the research base regarding adapting K-12 professional development literature into a collegiate setting.

Laurie J. Burton (burtonl@wou.edu) Western Oregon University

Mathematics for Elementary Teachers: Using Virtual Manipulatives

A variety of short hands-on activities designed for student exploration in a foundational Mathematics for Elementary Teachers course will be shared and demonstrated using a simple open access Flash Virtual Manipulative Kit. The kit is linked to a website and any student with an internet connection can access the virtual manipulatives for free. The kit includes a label tool that students can use to explain and illustrate their work. The virtual manipulatives available are pattern blocks, color tiles, polygons fro tessellations, attribute pieces, black and red tiles, fraction bars, numeration pieces, decimal squares, geoboards and spinners. Simple activities from some of the main areas of a foundational Mathematics for Elementary Teachers course will be shared: Activities will be drawn from Number Sense (including fractions and / or decimals), Probability and Statistics and Geometry Techniques for using the virtual manipulatives will be shared and student successes, issues and responses to using the virtual manipulatives regularly as part of their course will be shared and discussed.

Maria Fung (mfung@worcester.edu) Worcester State College
Christine Latulippe (cllatulippe@csupomona.edu) Cal Poly Pomona

Computational Estimation Focus in a Number Systems Course for Pre-service Elementary School Teachers

We describe how a thematic emphasis and focus on computational estimation across a number systems course for pre-service elementary teachers affects their number sense skills and mathematics attitudes. The estimation emphasis includes mental computation practice with natural numbers, decimals and rational numbers; student reflections on articles about number sense and estimation; discussions related to real-life estimation situations; and a focus on number sense development. We report results related to significant improvement in estimation skills and ability, together with an enhanced understanding of the importance of computational estimation. These results are obtained using a pre- and post- survey to collect both quantitative and qualitative data as it pertains to demographics, and to estimation understanding, skills and attitudes.

Virginia (Ginny) L. Keen (keenvirl@notes.udayton.edu) University of Dayton

Mathematics Tasks with Classroom Connections

My early childhood and special education students in foundational mathematics content courses become pen pals of children in local elementary schools. Students have children assigned as pen pals and write two letters (observing Language Arts Standards that the children are to follow) to their first-/second-grade pen pals: 1) a letter with a statistics question for the child to collect classroom data on, and 2) a letter with a probability experiment described that the child may carry out with a family member. I intentionally pair students studying to be Intervention Specialists with inclusion children. One partnering teacher has her children collect the data and then write informal letters to my students in which they share their results. For the culminating task, students create children's books related to geometry that they read to their pen pals when we visit the schools. When visiting the schools, my students read their books, ask the child(ren) about the results of their probability experiments, and possibly carrying out the experiment again. This "pen pal" project grew out of two primary concerns: 1) Concern over my students' discomfort with mathematics, and 2) the necessity for students to make career choices very early in their University experience. By having students develop books that can be used to introduce, support or assess mathematical knowledge, my students become more comfortable dealing with the mathematics. They also are relieved to find that they enjoy working with young children, reassuring themselves that they are preparing for a career they will enjoy.

Robert Ely (ely@uidaho.edu) University of Idaho
Jessica Strowbridge Cohen (jessicac@uidaho.edu) University of Idaho

Productive Discourse From Rich Tasks: The Example of the Double Spin Game

We propose a model and an example for how to (a) implement rich tasks in a mathematics class for pre-service elementary teachers and (b) orchestrate productive discourse by carefully using the work the students produced while doing the task. An example of a rich task is the Double Spin Game: Students pair up and they each design a spinner with sectors of any size and any number in each sector. Each player spins his spinner twice, adds the two numbers, and records the sum. They take turns doing this, and the winner of the round is the first to record each of the sums 2, 3, 4, 5, 6, 7, and 8 at least once. After the students play three rounds, each player revises his own spinner design, then they play again and revise again. Players record their original spinner design and their revised designs, and record their mathematical reasons for revising their designs. To orchestrate productive discourse about the mathematics of compound probability in the task, first the students hand in their record of their designs, revisions, and reasons. The teacher carefully chooses several designs and revisions and has these students present them the next day. This selection process should be done purposefully, following the steps outlined in Stein & Smith's model (1998), outlining a hypothetical learning trajectory. By doing this with the class we also model the method by which these pre-service teachers should themselves design rich tasks and orchestrate productive discourse in their future classrooms.

Paul McCreary (pmccrear@gmail.com) The Evergreen State College

One-Room School House Mathematics Class

How to help pre-service elementary and middle school teachers learn content while engaging in best teaching practices? Arrange for the pre-service teachers to enroll in a mathematics class in which there are multiple learning levels. Ideally the pre-service teachers will have classmates who are more advanced than they are and other classmates who have less preparation than themselves. This will place the pre-service teachers in the position of offering assistance to classmates while also seeking assistance from peers. From this unique perspective the pre-service teachers will practice delivering explanations while reflecting on the effectiveness of explanations made to them. As instructor of the course, I was amazed to witness how quickly students took to tutoring and being tutored by classmates in a class where there were clearly identifiable different levels of achievement. The result was increased reflection by the pre-service teachers on the process of teaching and on the dynamics among students in the classroom.

Zhongxiao Li (zli@ggc.usg.edu) Georgia Gwinnett College

The Structure of Student Dialogue in A Web-Assisted Mathematics Course

Fall term of 2006, a web-assisted undergraduate mathematics course was taught at the University of Idaho: Math 235 Mathematics for Elementary Teachers I. The course goals were: To foster a deep understanding of critical mathematical content; and to promote the development of mathematical communication and collaboration concepts, skills, and dispositions. Outside of regular class periods, students participated in an ongoing asynchronous mathematical dialogue using the Idaho Virtual Campus Discussion Tool. The structure of this dialogue was analyzed using graph theoretic methods associated with social network analysis.

Biomathematics in the Undergraduate Curriculum

Saturday, August 8, 1:00–4:15 PM

Steven McKelvey (mckelvey@stolaf.edu) St. Olaf College

Integrating Biology into a Math Major Track Calculus 2 Course

Many entering first year students begin their college mathematics careers with Calculus 2. It is important to design a course that serves several masters. We seek to offer a course that leads directly into the mathematics major and a course that utilizes examples from biology that will attract science students, particularly aspiring pre-meds, to the serious study of more advanced mathematics. The course described here is not a "calculus for biologists" course, but rather a general purpose course that provides a solid basis in Calculus 2 topics for potential math majors as well as acting as a demonstration of the power and applicability of mathematics to lower division science students. A secondary, but important, goal of this course is to recruit students interested in biology and other sciences, but who may not have considered further study in advanced mathematics, into the mathematics major. The means to this end is to teach a standard Calculus 2 course from a standard textbook, the same text used by our standard non-biology-based Calculus 2 sections, infused with illustrative examples taken from biological applications. These examples involve varying levels of biological sophistication. There is no biology-based prerequisite for this section, it is open to all incoming students ready for a Calculus 2 experience.

Timothy D. Comar (tcomar@ben.edu) Benedictine University

Motivating Mathematical Content in Biocalculus Courses Using Biology

Biocalculus courses are designed to provide quantitative techniques and approaches that will be useful to students majoring in the biological and health sciences in the future coursework and careers. The mathematics must be presented in engaging manner in which the students see the need for the mathematics. To achieve these goals, the mathematics must be appropriately motivated using concepts and problems from biology. We will present several examples of how this can be accomplished using material from our biocalculus courses.

Gretchen A. Koch (gretchen.koch@goucher.edu) Goucher College

Modeling in the Natural Sciences: Introducing Applied Mathematics at the Sophomore Level

This talk will describe a sophomore level applied mathematics course designed for chemistry and biology majors. Having completed Calculus II, students explored various biological models using dynamical systems, linear algebra, and differential equations. Several of the ESTEEM modules that are available through the BioQUEST Consortium (http://bioquest.org) were used as a way to explore the models at a deeper level in Excel. Students also completed paper presentations as a way to link mathematical modeling to their field of interest.

Borbala (Bori) Mazzag (borim@humboldt.edu) Humboldt State University

Introductory Mathematical Modeling at Humboldt State University

In this talk I will illustrate the way various mathematical topics are covered in Humboldt State University's MATH 361: An Introduction to Mathematical Modeling. MATH 361 focuses on various discrete and continuous dynamical systems, and the majority of the applications comes from biology. I will begin with a brief description of the content and organization of the entire course, and then give a detailed look at employing the example of a biochemical switch (based on Lewis, Slack and Wolpert, 1977) to teach bifurcations. I will touch on both the material presented in class, as well as the material in the related weekly computer lab. Computer labs primarily consists of modifying existing Matlab files, conducting numerical experiments and producing a written lab report. If time permits, I will share a summary of successes and limitations of the course in its current format.

Mazen Shahin (mshahin@desu.edu) Delaware State University

Explorations in Mathematical Models in Biology

In this paper we will share the pedagogy and methodology of a course on Mathematical Models in Biology. The course utilizes difference equations and matrix algebra as the main mathematical tools and integrates a computer algebra system and cooperative learning. The main theme of the course is modeling of biological and ecological systems using linear and nonlinear difference equations as well as systems of difference equations. The target audiences of this course are freshmen life science and mathematics majors. Students work in small groups on carefully designed activities that guide them to discover mathematical concepts on their own and explore the connection between biology and mathematics. The contents of the course follow the recommendations of the Curriculum Foundation Project (Biology) of the MAA's Committee on the Undergraduate Program in Mathematics (CUPM). Major portion of this course material has been introduced in a liberal arts core course for undergraduates for several years. In addition, portions of this course were successfully introduced to highly motivated high school students in the summer enrichment programs Explorations in Mathematics and Biology (EMB), and Girls' Explorations in Mathematics and Science (GEMS) for 13 years. We will give a few samples of the instructional material to illustrate the teaching/learning approach and we will share the challenges in implementing such course.

Majid Masso (mmasso@gmu.edu) George Mason University

Novel Pedagogical Resources Based on Protein Structure Analysis

Life science applications in undergraduate mathematics classes typically focus on ODE and PDE modeling of biological systems. Biomathematics courses may also illustrate the utility of graph theoretic, numerical, and statistical approaches in the study of DNA and RNA molecules. However, in-class examples and homework exercises based on protein structure analysis are rarely exploited despite an inherently large number of quantitative applications. This talk will begin with a basic overview of proteins and their 3D structures, followed by a description of one particular approach to protein structure analysis that draws on methods from computational geometry, finite mathematics, probability theory, computer programming, and statistical mechanics. The example provides a rich new source of pedagogical tools that are easily implemented in the mathematics classroom.

Jennifer Marie Franko (frankoj2@scranton.edu) The University of Scranton
Rachel Schwell (schwellrac@ccsu.edu) Central Connecticut State University

Geometry and Biosurveillance

This talk will present a module which incorporates non-Euclidean metrics into biosurveillance through spatial scan statistics. This module may be included in a biomathematics course, as an application in a geometry course, or adapted for a lower level critical-thinking course.

Yves Nievergelt (ynievergelt@ewu.edu) Eastern Washington University

Real and Generic Population Data Without Best-Fitting Verhulst Growth Curves

Myths must be debunked in the first undergraduate mathematics course, because it is the only one some scientists ever take. For example, some population growth modeling methods lack scientific foundations. Specifically, in the topological space of all triples of points in the plane, there exists a non-empty open subset where each triple of points does not admit of any best-fitting growth curve of Verhulst's type for any regression method (ordinary, multiple, orthogonal, weighted, correlated, least squares, least absolute values, etc.). Hence there is no maximum likelihood estimator either. Such an example with real data will be presented. This counter-example demonstrates the need in every application for either theorems to guarantee the existence of a best-fitting curve or surface, or conclusions based on arguments other than best-fit and maximum likelihood.

Daniel Kim (kimd@sou.edu) Southern Oregon University

A Discussion between Lotka-Volterra Model and Elliptic Regression

Lotka-Volterra differential equation model is one of typical undergraduate biomathematics curriculum. This model is useful to relate multiple time-dependent variables with periodic motions. In this talk we first look at a case where Lotka-Volterra model has a limitation, and introduce an effective alternative method called elliptic regression. This is not a typical in undergraduate biomathematics, but it has a framework that can fit into undergraduate biomathematics program in line with Lotka-Volterra model. Theory and an example of the method will be introduced.

Atabong Timothy Agendia (agendia@yahoo.com) Madonna University
M. O. Oyesanya M. O. (moyesanya@yahoo.com) University of Nigeria

Teaching Bioinformatics/Biomathematics at the Undergraduate Level

Biomathematics, as well as Bioinformatics sciences, are gaining grounds amongst mathematicians in the developing (less industrialized) countries. This is so because these sciences try to bring mathematics to reality. However, specializing in these areas is only possible after a first degree in mathematic, computer science and to a lesser extent, biology. Considering a carrier in biomathematics at the undergraduate level is very important for this will produced good biomathematicians. To this end, important questions arise in teaching biomathematics to undergraduate mathematics, biology and computer science students: Can a single course satisfy the demand for professional applications in Medicine, Biology, and Biochemistry? How can the subject be made interesting and What textbooks are to be used? What is the role of the lecturer and at What biological, mathematical and computer science levels should the subject be taught? What physics and chemistry should be incorporated in subject? What practical data should be used that will best validate or consolidate the theoretical concepts? Is there a necessity for tutorials and what courses should be considered as pre-requisites for the subject? In this paper we prepare a view point to some of these problems.

Getting Students Involved in Writing Proofs

Session 1: Friday, August 7, 8:30–11:30 AM

Tom McNamara (thomas.mcnamara@swosu.edu) SWOSU

Experiences Using a Wiki

We will discuss the use of a wiki that allows students in an Advanced Calclulus/Real Analysis course to post, edit and comment on their proofs. An overview of the technical considerations will be given. We will detail the successful outcomes and also look at areas that have room for improvement.

Robert Roe (rroe@mst.edu) Missouri University of Science & Technology

Video Examples of Students Writing Proofs

For many years my department has offered a course to help mathematics students, entering into upper level work, learn to write mathematical proofs. Traditionally this course has used an Inquiry Based Learning approach. This past spring semester the Educational Advancement Foundation funded video recording of the course. I will talk briefly about how the course is run and show video clips showing the progress students make in their proof writing abilities during the semester.

Hossein Behforooz (hbehforooz@utica.edu) Utica College

How Difficult is Writing in Mathematics?

We all know that MATH for many students is another FOUR letter word. Our students have serious problems in doing math, proving theorems, solving math problems and finally writing the proofs or solutions. The fact is that, most of the math departments face the challenges of getting math students to learn how to prove and how to solve math problems on their own way and how to write the solutions or proofs. I have kind of a successful method to solve this problem in my upper level courses and reduce the math fears, frustrations and anxieties of the students. In this presentation I will discuss on my own methods on teaching math with writing projects and I will share the results with you.

Sarah Eichhorn (sfrey@math.uci.edu) University of California, Irvine

Learning Proof Writing by Grading Incorrect Proofs

Asking a student to write a proof can be an insurmountable task if that student does not understand what constitutes a valid proof. Although students may be able to follow and understand a proof given by an instructor, they may not necessarily be able to distinguish a valid proof from an invalid one. Before addressing their ability to generate original proofs, it may be beneficial to give students opportunities to explore incorrect proofs in order to gain a better understanding of what makes a good proof. In my Introductory Real Analysis course, I periodically presented students with a simple theorem and sample "proofs" of the theorem. These "proofs" were actually incorrect in various ways. Students were asked to evaluate and grade the "proofs" in groups. After this exercise, I had them try to write a valid proof of the theorem on their own. In my presentation, I will discuss this activity in further detail and discuss its impact on the students' proof writing abilities. The student impact will be discussed using informal, indirect and anecdotal measures gathered from 120 students over two course offerings.

Nathan Carter (ncarter@bentley.edu) Bentley University
Kenneth Monks (ken.monks@gmail.com) University of Scranton

Lurch: Software for Teaching and Writing Mathematical Proofs

If your word processor can check your spelling and grammar, shouldn't a math word processor be able to check your computations and inferences? Although this is a tall order, it is the aim of the Lurch Project (http://lurch.sourceforge.net), a new piece of mathematical software under development. The developers will share their goals for the software, as well as how they have used it in their upper-division math classes to introduce students to proof-writing. We will address the benefits and challenges of using software when teaching proof-writing, and the degree to which the current version of Lurch meets those challenges. The talk will also address how individual instructors can customize Lurch to the needs of their textbooks and courses.

Sam Vandervelde (svandervelde@stlawu.edu) St. Lawrence University

Polishing (Off) Proofs with $\LaTeX\,2_\varepsilon$

Students face a daunting array of potential barriers as they begin to compose their own proofs. They must learn how to use notation correctly, absorb a new vocabulary, master a set of proof techniques, reason with increased rigor, and more. In this talk I will present a pedagogical tool that I call "Polished Proofs" in which students prepare a proof using $\LaTeX\,2_\varepsilon$ as part of a problem set; then receive comments, revise their proof, and resubmit it until the proof is polished. (At which time it is awarded a bonus point.) I will share examples of student work as it progresses through the various stages of revision. I will also discuss student feedback regarding the efficacy of this activity for overcoming the hurdles to developing and writing proofs. Finally, I will outline how I introduce and augment this activity over the course of a semester in my Bridge to Higher Math course.

Doug Ensley (deensley@ship.edu) Shippensburg University

Proof Across the Major: Curriculum, Assessment and Discussion

This presentation will discuss the efforts of Shippensburg University to integrate a tangible proof thread throughout our mathematics major courses. Two important components of these efforts are our department's assessment plan (which establishes proof benchmarks in three specific courses) and the ongoing dialogues between faculty and students on the effectiveness of lower-division content in preparing students for upper-division proofs. The department's assessment plan consists of tracking final exam answers to specific proof questions in three courses: Discrete Mathematics (freshman), Introduction to Abstract Algebra (sophomore/junior), and Advanced Calculus (junior/senior). From these questions, each mathematics major is assessed on specific areas of success as well as on continuing maturity in dealing with abstraction in mathematics. In addition to these assessment benchmarks, we attempt to reinforce and build on proof ideas through targeted exercises and–labs" in other courses as well. This presentation will include an overview of our philosophy on teaching proof, specific proof-building activities form lower-level courses, a summary of our assessment data, and anecdotes from upper-level students on how we are doing.

Yun Lu (lu@kutztown.edu) Kutztown University of Pennsylvania

Proof Writing in Abstract Algebra

I teach Abstract Algebra regularly in Kutztown University of PA. It is one of the core courses for math majors and secondary mathematics education majors. In the last couple of years, I've tried different methods to get students motivated. For example, interactive lecture, in-class work, group projects, presentations, etc. Some worked, while others didn't.In this talk, I will share some of my successful experience with the audience along with students' feedback.

Linda McGuire (lmcguire@muhlenberg.edu) Muhlenberg College

Proving in the Right Circles

This talk would address the use of what this presenter calls "Proof Circles'" as an approach to helping students develop and deepen both proof writing and problem-solving skills. Proof Circles are group work structures (similar to literary circles) with a decidedly mathematical focus and format. This method has been successfully implemented in both a sophomore-level course devoted to teaching basic proof techniques and upper-division mathematics classes that are content specific. In a Proof Circle, students are organized into working groups and each person is assigned a specific role. While it often varies depending on the context in which this technique is used, the functioning of this structure may be described as follows. A group is given a problem to solve and a subsequent proof of their result is required. Each individual student then assumes their role (from Ringleader to Devil's Advocate) and approaches the problem from the perspective that their assigned part dictates. They have specific information to gather, ideas to formulate, and tasks to complete. The group members then report back to each other and begin to craft solutions and proofs based upon their pooled information. Ultimately, proofs are presented to the rest of the class. This approach has proven to be very successful in generating student interest, promoting student focus, developing mathematical (as well as general) communication skills, and improving individual as well as collective output. During the presentation the idea of a Proof Circle would be introduced and defined. Sample problems from various courses would be discussed as well as several examples of student work. Assessment tools used to measure efficacy and student reactions upon course completion would also be addressed.

Session 2: Saturday, August 8, 8:30–11:50 AM

Connie Maude Campbell (campbcm@millsaps.edu) Millsaps College
Kay Somers (mekbs01@moravian.edu) Moravian College

Engaging Students in Critical Thinking about the Proof Writing Process

As students learn to write mathematical proofs there are a number of skills they must develop beyond basic logic skills. These include (but are not limited to): (1) the ability to understand new definitions; (2) strategies for getting "unstuck" when an idea or proof technique does not seem to work; (3) making the transition from the idea of the proof to a well-written proof with an appropriate amount of detail. The authors have been involved in a project that utilizes video case studies as one tool to help students develop these skills. Each video case study shows two or three students working together to prove a theorem that is new to them. Our students watch the students in the videos work to understand and develop the ideas needed in their proofs. By pausing the videos and asking carefully designed questions, our students are able to anticipate next steps and reflect on the proof process to help them develop their own skills. While students are sometimes reluctant to critique the work of their own peers or the work of their instructor, they seem to be very willing to engage in critical analysis of the thought processes of the students in the videos. As such, these case studies offer rich opportunities to give our students permission to think critically about the process of proof writing. We will demonstrate and describe our experiences in using the videocases with our classes.

Tracey McGrail (tracey.mcgrail@marist.edu) Marist College

Proof by Tribal Narrative

The Marist College *Introduction to Mathematical Reasoning* course is a standard first look at proof writing. The audience is for the most part students who have taken calculus and are unfamiliar with formal proof. In order to ease this transition, students are asked to construct proofs as a group exercise. This assignment encourages active participation, attention to detail, and a sense of ownership of the material. The speaker will describe the nature of the exercise and discuss the relative advantages and disadvantages to this approach in comparison to a traditional lecture.

Theron James Hitchman (theron.hitchman@uni.edu) University of Northern Iowa

Proof in Geometry: Euclid and a Class Journal

We describe a method for introducing students to making their own arguments in a geometry class by using a combination of Euclid's Elements as text and a fictitious class journal for student work. We discuss the issues involved and the level of success, with some student work as evidence.

Leon Brin (brinl1@southernct.edu) Southern CT State University

Proof-Starters

One of the greatest hurdles for certain learners of proof writing is Mount I-don't-know-where-to-start. This session will describe two techniques for making this uphill climb. First, putting to use the formal logic that precedes writing proofs (and is then promptly forgotten) is a great way to get students involved. Second, beginning with a proof shell gets the writing started and helps students keep their focus as they write.

Erin Terwilleger Mullen (terwilleger@math.uconn.edu) University of Connecticut

Teaching an Engaged Analysis Class

The first time I taught an undergraduate analysis class, I conducted a traditional class which consisted of lecture for three hours a week and plenty of assigned homework problems. I found that the students did not absorb much from my lectures and were not able to do a large portion of the homework which involved proofs. To combat this in my analysis class this past semester, I decided to take a more interactive approach. This included daily reading assignments from the text where they had to read proofs of theorems and examples on their own and weekly homework sessions where the students discussed some easier problems with each other. Then I assigned just a few key problems to be graded. I found that the homework and exam grades improved dramatically over the previous time I taught the course. These ideas were inspired by a talk given by David Pengelley of New Mexico State University at the Joint Mathematics Meetings in January 2009.

Jason Morris (jrmorris@brockport.edu) The College at Brockport (SUNY)

Teaching Students the Habit of Using Outlines to Construct Proofs

We discuss the difficulty that students often have with getting started on a proof, and with knowing what to do next. We know that certain aspects of setting up a proof are "routine" or even "mechanical", but it can be very difficult for many students to appreciate this. This fact is particularly true once we have made the determination that our class is ready for proofs that are communicated in concise English. Students who are still struggling with the basic logic of proofs may feel that they can follow the instructor's proofs, but will be remarkably unable to handle their own. We will discuss how the common proof strategies can be presented to students in a schematic way that the students can use to understand how to "set up" their own proofs. (Our own method of outlining a proof is based on the the "Fitch method" of subordinate proofs.) Rather than just teaching the technique once, we should convey to our students the practicality of using such a technique throughout their studies and even their careers. Indeed, most of us employ some kind of proof outline not just when introducing students to proofs, but also when conducting our own work. Yet we may ultimately de-emphasize this important skill in class, perhaps employing an outline only for proofs in which the logic is technically demanding. It may well be better to emphasize the utility of outlines.

Scott Beaver (beavers@wou.edu) Western Oregon University

The Natural Role of the Sequences and Series Calculus Course

The Sequences and Series calculus course (S&S) can be structured to provide students with a unique opportunity to build their proofs skills prior to, or concurrently with, a bridge course. Although students in S&S do not prove broadly applicable theorems, the course content demands that students argue the truth or falsity of statements about particular sequences or series. Faithful adherence to this guiding principle in S&S gives instructors two critical tools which can have positive implications for students well beyond S&S. Teachers can begin to familiarize students with the process of applying elementary logical principles to mathematics and can acclimate students to the typical proof structures while the problems are still relatively simple, providing a scaffolding on which students' skills can be built as they progress through their undergraduate years. In this talk, I'll offer a framework for S&S which places logical reasoning on equal footing with content by employing the theorems and convergence tests as axioms, and give sufficient supplementary theorems and definitions to ensure that students can construct appropriately rigorous arguments.

Aloysius Bathi Kasturiarachi (akasturi@kent.edu) Kent State University

Writing Proofs Across the Undergraduate Mathematics Curriculum

For many undergraduate mathematics students the need for rigorous mathematical proof comes as a surprise, often in the middle of an upper level mathematics course. In this presentation, I will share some ideas on how to infuse methods of proof in courses such as College Algebra, Trigonometry, Calculus, Linear Algebra, Fundamental Geometry, and Matrix Theory so that students are better prepared when they enter upper level courses. Pedagogical best practices that have worked well in the past will be shared.

Mary K. Porter (mporter@saintmarys.edu) Saint Mary's College

Writing to Understand Mathematical Proofs

In Foundations of Higher Mathematics, a transition to higher mathematics course at Saint Mary's College, students learn to read, write, and understand proofs. As part of this course, students are required to write a four-page expository paper on a particular proof technique. This year, students in one section of the course wrote papers on mathematical induction, whereas students in the other section wrote about proof by contradiction. In this presentation, I will describe each of the two assignments and discuss the benefits (to students and instructors) and challenges associated with these writing assignments. I will also discuss my current research project exploring the impact of each of these writing assignments on students' understanding of the two proof techniques.

Usha Kotelawala (kotelawala@fordham.edu) Fordham University
Rommel Fernandes (rommelf@u.washington.edu) University of Washington

Student Experiences in Understanding the Processes and Purposes of Proving

Research has found that college students and their mathematics professors often have very different views of mathematical proof (Raman, 2003). This work aims to investigate student experiences with proving and to offer a less traditional learning experience with proving which includes the conjecturing process. A survey was created as a preliminary study to investigate student experiences and habits with proving and their conceptions of the role and purpose of proving. A combination of multiple choice questions and free response items were used to anonymously gather data. The goal in analyzing the qualitative free response questions is to determine what categories emerge as student perceptions of the purpose and role of proving. Multiple-choice items were used to collect background information on respondents and to gain information on student experiences in mathematics courses. Questions focused on determining whether or not students felt that generation of proof was generally obtaining a pre-determined path or whether experiences with proof had allowed open-ended results. This presentation will also share results from an approach to teaching proving with simple but non-routine problems. Students were given a problem with an unknown solution. After exploring the problem, they came up with their own conjectures and worked to develop proofs for these conjectures. This activity was used in an undergraduate course in proving and in course for pre-service mathematics teachers.

Effective Ways to Teach Upper Level Mathematics Courses for Secondary Mathematics Education Majors

Session 1: Friday, August 7, 1:00–3:00 PM

Patricia Baggett (baggett@nmsu.edu) New Mexico State University Dept of Mathematical Sciences
Andrzej Ehrenfeucht (andrzej.ehrenfeucht@colorado.edu) University of Colorado

A Calculus Course for Secondary Mathematics Education Majors without Algebra and Precalculus Prerequisites

We describe a one-semester upper division mathematics course that has been taught four times at New Mexico State University. It teaches both differential and integral calculus through hands-on applications. It has no prerequisites except consent of the instructor, and has therefore been taken by students with very different backgrounds. Students are presented with a design problem whose solution requires the use of calculus. They build a mathematical model of the task, find a solution to the problem involved, use graphing calculators to carry out the necessary computations, and finish by constructing the designed object. The course has mathematics content that is significantly different from most calculus courses (e. g. it uses the Lebesgue rather than the Riemann integral, discusses Newton's method of solving equations, and omits most algebraic formulas typically used to compute derivatives and antiderivatives). It relies more on numerical methods than on the algebraic techniques that are usually presented in calculus, employing algebra mainly during the creation of mathematical models of physical objects. Overall the course covers much less material, but provides a much broader view of the two basic concepts, derivative, and integral. The course is highly motivating for students and gets excellent evaluations. We will describe the content and format of the course and several examples of tasks. We will discuss the main difficulties students encounter in doing the tasks, and the role played by their mathematical background, as measured by courses they have previously taken.

Barbara Shipman (bshipman@uta.edu) University of Texas at Arlington
James Anthony Mendoza Epperson (epperson@uta.edu) The University of Texas at Arlington

Empowering Student Learning in Mathematical Analysis

This talk highlights textbook-independent learning materials for a first course in real analysis that are being developed through an NSF-funded CCLI project (DUE #0837810). The materials aim to facilitate the use of research-based methods in the teaching of analysis, empowering students to gain a deep understanding of the subject and to communicate this understanding effectively. In most states nationwide, an undergraduate degree in mathematics, including a course in real analysis, is required for teaching mathematics at the secondary level. Research affirms, however, that traditional instruction in analysis bypasses much of the process of mathematical reasoning, preventing students from creating and communicating self-generated understanding. Traditional instruction also overlooks specialized mathematical content knowledge pertinent to teaching secondary mathematics. With this in mind, the learning materials are structured into five components, each with a specific purpose. The concept tests are short, focused questions that stimulate class discussions around potential misconceptions; guided discovery exercises lead a class effort to build definitions of concepts and connections between concepts, scaffolded collaborative tasks develop students' problem-solving and communication skills, historical vignettes enhance their appreciation and perspective of the material, and capstone connections highlight interconnections between analysis and other areas of mathematics. These materials are being refined and tested in classrooms with a high proportion of prospective secondary teachers. Upon completion, they will be made available to the mathematical community on a website and on CD.

Thomas Q Sibley (tsibley@csbsju.edu) St. John's University

Geometry for Majors and Future Teachers

Geometry is an ideal course to build mathematical intuition and visualization skills and link them to mathematical reasoning. These skills and links will enable future teachers to help their students fulfill the NCTM Standards in geometry. We describe hands-on experiences (and some computer experiences) that foster intuition and visualization skills for teachers and for their future students.

Angie Hodge (Angela.Hodge@ndsu.edu) North Dakota State University

Making History of Mathematics Relevant to Pre-Service Teachers

A history of mathematics course can be a very valuable course for pre-service secondary mathematics teachers (PSMTs). However, if the course is not made relevant to the PSMTs, they may not understand how it could be used in their future teaching beyond telling stories about mathematicians. Although storytelling about the lives of mathematicians is a valuable gain from a history of mathematics course, I found another way to make the course relevant to PSMTs. As an alternative to writing summaries on "sections" of the textbook, PSMTs were allowed to create a lesson plan that they could use in the 7-12 curriculum that related the history of mathematics to the current curriculum. In this talk, I will discuss how the PSMTs integrated 7-12 curriculum with the curriculum found in a 400-level history of mathematics course. The challenges and successes of the students will also be discussed, so that others can implement a similar project in their history of mathematics classrooms.

Paul Taylor (pttaylor@ship.edu) Shippensburg University

Introducing Secondary Certification Students to Mathematical Modeling

The junior level course in Mathematical Modeling at Shippensburg University is required for Secondary Certification majors, which comprised approximately 2/3 of the audience. Students are introduced to a wide variety of modeling techniques applied to problems from physics, biology, business and mathematics, programmed in Microsoft Excel following a book by Neuwirth and Arganbright. Students spend most of the class working on their own, expanding their model to answer exploratory questions. This course provides a strong introduction to both the wide applications of mathematics and computational sciences and the abilities of Microsoft Excel.

Joyati Debnath (jdebnath@winona.edu) Winona State University

Empowering Secondary Teachers with the Power and Beauty of Mathematics

Students in undergraduate curriculum, whether majoring in mathematics or not, should be exposed to the extensive power and beauty of mathematics. Our goal is to elevate students' knowledge in mathematics so that they can then venture out to any area of their choice. It is important that we seek to implant this mathematical understanding as early as possible. One way to do that is to empower our secondary mathematics majors in our undergraduate curriculum. Mathematics education majors at Winona State University are required to take an array of mathematics classes including a semester of Abstract Algebra and a semester of Advanced Calculus. In this presentation, different topics of higher-level mathematics courses will be discussed that can instill the pure and deeper understanding of mathematics. The concepts of these topics are easily transferable to the secondary level.

Session 2: Saturday, August 8, 8:30 AM–12:05 PM

Paul Seeburger (pseeburger@monroecc.edu) Monroe Community College

Dynamic Visualization Tools for Multivariable Calculus

A tour of an NSF-funded project that seeks to develop geometric intuition in students of multivariable calculus. This online exploration environment allows students (or instructors) to create and freely rotate graphs of functions of two variables, contour plots, vectors, space curves generated by vector-valued functions, regions of integration, vector fields, etc. A series of assessment/exploration activities has also been designed to help students "play" with the 3D concepts themselves, and to assess improvements in geometric understanding gained from these activities. The results of the first two semesters of this assessment will be shared. The grant project is titled, Dynamic Visualization Tools for Multivariable Calculus (NSF-DUE- CCLI #0736968). See http://web.monroecc.edu/calcNSF/.

Tamas Szabo (szabot@uww.edu) University of Wisconsin - Whitewater

Making Future Teachers Want to Learn College Mathematics

Prospective secondary teachers often complain about their content courses being "useless" for their future profession. We will share ideas on changing the way of presenting materials so that students can see the relevance of their mathematics courses. Most examples will come from Geometry and Number Theory, but are applicable in other courses as well.

Jeanette Palmiter (palmiter@pdx.edu) Portland State University
Jennifer Noll (nollj@pdx.edu) Portland State University

Modeling Effective Teaching of Probability and Statistics for Prospective/Current Teachers at Portland State University

For over three decades Portland State University's Mathematics and Statistics Faculty have offered upper division courses for pre-service and in-service high school and community college mathteachers to improve their understanding and to model effective teaching of Probability, Statistics, Geometry, Algebra, Calculus, Math History, and Discrete Mathematics based on the latest educational research. For example, in the Probability course we use a medical study in which doctors were asked the following: "The probability that a woman has breast cancer is 0.8%. If she has breast cancer, the probability that a mammogram will show a positive result is 90%. If a woman does not have breast cancer the probability of a positive result is 7%. Say a woman has a positive result. What is the probability that she actually has breast cancer?" The study found doctors with training in probability could not give the correct answer of about 8%, nor could our AP Statistics teachers. Motivating students through captivating problems, followed by simulations using manipulatives and software packages, helps in understanding conditional probability and Bayes Theorem, often a very difficult concept. In addition, in our Statistics course we provide opportunities to investigate data with a focus on describing important shapes in data sets, explaining and accounting for variability, and drawing inferences from data. We present ways to engage students in "data detective" work and methods for making sense of traditional statistics techniques and procedures. Our courses are fashioned and driven by educational research conducted at PSU and nationally.

Brian Matthew Loft (loft@shsu.edu) Sam Houston State University

Some Useful Models to Use in a Euclidean Geometry Course

A course in Euclidean geometry is valuable to our future educators not only for its content, but also for the proof techniques the students develop during the semester. However, an axiomatic approach can have limitations when trying to reach this second goal, especially when the students forget that there are models of geometry other than the Euclidean plane. Here I will present a family of very familiar geometric models that can be used to remind students that verification of a theorem in ¡math¿ $\mathbb{R}^2$ ¡/math¿ is not a ¡math¿ *proof* ¡/math¿ of the theorem. This infinite family also has as its limiting case a geometric model familiar to most college geometry classrooms.

Ernest Boyd (ernest.boyd@mnsu.edu) Minnesota State University, Mankato

Senior Capstone Experience for Mathematics Secondary Education Majors

Students majoring in mathematics education at MSU Mankato take senior-level courses in teaching pedagogy and classroom technology at the same time they take the capstone experience. The capstone experience tries to develop advanced mathematics using courses in geometry, real analysis, linear algebra and discrete math as prerequisites. The culmination of the capstone experience is a written paper and oral presentation given by each student. We present topics for this capstone experience in applied mathematics that build on connections between the prerequisite courses, the history of mathematics and the courses in teaching pedagogy and technology. Students gain more skills with technology and algebraic manipulations while studying historically important problems in optimization, and both discrete and continuous dynamical systems applied to economics and the natural sciences.

Edwin Herman (eherman@uwsp.edu) University of Wisconsin-Stevens Point

Using End of Semester Projects: Students as Teachers

For many years I have utilized capstone projects in upper division courses. Student groups choose a topic we did not cover in class and become experts on the material. In the last few weeks of the course, groups present the material in lecture format to their classmates (and me). Typically, this material appears on a later exam, so the students watch other groups' lectures quite intently. By teaching others, students learn the material much better and gain a stronger ownership of it. This is a benefit not only to Secondary Education Majors, but to all Majors in Mathematics. I will describe my experiences with the project, including methods of assessment and student feedback.

Kathryn Ernie (kathryn.t.ernie@uwrf.edu) University of Wisconsin - River Falls

Using Personal Response Systems to Focus Teaching on Representations and Conceptual Understanding of Mathematics

Personal response systems or clickers have been used to engage students in class discussion and peer teaching, particularly in large lecture classes. In math content courses for future math teachers, using personal response systems allows instructors to model higher levels of questioning and a method to focus students' attention on common misconceptions that their own future students may have. The use of clickers also provides a way to launch a discussion on mathematical reasoning. This semester, voting was used as a communication and active learning tool in math content courses for future middle school teachers. In Wisconsin students may teach middle school mathematics through either a grades 1 - 8 or 6 - 12 certification. The model used emphasized the development of key conceptual items incorporating common misconceptions as the alternative responses. Usually one or two deep conceptual items were addressed at the beginning of class. These items were developed to help structure the major representations and concepts that were to be discussed that day. A dialog began to emerge related to the misconceptions themselves. What might be the reasoning of a student that selected response a? How do you explain that response b is correct? What mathematical representations/tools could be used to answer this question? Are there alternative methods of reasoning that may be used? Examples of clicker items developed and discussed this semester will be shared from this integrated math content course for future middle school teachers.

Yun Lu (lu@kutztown.edu) Kutztown University of Pennsylvania

Teaching Abstract Algebra for Secondary Mathematics Education Majors

Abstract algebra is one of the core courses required for the secondary mathematics education majors in Kutztown University of PA. A lot of students have trouble with it since it is so different from the other courses. I've tried different methods to help them build confidence in doing some proofs. In this talk, I will talk about some of these methods with the feedback from my previous students.

Manan Shah (manyshah@gmail.com) None

The Monte Carlo Method as an Educational Tool: Grade School to Graduate School

The Monte Carlo method has become popular in computational finance and other fields in which solving high-dimensional problems by deterministic methods is cumbersome. While the Monte Carlo method is a fairly advanced problem solving tool, the concepts and techniques underlying it are well within the reach of grade school, high school, and undergraduate students. In this paper, we discuss how the Monte Carlo method can be woven into existing traditional courses through several hands-on projects that are more than just flipping coins and rolling dice. These projects can help to reinforce mathematical topics such as arithmetic and geometry and introduce new topics such as integration and probability/statistics. In addition to helping tie together several areas of mathematics, a probabilistic approach to problem-solving may help to maintain or further promote a student's interest in mathematics.

Mu-Ling Chang (changm@uwplatt.edu) University of Wisconsin-Platteville

What is the Prime Factorization of a Factorial?

The origin of mathematics started with integers. To understand the structure of integers, we need primes and prime factorizations. It is very challenging to determine whether a particular integer is prime, and this leads to the difficulty of finding the prime factorization of a huge number. For example, $87173993000 = (2^3)(5^3)(8741)(9973)$ where 8741 and 9973 are primes. I believe that the easiest way to make our students understand a mathematical idea is to give them a fun problem to explore. When I introduce prime factorization in class, my fun example is n!. The method for finding the prime factorization of n! forms a great model that helps our students develop a logical thought process toward problem solving. The best part is that any high school and/or middle school student can understand this problem and its solution method without any advanced mathematics.

Teaching Numerical Methods

Friday, August 7, 3:15–6:30 PM

Jeffrey L Stuart (stuartjl@plu.edu) Pacific Lutheran University

Teach Ill-Conditioning in a Single Lecture!

Many students complete their one college course in linear algebra with the mistaken view that all invertible linear systems of equations behave the same way: that there is a unique solution, that Gaussian elimination finds it, and that there is no more to be said. We offer an easy, student-centered classroom activity that demonstrates that the solution behavior of small, invertible linear systems can be complicated, showing high sensitivity to small perturbations. The demonstration consists of four parts: a simple physical demonstration using commonly available items, student-performed computations, computer-based computations, and a final algebraic analysis. The aim of this lesson is to develop an appreciation for the idea that linear systems can be ill-defined and for the role of geometry in ill-conditioning. Supplemental homework exercises demonstrate that poor scaling can also contribute to ill-conditioning. In the author's experience, students are consistently surprised in the differences of behavior that can occur even in linear systems consisting of two equations in two variables. This lesson can be embedded within a first course in linear algebra or within a short unit on numerical linear algebra.

Tim Chartier (tichartier@davidson.edu) Davidson College

Google Power

You submit a query to Google and soon the search engine returns an ordered list of pages. The page listed first is considered, loosely speaking, the best web page related to your query. This talk will discuss the role that the Power method can play in creating such an ordered list of ranked web pages. The talk will also discuss a formulation of the same problem that requires solving $A\mathbf{x} = \mathbf{b}$ rather than computing a dominant eigenvector. Finally, the talk will offer resources suitable for student exploration in this topic.

Kathie Yerion (yerion@gonzaga.edu) Gonzaga University

How Animals Get their Stripes? An Interdisciplinary Module for Numerical Analysis

J. D. Murray (1981) proposed a mathematical model for the biological process that produces animal coat patterns. The model consists of a system of nonlinear partial differential equations. He approximated solutions by the method of finite elements. The author developed a finite difference method to approximate solutions. This method requires the solution of large systems of algebraic equations by the common technique known as Gaussian elimination. However, these systems (although close to upper triangular) are highly susceptible to large round-off error without the technique of Gaussian elimination with partial pivoting. This teaching module is to give students in mathematics and computer science an interdisciplinary project from biology for which: the chosen computational algorithm must handle round off error, analysis of the errors yield some interesting dynamical systems, and the resulting program must take significant time to reach a steady state. Experimental runs of the final computational algorithm do yield coat patterns found in zebras, raccoons, and snakes.

Michelle Ghrist (michelle.ghrist@usafa.edu) U.S. Air Force Academy

Finding Error Terms for Numerical Integration: Mean Value Theorem vs. Taylor's Theorem

Many books use Taylor's Theorem to derive error terms for numerical differentiation formulas but then switch to applying the Mean Value Theorem for finding error terms for numerical integration formulas. In this talk, I discuss several options for finding error terms for numerical integration formulas, to include Newton-Cotes and Gaussian quadrature methods. In particular, I present two techniques which are not widely known but offer alternatives to the usually tedious Mean Value Theorem approach. One involves applying Taylor's Theorem, while the other uses an extension to the theorem giving the error of interpolating polynomials. In general, I observe that students find both of these approaches more palatable and understandable than the approach taken by most textbooks.

Atabong Timothy Agendia (agendia@yahoo.com) Madonna University
Benjamin B. Ibeabuchi (timedith@yahoo.com) Madonna University OKIJA Nigeria

Numerical Approximation of Taylor Series in the radius of Convergence

Unlike most simple functions appearing in pure mathematics which can be integrated, differentiated and solvable in differential equation by straightforward methods, economical, engineering and numerous applied mathematical functions are complicated and cannot be generally integrated or solvable in equation (linear, differential and integral equation) by straight forward methods. Numerical analysis provides computational tools of evaluating these complicated functions. It can be used to determine the roots of equations, the approximate solution of differential equations, approximate values of integrals and derivatives, eigenvalues of matrices and many other problems in mathematics. Before the advent of high speed computers for numerical analysis, Taylor series were used to approximate non-polynomial functions having derivative of all order in the neighborhood of a point within the interval of convergence, with a polynomial function for easy manipulation. In this research a computerized Taylor series expansion software in conceived and implemented for the advancement of numerical analysis.

Dick Jardine (rjardine@keene.edu) Keene State College

Learning Numerical Methods Through Its History

Including the history of numerical methods as part of the learning process is intended to deepen student understanding of the subject. Various student activities and projects are described, ranging from presentations of biographies to reading original works, all connected to the specific topic in numerical methods that students are learning at the time.

Matthew Glomski (Matthew.Glomski@marist.edu) Marist College

Proof by Computer: Logic and its role in Numerical Analysis

With an example from classical thermodynamics as a starting point, we will consider the idea of computability in numerical analysis. Specifically, we will address one method of computer proof in an environment where exactness is not always a possibility.

Aaron Melman (amelman@scu.edu) Santa Clara University

A natural derivation of Laguerre's method

Most if not all texts that treat Laguerre's method for the computation of zeros of a polynomial do not satisfactorily explain why the method should make sense. We present a natural explanation of why it does.

Adrian Ionescu (ionescu@wagner.edu) Wagner College

Integrating Some New Methods in a Numerical Analysis Course

This project presents some applications of our new C software in the Numerical Analysis course at Wagner College, which is mainly taken by seniors majoring in mathematics (and computer science). The students majoring in mathematics and/or computer science have a good programming background (they take at least one course in C/C++). Our C software is designed to solve numerically ordinary differential equations using the classical Runge-Kutta methods as well as the new Goeken-Johnson methods. We have also developed some interesting worksheets incorporating Maple into our Numerical Analysis course. For example, our software and Maple are used to analyze some classical examples in the specialty literature, such as, the predator-prey population analysis.

Ken Wiggins (ken.wiggins@wallawalla.edu) Walla Walla University

Computer Programming in Numerical Analysus

This talk will be a review of programming projects that I have used over 30 years of teaching and changing student attitudes toward programming. A survey of several universities will be included, and this survey will include the type of programming assignments given to students in numerical analysis classes and the expectation of engineering schools regarding student programming skills. Finally, suggestions will be made for appropriate programming assignments for students in numerical analysis classes.

Current Research in Mathematics Education for In-service Teachers

Thursday, August 6, 1:00–3:00 PM

Cheryl McAllister (cjmcallister@semo.edu) Southeast Missouri State University
Cheryl Beaver (beaverc@wou.edu) Western Oregon University

Common Error Patterns in Pre-service Teachers' Attempts at Writing Fraction Word Problems

It is important for teachers to have a deep conceptual understanding of the mathematics they are teaching. The learning of fractions can often become rote and turn into a task to be memorized rather than understood. This talk will give the results of a study analyzing error patterns when pre-service teachers were asked to create story problems for specific fraction operations. Students were given a particular single-operation fraction expression, asked to do the calculation, and then create a story problem that would require the use of both the expression and calculation to answer the story problem. Distinct error types, determined by mathematical and grammatical characteristics, were identified. As teacher educators work with their students, they need to design instructional activities that will address the underlying misconceptions that generate these errors. If teachers in American classrooms cannot fashion simple story problems when teaching fractions to their students, their ability to teach this difficult topic, and in particular to impart robust concept images, will be limited.

ivona grzegorczyk (ivona.grze@csuci.edu) California State University Channel Islands

Factoring Polynomials

California standards include factoring polynomials introduced already in middle school curriculum. However, test results show that even high school students have problems with factoring polynomials in one variable, including quadratic equations. We will present interesting results from the special workshop for in-service teachers that address their understanding of the subject, misconceptions and missed connections to other topics, curricular requirements and methodology used in the classroom. We will suggest new graphing based pedagogy and show activities that require in-depth understanding of factoring and are accessible to children.

James Anthony Mendoza Epperson (epperson@uta.edu) The University of Texas at Arlington

Can Inservice Mathematics Teachers Answer Mathematical Questions that Arise from Classroom Use of Dynamic Software?

Using dynamic software in the teaching and learning of mathematics offers new opportunities for students to construct their own mathematical knowledge. The National Council of Teachers of Mathematics Principles and Standards for School Mathematics (2000) describes technology as "essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." (NCTM, 2000, p. 11) As one of the six principles for school mathematics, the Technology Principle includes a vision that all students have access to technology to facilitate their learning of mathematics (p. 24). The author highlights pre-test and post-test results from questions that assessed inservice teachers' mathematical understanding in the area of function transformations and their ability to answer mathematical questions that may be posed by their students, particularly those questions that would arise in a lesson that uses dynamic software. In the two-week period between the pre- and post- tests, inservice teachers worked on transformation problems using dynamic software and animations in which they confronted many of the mathematical and pedagogical issues that the pre-test meant to expose. Whereas the Principles and Standards support the idea that teachers must experience how technology can enhance learning and investigate models for integrating it in their classroom practice (NCTM, 2000, p. 373), our results suggest that the use of dynamic software in instruction may also create new mathematical misconceptions or prompt mathematical questions that secondary mathematics teachers are unable to answer.

John C Mayer (mayer@math.uab.edu) University of Alabama at Birmingham

Implementation of Inquiry-Based Pedagogy Significantly Improves Student Achievement

The Greater Birmingham Mathematics Partnership teaches summer mathematics courses for in-service teachers modeling inquiry-based pedagogy. Students whose teachers provided a high level of implementation of GBMP pedagogy showed significantly more gains statistically in student achievement in mathematics on the SAT-10 than students whose teachers provided a moderate or low level of GBMP inquiry-based instruction. These findings were consistent across diverse school districts and grade levels. This research is supported by the National Science Foundation through the Mathematics and Science Partnership program. The Greater Birmingham Mathematics Partnership is a targeted partnership among 9 school districts in the Birmingham area with total student enrollment over 85,000, the University of Alabama at Birmingham, Birmingham Southern College, and the Mathematics Education Collaborative of Bellingham, WA. Our research and professional development is targeted at middle school grades (5-8), though our project involves professional development for in-service teachers in grades K-20, as well as pre-service teachers at UAB and BSC.

Ronald F. Barnes (barnes@dt.uh.edu) Univ of Houston-Downtown

Clicking with Class

As schools adopt student response system technology, clickers, teachers are spending a considerable amount of time and effort developing appropriate clicker questions. The authors are developing a set of clicker questions on descriptive statistics. These clicker questions will be integrated into our in-service courses for teachers which contain modules in statistics. The presenters will share insights, questions and results of course experiences. This work extends an earlier study by the authors integrating clickers into their business statistics courses. Participants are encouraged to share and submit their favorite questions for a Library of Clicker Questions maintained by the authors. Clicker questions can be submitted to either author at their email addresses.

Susan Lea Beane (beanes@uhd.edu) University of Houston-Downtown

Using Learning Groups to Remove Reasoning Gaps: Implications of Piaget's Theory of Numbers, Vygotsky's Social Theory, and Glasson's Choice Theory.

These theories allow teachers to understand why some students struggle in math no matter how many times the concept is repeated or explained. When we teach mathematics, what we are really teaching are only the most superficial aspects of math, i.e. specific values, formulas, and methods. Applying these theories in the classroom will help students construct a better, thorough understanding of mathematical relationships and the reasoning behind the relationships.

General Contributed Paper Sessions

Session 1: Thursday, August 6, 8:30–10:30 AM

Kevin Charles Moore (kmzipsgolf@gmail.com) Arizona State University

An Investigation into Precalculus Students' Conceptions of Angle Measure and Trigonometric Functions

Trigonometric functions and angle measure are standard topics for mathematics students ranging from middle school to graduate school. Also, various topics of physics, engineering, and other sciences are reliant on understanding the periodic behavior of sine and cosine functions (e.g., wave behavior). However, students are frequently observed as having difficulty reasoning about trigonometric functions. Recently, it has been argued that it is possible to develop more coherent student understandings of trigonometric functions by developing understandings (e.g., conceptions of angle measure) that are foundational to the various settings of trigonometry. The presentation will report results from an investigation of precalculus students' conceptions of angle measure, radian as a unit of measurement, and trigonometric functions. The study involved multiple teaching experiments and individual exploratory interviews focused on gaining insights to the understandings students develop as they interacted with research designed curricular materials. The subjects of the study were enrolled in an undergraduate precalculus course that focused on developing students' quantitative and covariational reasoning abilities. These reasoning abilities have been determined to be foundational for understanding ideas of variable, rate of change, and function. Curricular activities engaged students in making meaning of applied problems, including the ability to identify varying and fixed quantities in an applied context, and formalizing the quantitative relationships expressed in the problems' context. Results from this investigation revealed that ideas of angle measure and the radian as a unit of measurement are foundational for developing understandings of trigonometric functions across the various contexts of trigonometry.

Stephen Davis (stdavis@davidson.edu) Davidson College
Michael Boardman (boardman@pacificu.edu) Pacific University

AP-Calculus: What college faculty need to know, I

The growth of the AP-Calculus program presents special challenges for colleges and universities. An informed understanding of the AP-Calculus program is needed for mathematics departments to address these challenges. In the first part of this presentation, we document the growth of AP-Calculus, present an outline of the course, and describe the AP-Calculus exam. In the second part, we illustrate the exam through example questions from recent exams and discuss the methodology for determining grades. Finally, we offer insight into ways higher education faculty can be involved in and have influence on the future of the AP-Calculus program. The presenters are the Chair of the Development Committee and Chief Reader for the AP-Calculus program.

Michael Boardman (boardman@pacificu.edu) Pacific University
Stephen Davis (stdavis@davidson.edu) Davidson College

AP-Calculus: What college faculty need to know, II

The growth of the AP-Calculus program presents special challenges for colleges and universities. An informed understanding of the AP-Calculus program is needed for mathematics departments to address these challenges. In the first part of this presentation, we document the growth of AP-Calculus, present an outline of the course, and describe the AP-Calculus exam. In the second part, we illustrate the exam through example questions from recent exams and discuss the methodology for determining grades. Finally, we offer insight into ways higher education faculty can be involved in and have influence on the future of the AP-Calculus program. The presenters are the Chair of the Development Committee and Chief Reader for the AP-Calculus program.

Martha Allen (martha.allen@gcsu.edu) Georgia College & State University
Amy Kelley (amy.kelley@gcsu.edu) Georgia College & State University

Find Your Team: Content Team-Forming Activities for Calculus Class

Given that there is a large body of evidence that active learning methods yield improved learning outcomes and retention, we strive to make our calculus courses more active. Towards this goal, we employ in-class cooperative learning activities in which students work in teams to solve fundamental problems in calculus. In this talk, we will focus on a particular method for team selection that requires the students to exhibit an understanding of specific calculus concepts. We will describe some of the team-forming activities that we have created and implemented in our calculus courses.

Jane McDougall (JMcDougall@ColoradoCollege.edu) Colorado College

Global Warming in Calculus One

A surprising number of concepts in first semester integral calculus can be approached through a study of Planck's curve and the temperature of planet earth. This talk is based on ideas that were implemented in the classroom during a 2007 course which simultaneously covered the topics of global climate change and first semester calculus.

Richard Werthamer (rwerthamer@nyc.rr.com) retired

The Analysis of Casino Blackjack as a Supplement to the Teaching of Calculus

It has been known for decades how a blackjack player can use the information he has available to improve his odds: he can practicably achieve an edge in his favor of as much as 1-2% per round. Such tactics were originally found semi-heuristically, and have since been verified and reinforced primarily by computer simulations. I have explored the analytic derivation of an optimal blackjack playing strategy and its performance. Although the problem is characterized strictly by discrete variables, they can be closely approximated by continuous ones; the machinery of vector calculus then suffices to obtain a full solution, much more wide-ranging than with the previous methods. This talk will focus on the numerous mathematical tools that prove useful for the analysis, including matrix algebra, functional maximization, the Central Limit Theorem, Taylor and Hermite expansions, and partial differential equations. The analysis is presented in a new book on the science of casino blackjack which, as an application of calculus to optimizing play of the game, could be an appealing supplement to traditional classroom teaching.

Kimberly Anne Roth (roth@juniata.edu) Juniata College

Assessing Clicker Examples Versus Board Examples in Calculus

The combination of clicker questions and peer instruction has been shown to increase student learning. While implementations in large lectures have been around for a while, mathematics has been increasingly using clickers in classes of smaller size. In the fall of 2008 I taught two sections of Calculus I, one with 26 students and the other with 14. Each section used clickers daily, always for a warm-up question and usually for several questions during lecture. Warmup questions were either review from previous day's material or preview of today's. The students see the question when they arrive in the classroom and the question is discussed after 10 to 15 minutes of homework questions. Then lecture occurs for the rest of the 55 minute class with interspersed questions. The clicker questions during lecture follow the more traditional format of answer, pair and share, answer again, and the discuss as a whole. For each exam I picked two topics that I had clicker questions for. In one section I did clicker questions with board examples and in one section I did only board examples except for the warm-up clicker question on the previous day's material. The sections were reversed for the other topic. On the exam there were questions on the topics. The final had questions on all of the topics. I will present a report about the collected data and attempt to answer : Does doing questions by clicker have an effect on exam performance?

Ralph Alan Boedigheimer (Ralph.Boedigheimer@usafa.edu) United States Air Force Academy
Michael Brilleslyper (Mike.Brilleslyper@usafa.edu) United States Air Force Academy
Kenneth Horton (Kenneth.Horton1@usafa.edu) United States Air Force Academy
Scott Williams (Scott.Williams@usafa.edu) United States Air Force Academy
Hortensia Soto-Johnson (Hortensia.Soto@unco.edu) University of Northern Colorado

What's on Your Card? Information Students Perceive Will Help Them on a Calculus Exam.

Sometimes as educators we allow students to use a note-card on mathematics exams in order to alleviate memorization and stress. But, what do students record on note-cards and is the information beneficial for the exam? In this mixed-methods exploratory study, we investigate various concepts that students document on a note-card for a calculus final. We provide some results regarding themes that appear on the students' note-cards and the relationship between the themes, the students' grade prior to the final, and the students' course grade. We also explore possible distinctions between advanced placed students and those students following the general track.

Session 2: Thursday, August 6, 8:30–10:30 AM

Leslie Horton (lhorton@deltastate.edu) Delta State University

Cotton Bolls and Boll Weevils—A Summer Mathematics Institute in the Mississippi Delta

High schools in the Mississippi Delta area have the highest drop out rates in the state of Mississippi. Mississippi has one of the highest drop out rates in the country. Ten school districts in the Mississippi Delta are among the 20 lowest scoring districts on the state wide tests. Scores of proficient on these state wide tests would rate at most a basic classification on the NAEP. More statistics could be cited, but the point is clear: students in this area are in desperate need of help. An NCLB-funded summer institute in mathematics at Delta State University seeks to help mathematics teachers in grades 4-8 help their students. For twenty days, teachers from area schools work to prepare themselves to be better teachers. The results have been good. Attitudes as well as content knowledge have been positively impacted. My session would present activities (including use of technology) and results from the institute

Amy Spears (aspears@lindenwood.edu) Lindenwood University

Technology and the Classroom. Teachers Discover How to Enhance Learning With Strategies.

This presentation will be focusing in on strategies of implementing technology into the math lessons and classroom. A variety of strategies will be presented for math topics especially at the elementary and middle school level. Technology plays a major role in the lives of students today. The goal of educators needs to make sure that all students are able to learn at their maximum potential. Students need multi-dimensional learning tools to help them gain complete understanding. Students often learn in multiple ways simultaneously and thus require some if not all of the major learning styles (auditory, visual, kinesthetic) to be active to grasp a concept. Technology is a means to addressing student needs by providing the multi-dimensional learning tools. Previous methods of instruction do not engage current generations of students nor enable them to learn to the best of their abilities. Students would have a higher appreciation for learning if the technology they use outside of school were implemented in the classroom. Therefore teachers need training and instruction in the use of current and future technologies and how to implement them in the classroom in order to better instruct students. By the time students in the 7th grade graduate from high school and college, there will be jobs available that do not exist today. To best prepare them for the future we must plan ahead and incorporate current and future technologies in the classroom enabling the development of critical thinking and problem solving skills.

Charlotte Ann Knotts-Zides (knottszidesca@wofford.edu) Wofford College

Math, Murder, and Mystery: Mathematics and Detective Fiction

This spring, a colleague in the English department and I are teaching linked courses around the theme of mathematical logic and detective fiction. Concepts common to both classes include thinking critically, analyzing arguments, and solving problems. Students grapple with the idea of absolute certainity in mathematics and/or life as they read an engaging mathematical mystery novel. This presentation will explain the logistics of the class, its common goals, and some of the surprises along the way!

Elana Epstein (eepstein@sjcny.edu) St. Joseph's College

Mathematics in Movies and Television Shows

It is becoming more and more common for different mathematical topics to get mentioned in television shows and movies. As a math teacher, you can capitalize on this inclusion of math in the popular culture in order to make math engaging to your students. This presentation will cover some ideas of ways to use the math seen in this media in your classroom.

Mary B. Walkins (leolawalkins@yahoo.com) Lee University

A Perspective on the History of Mathematics

In this presentation, I seek to share my personal experience teaching the History of Mathematics course at Lee University, Cleveland, TN. Thus far, I have taught this course twice (in fall 2005 and 2007), and I am scheduled to teach it again in the fall 2009. Primarily, I definitely focus on the "history side" of the mathematics, and students are encouraged to read the text to facilitate discussion in class. Starting from the early number systems to the modern thoughts in mathematics, I encourage students to look at what has been done in the past, by notable mathematicians, with the hope that it will inspire them to learn mathematics and to seek new ways of solving problems. Since mathematics is still being created, students can be motivated to study the subject. Secondarily, I require students to submit solutions to "actual problems" in mathematics. These exercises persuade students to critically think through the problem solving, as they use old and new methods of thought.

Madeleine Jetter (mjetter@csusb.edu) California State University San Bernardino

Bounds on the Roots of the Steiner Polynomial

Every convex body in n-dimensional Euclidean space is associated to a polynomial of degree n , the Steiner Polynomial, whose coefficients encode important geometric information about the body. For example, a convex region K with area A and perimeter L has the Steiner Polynomial $A + Lt + \pi t^2$. Surprisingly, the roots of the Steiner Polynomial in two dimensions are geometrically significant: the fact that the Steiner Polynomial in two dimensions has negative real roots is equivalent to the isoperimetric inequality, which says that of all closed curves with a given length L, the circle encloses the largest possible area. In higher dimensions, the roots of the Steiner Polynomial need not be real. Teissier posed the question of whether the real parts of the roots must be negative. Let K be a C^2 convex body such that its principal radii of curvature have a minimum value of $\rho_{\min}$ and a maximum value of $\rho_{\max}$. We can show that if the roots of the Steiner Polynomial of every convex body in R^n lie in the left half-plane (this holds for $n \leq 5$), then the real parts of the roots of the Steiner Polynomial for K are bounded above by $-\rho_{\min}$ and below by $-\rho_{\max}$. This is a partial generalization of the situation in two dimensions.

Howard Penn (hlp@usna.edu) U.S. Naval Academy

Did Humidifying the Ball Cut Down on Home Runs at Coors Stadium?

Coors field in Denver Colorado has always been a ballpark where there have been a lot of home runs. This is despite the fact that the distances to the fences are longer than most ballparks. The primary reason is the altitude. In fact, it is estimated that the ball will travel 10% further in Denver than at sea level. In 2002, the Colorado Rockies began keeping the balls in a humidified room for a month before using them in a game. In this paper, we will study whether or not this change made a significant difference in the home runs hit there. We will also look at the opposite problem in Detroit, where one of the fences was moved in to make it easier to hit home runs.

Thomas Schroeder (TLS7@buffalo.edu) Univ @ Buffalo - SUNY

What Mathematics Should Be Taught to Gifted Secondary School Students, and How?

The Gifted Math Program at the University at Buffalo has negotiated answers to these questions over the three decades it has been in operation, and those answers continue to evolve. In this session we will share our experiences based on two guiding principles: (1) for gifted students the typical school mathematics curriculum should be both accelerated and enriched, and (2) gifted students should be expected to solve challenging non-routine problems and prove theorems in order appreciate the nature of mathematics (as distinct from typical "school mathematics"). We hope to generate an exchange of ideas not only during the session but also subsequent to it.

Session 3: Thursday, August 6, 1:00–6:30 PM

Erin Skjelstad (erin.skjelstad@ttu.edu) Texas Tech University
Jerry Dwyer (jerry.dwyer@ttu.edu) Texas Tech University

Calculus II Students' Motivation and Instructors' Teaching Styles

Calculus II students' motivation and perceptions of their instructors' teaching styles were investigated. Instructor nonverbal and verbal immediacy behaviors as well as instructor positivity, and use of group work and application problems were studied. Students in a large research university were surveyed at the start and end of a semester. A significant positive correlation was found between student motivation and instructor behaviors and teaching styles. The strongest correlation was between student motivation and teacher affect. Verbal and nonverbal immediacy behaviors also correlated well with student motivation. The results have implications for Calculus II instructors who may be advised to exhibit more positive immediacy behavior.

Martha Ellen Waggoner (murphy.waggoner@simpson.edu) Simpson College

Using Toilet Paper to Help Students Make Generalizations

When students are given a specific problem to solve, they do not naturally create a general solution method that could be applied in other situations. In this presentation, I will discuss a project that I use to help students learn the value of generalization and give them an introduction to sensitivity testing. The project starts by having students find the number of sheets of paper on a specific sealed roll of toilet paper, but they must take that method and produce a formula that could find the number of sheets of paper on a general roll of perforated paper. They then test the various models created by the class for sensitivity to measurement error to find the "best" method. I have used this assignment in an introductory modeling course with a calculus prerequisite, but the material could be adapted for use in pre-calculus courses. Students find the assignment engaging and surprising. Some of the students struggle with the process of creating a single, simple formula based on measurements. The students always want to know the "real answer" so they can check their work, but instead of telling them the answer I use this opportunity to introduce them to measurement error and sensitivity testing. In this talk, I will give the details of the project, which includes in-class work, use of Excel and a final written report. I will also discuss the pitfalls and surprises the students encounter. Finally, I will give a brief description of how I plan to use the project in Quantitative Reasoning in the fall.

Salar Alsardary (s.alsard@usp.edu) University of the Sciences in Philadelphia

Primary Trait Analysis to Assess a Learner-Centered, Upper-Level Mathematics Course

This study presents a primary trait analysis of a learner-centered, discrete mathematics course based on student-to-student instruction. The authors developed a five level scoring rubric for each of the primary traits: conceptual knowledge, procedural knowledge, application of understanding, and mathematical communication skills. The instructor developed seventeen performance-based problems that do not have a single best answer. Eleven students took this exam as a pre-test and post-test to measure their gain in knowledge and skills and every student showed improvement in the four traits. The Sign test showed significant (p-value ¡ .0001) score improvement.

Thai-Duong Tran (tdtran@htu.edu) Huston-Tillotson University

Numerical Calculations With Internet Explorer Scripts and Google Flot

Students learn the algorithms of numerical methods and do some "short" exercises with paper and pencils. The more "realistic" problems are done with computers. It is important that the students can easily implement the formulas (that they use for paper-and-pencil exercises) into programming scripts that "run" on (almost) any PCs. For the class, we do not need robust and elegant programs. It is more important to have a "straght forward" transition from the formulas to the computer scripts that can work right away (with minimum technical difficulties). These scripts are fast enough for class work, easy to modify and see input, output. Google "flot" will give pretty graphs to show the results even when the results are calculated by paper and pencils This presentation will try to show how to use these scripts and Google flot for numerical methods classes

Doreen De Leon (doreendl@csufresno.edu) California State University Fresno

An Euler-Cauchy Surprise

Since the nonhomogeneous second order Euler-Cauchy equation,

$$t^2y'' + aty' + bty = f(t), t \neq 0,$$

is not a constant-coefficient equation, the method of undetermined coefficients should not be expected to correctly give a particular solution, regardless of the right-hand side function. Surprisingly, however, some students successfully applied the method of undetermined coefficients to solve exam problems in which $f(t)$ was a monomial. The fact that direct application of the method of undetermined coefficients works for such problems is perhaps not too surprising if we consider that the Euler-Cauchy equation (assuming $t > 0$) may be transformed through a change of variables into a constant-coefficient equation to which the method of undetermined coefficients can be applied. Here, we investigate when we can we apply the method of undetermined coefficients (after perhaps a bit of modification) directly without performing this transformation.

Nathan Carter (ncarter@bentley.edu) Bentley University

Visual Group Theory

Group theory, the study of symmetry, is usually reserved for upper-division undergraduate courses. The book *Visual Group Theory*, published earlier this year by the MAA, introduces group theory from a thoroughly visual perspective, and is intended to bring the subject within reach of recreational readers. This talk will tour that book's visual approach to the subject, which uses Cayley diagrams to display both the beauty and structure of groups. Someone with no math background beyond algebra can inspect Cayley diagrams to learn about subgroups, homomorphisms, quotients, and even Sylow theory. And perhaps most importantly, visualization puts the intricate and symmetrical beauty of the subject on center stage.

Jiyeon Suh (suhj@gvsu.edu) Grand Valley State University

Estimating the Hidden Shape

Given a collection of concepts(shapes), it is known that as long as its VC dimension is finite, these concepts are learnable. In this talk, we will understand what is VC dimension through the study of finding the VC dimension of the collection of rectangles, and also understand the meaning of "learnable". All contents will be tried to be presented through simple visual explanations to reach the general audience including undergraudate students. Knowledge about probability distribution and random number will be helpful.

Min-Lin Lo (mlo@csusb.edu) California State University San Bernardino

Investigating the Parameters $\mu(D)$ and $k(D)$ of Distance Graph Coloring with Undergraduates

Let D be a set of positive integers. The distance graph generated by D , denoted by $G(\mathbb{Z}, D)$, has the set $\mathbb{Z}$ of all integers as the vertex set, and two vertices x and y are adjacent if $|x - y| \in D$. The chromatic number, fractional chromatic number, and circular chromatic number of distance graphs have been studied greatly over the past two decades. Closely related to these parameters is the density of a distance set D , $\mu(D)$, and the parameter involved in the Lonely Runner Conjecture, $k(D)$. It is known that $\mu(D) \geq k(D)$ for all distance sets D ; in addition, $\mu(D) = k(D)$ for all distance sets D where $|D| \leq 2$ and $gcd(D) = 1$. So far, all known examples for $|D| = 3$ conclude that $\mu(D) = k(D)$. Whether this identity is true for all 3-element distance sets remains an open problem. In summer 2008, I selected two undergraduate students to investigate this open problem with me. We proved that $\mu(D) = k(D)$ for two more types of 3-element distance sets D in addition to the known types such as "punched sets" and "union of two intervals". In this talk, I will present the results we obtained and if time permits, I will relate my experience of working with undergraduates on this research venture.

Sara Crawford (Sara.Crawford@valpo.edu) Valparaiso University

Lessons Learned from Leading First Undergraduate Research Team in Applied Statistics

This past academic year, I led an undergraduate research team for the first time in applied statistics that explored a departmental data set looking at the association between a change in teaching load and end of the semester teaching evaluation scores. The research team consisted of three undergraduate students with varying statistical backgrounds, including one freshman, one sophomore, and one junior. Throughout the year, the students conducted a complete analysis of the data set using previously learned statistical methods that culminated in a poster presentation of the research as well as a final written report. They also explored some new ways to approach the statistical analysis that were designed to accommodate data that violated certain assumptions. During the talk, I will discuss some of the lessons learned from this experience, including time management, incorporating students with varying backgrounds, and directing the independent study of advanced topics.

Alex Michael Turzillo (aturzillo@gmail.com) No Affiliation

Elementary Geometry in Parabolic Reflections

In this paper, we approximate the shapes, sizes and positions of optical focal volumes for collimated light shining into a parabolic mirror by modeling how rays reflect and intersect within the plane of a parabola. When light rays shine into a parabolic mirror at an angle normal to the directrix of the paraboloid, they intersect after reflection at a focal point. We demonstrate that parallel rays of light shining into a parabolic mirror at angles not perpendicular to the directrix intersect each other in focal volumes rather than at a point. By interpreting the scenario in terms of elementary geometry, we develop a measure of the minimum distance across the focal volume in terms of light angle and curve parameters. We use a symbolic geometry system in our investigation.

Philip Todd (philt@saltire.com) Saltire Software

From Inductive Reasoning to Proof by Induction With Geometry Expressions

In this lesson, we investigate the radii of the circles in a Pappus Chain using the symbolic geometry software Geometry Expressions. Class members self-select into "doers" and "thinkers". Doers contribute the next radius value by constructing the geometry using the software. Thinkers attempt to infer the pattern and work out the next value before the doers can measure it. The race is on. How many measurements do we need to make before the thinkers overtake the doers? Having inferred a pattern, can we prove it? Geometry Expressions differs from traditional dynamic geometry software by allowing the input, manipulation and output of symbolic quantities. Hence an expression involving an indeterminate n can be entered for the radius of one circle, and an expression for the next circle in the sequence output by the program. In this way the class will perform the steps of an inductive proof. The lesson hopes to draw the important distinction between inductive reasoning (encouraged, facilitated, and arguably overused with traditional dynamic geometry environments) and proof by induction, which the new capabilities of a symbolic geometry system enable.

James Dale Harper (harperj@cwu.edu) Central Washington University

Ramanujan, Quadratic Forms and the Sum of Three Cubes

We all know $3^2 + 4^2 = 5^2$. But did you know that $3^3 + 4^3 + 5^3 = 6^3$? Isn't that interesting! Euler had a non-homogeneous parametric solution to the Diophantine equation $A^3 + B^3 + C^3 = D^3$. Ramanujan discovered a quadratic form representation that satisfies this equation and proffered others to"find other quadratic expressions satisfying similar relations." Using this cubic identity as a seed I will show how to construct a quadratic form representation for our Diophantine equation. Different seeds will generate different quadratic forms. Surprisingly, Fermat's method of making a quadratic a square will play an important role.

Unlike its Pythagorean cousin, it is not known whether there is an integer formula that will generate all integer solutions to the aforementioned Diophantine equation. The prerequisites to understand this talk are minimal: All you need to bring is a strong set of pre-calculus skills. Students are welcome.

Fangyun Yang (fangyun.yang@gmail.com) University of California Riverside

Index Theory and Its Applications

Given a linear partial differential system, usually it is very hard or impossible to find the solutions. The critical insight of Atiyah and Singer is to ask a slightly different question: how many solutions are there? And The Atiyah Singer Index Theorem gives a nice answer: the number of solutions depends on the shape (topology) of the region we are working on. In this talk, I will first explain Atiyah Singer Index Theorem. Then I will talk about some of its generaliztions on manifolds with singularities.

Shishen Sam Xie (xies@uhd.edu) University of Houston-Downtown

A Numerical Algorithm for Solving Nonlinear Integral Equations

A numerical algorithm based on decomposition technique is presented for solving the nonlinear Volterra-Fredholm integral equations arising in the modeling of many applications. The algorithm is applied to several problems of integral equations and yields fast converging series solutions. The approximate solutions are compared with exact solutions, and it is shown that only a few terms are needed in the series to yield accurate and computable solutions with a fairly reasonable error.

Gianluca Caterina (gcaterin@endicott.edu) Endicott College

A "No-Go" Theorem for the Existence of a Discrete Action Principle

In this paper, we study the problem of the existence of a least-action principle for invertible, second-order dynamical systems, discrete in time and space. We show that, when the configuration space is finite and arbitrary state transitions are allowed, a least-action principle does not exist for such systems. We dichotomize discrete dynamical systems with infinite configuration spaces into those of finite type for which this theorem continues to hold, and those not of finite type for which it is possible to construct a least-action principle. We also show how to recover an action, by restriction of the phase space of certain second-order discrete dynamical systems. We provide numerous examples to illustrate each of these results.

Vladimir Riabov (vriabov@rivier.edu) Rivier College

Non-Trivial Motivational Study Cases for the College Courses in Introductory Mathematics

Traditional textbooks written for Introductory Mathematics courses contain basic algebraic principles, simple algorithms, "drilling" exercises, and trivial applications and examples that are mostly oriented on "average" students, giving no opportunities for advanced students in exploring the beauty of abstract mathematical concepts and modern applications of these concepts in applied mathematics, computer science, networking technologies, biology, sociology, business, and other disciplines. A different approach has been developed and introduced to the advanced students. We encourage students from local high schools, college freshmen, and juniors in participating in Mathematical contests; in taking the specially-designed course, "Modern Applications of High-School Mathematics" in the Rivier College Challenging Program; in participating in extracurricular activities, such as the Mathematical Seminar and Students' Mathematical Club; and in exploring the challenging study cases offered in traditional classes. In the present paper, we introduce several non-trivial motivational study cases (with solutions), including "Restoring Digits in Arithmetic Calculations", "Evaluating Infinite Square-root Sequences", "Factor Analysis of Large Numbers", "Estimating the Last Digit of a Number in a Large Power", "Applying Simple Trigonometric Formulas in Evaluating Complex Expressions", "Resolving Sophisticated Logical Problems", "Sports Combinatorics", "Examining Coupling Positions for the Minute and Hour Clock-arrows", "Recursion and Fibonacci Numbers", and "Digital Geometry & Computing in Structural Molecular Biology". The problems have been offered for students who are familiar with basic algebraic and geometrical concepts and are looking for modern applications of these concepts in advanced areas of mathematics (e.g., Theories of Numbers, Logics, and Algorithms), biology, and computer science (e.g., Encryption Algorithms).

Keith James Coates (kcoates@drury.edu) Drury University Department of Math and Comp. Sci.

An Intuitive Proof of the Singular Value Decomposition of a Matrix

We view the singular value decomposition of a matrix from a geometric viewpoint, leading to a proof that is easy to visualize. The approach makes use of a simple mathematical limit: $\lim_{\theta\to 0} \frac{1-\cos\theta}{\sin\theta} = 0$.

Joe Latulippe (jjlatulippe@csupomona.edu) Cal Poly Pomona
Randy Sierra (rasierra@csupomona.edu) Cal Poly Pomona

A Mathematical Model for the Healing Process of Wrist Injuries

Scapholunate ligament injuries often occur when the wrist is hyperextended, causing either a partial or complete tear of the ligament. When a tear occurs, surgical repairs aim at reducing the width of the scapholunate gap. Over time the repaired ligament will weaken causing the gap to widen. During this period the range of motion of the wrist will increase as the ligament stretches. As the amplitude of the range of motion increases the scapholunate gap will also increase. To better understand this, we present a mathematical model that replicates the varying range of motion observed in the healing process. Due to the movements of the wrist, the motion is assumed to be oscillatory. To better understand the general behavior of the model, we present the model as a system of non-linear ordinary differential equations. Using our model we investigate the long-term effects on the ligament, angle of dissociation, and range of motion. These investigations are presented as both analytical results and numerical simulations.

Anand Kumar (anandkumar@super30.org) Ramanujan School of Mathematics

Inequalities through Geometry

The main goal of all prestigious undergraduate level mathematics competitions around the world is to present a good mathematical challenge to brilliant young minds. Problems based on concepts of inequalities are vital source of this challenge. From the beginning of our school days, we are gradually trained to learn about basic concepts of inequalities. But it is often said, "A picture is worth more than a thousand words". Therefore, it will be better to understand, as well as explain, some of these inequalities with the aid of drawing pictures. Several times, we can be able to easily solve the inequality problems via geometrical methods. In this talk I would like to discuss some examples of this nature.

Mohan Tikoo (mtikoo@semo.edu) Southeast Missouri State University

Separating H-sets With Open Sets

In this paper, we provide a systematic study of those spaces in which disjoint H-sets can be separated by disjoint open sets in a topological space. This introduces four new separation axioms that lie strictly between T2 and T3 spaces. This is joint work with Professor Jack Porter.

Md. Ziaul Haque (zhaque@fisk.edu) Fisk University

Error Estimators in Solving Systems of Second-Order Hyperbolic Partial Differential Equations by Finite Element Method of Lumped Masses

To reduce the computational cost, explicit methods are generally preferred for hyperbolic problems. As a result the finite element method of lumped masses instead of consistent masses has been implemented to discretize the wave equation. The diagonal mass matrix resulting from lumping leads to a special system of second-order ordinary differential equations. Instead of transforming this system to an equivalent first-order system as is typically done, I solve this directly by using explicit Runge-Kutta-Nystrom method that offers improved efficiency and less memory. Explicit and implicit a posteriori error estimators developed for elliptic and parabolic problems (for non-lumping case) have been extended to hyperbolic problems and used to estimate the spatial discretization error of the wave equation. They are seen to be asymptotically exact for the method of lumped masses for second-order hyperbolic problems. Computational results demonstrate the effectiveness of both estimators and make possible a comparison of them.

Session 4: Friday, August 7, 8:30 AM–12:00 PM

Ronald F. Barnes (barnes@dt.uh.edu) Univ of Houston-Downtown
Linda Becerra (becerral@uhd.edu) University of Houston Downtown

Set Theory: Cantor's Creation and Its Ramifications

Georg Cantor created set theory, which set in motion a re-evaluation of the foundations of mathematics. With Cantor, infinity—the whole basis of analysis and irrational numbers - was finally put on a sound foundation. By 1900, the branch of mathematics known as axiomatics was developing with the aim of making mathematics more rigorous. This branch prescribes the methods and techniques of establishing consistent, complete systems each based on a set of independent and non-contradictory axioms. Even the principles of logic had been extended to accommodate the types of reasoning that mathematicians had been using informally and often implicitly. But as Frege put it, "Just as the building is completed, the foundation collapses.“ Set theory was instrumental in this collapse. It gave rise to contradictions and opened men's eyes to contradictions in older branches. Cantor's theory of infinite sets gave birth to many paradoxes and antinomies. There are those who say the paradoxes or antinomies arising from Cantor's theory of the infinite can be resolved and those who maintain that we must discard infinity and the infinite processes, for they lead inevitably to paradoxes. Felix Hausdorff characterized set theory as "a field in which nothing is self evident, whose true statements are paradoxical, and whose plausible ones are false." This paper will consider some of the reasons for the controversy created by set theory and describe what is at stake in the debate. The debate continues among the formalists and intuitionists.

Kyle Riley (kriley@taz.sdsmt.edu) South Dakota School of Mines & Technology

The Allure of Numerical Methods for PDEs

Almost any topic in a numerical methods course can be fun, but there is just something about Partial Differential Equations that really stimulates student interest. It could be the wonderful applications, the ingenious approach of the techniques, the rich array of applications, or maybe it is just the really cool pictures. However, the challenge of teaching, and learning, numerical methods for PDEs is the inherit complexity of the topic. Students rarely have the experience of working with PDEs before a numerical methods course and the choice of solution technique is never trivial. This talk will present a few ideas and examples of what can be done with this wondrous subject to make it accessible for a numerical methods class.

Thomas Koshy (tkoshy@framingham.edu) Framingham State College

Lobb's Generalization of Catalan's Parenthesization Problem Revisited

In 1999, A. Lobb investigated the following generalization of the celebrated parenthesization problem, studied by E. C. Catalan in 1838: Find the number $L_{n,m}$ of arrangements of $n + m$ positive ones and $n - m$ negative ones, such that every partial sum is nonnegative, where $0 \leq m \leq n$. Clearly, $L_{n,0} = C_n$, the nth Catalan number. The Lobb numbers $L_{n,m}$ can be obtained as differences of adjacent binomial coefficients, to the left of or on the middle column, in even-numbered rows of Pascal's triangle. Consequently, every Catalan number C_{2n} can be expressed as the sum of $n + 1$ squares. Interestingly, an investigation of the differences of adjacent binomial coefficients to the left of the middle column in odd-numbered rows of Pascal's triangle yields a number of delightful results involving Catalan numbers. For example, every Catalan number C_{2n+1} can be expressed as the sum of $n + 1$ squares. Thus, every Catalan number C_n can be expressed as the sum of $\lfloor n/2 \rfloor + 1$ squares.

Jay Lawrence Schiffman (schiffman@rowan.edu) Rowan University

Divisibility And Periodicity Ideas In The Fibonacci And Lucas Sequences

The question of divisibility is essential in mathematics. With regards to the Fibonacci sequence, it is well known that every third Fibonacci number is even and every fifth Fibonacci number is divisible by five. In contrast, some easily derived results may be less well known including no odd Fibonacci number is divisible by certain primes including 17 and 61 and no Fibonacci number is congruent to 4 or 6 modulo 8. With regards to the Lucas sequence, the facts that no Lucas number is divisible by five or any power of two beyond the second are well known. We will prove that certain primes as well as composite numbers never enter the Lucas sequence in contrast to the Fibonacci sequence. For example, it is assured that at some point among the initial one hundred Fibonacci integers, the integer ten will appear as a factor. Mathematical Induction and technology will be employed to help participants discover signs of divisibility and periodicity ideas pertaining to these two fascinating recursive sequences.

Gowribalan Vamadeva (gowribalan.vamadeva@uc.edu) University of Cincinnati

Incorporating a Functions Approach in a Traditional Freshman Algebra class

Are you afraid of being the lone passenger on the reform train? This session introduces effective methods used to supplement a traditional curriculum with conceptual ideas. Faculty will be given strategies to have an enlightening classroom environment that balances the needful skills with much needed theory. Many departments across the country are still using traditional textbooks and curriculum in their algebra classrooms. Every day, Faculty find a hard time having their voices heard in a departmental setting to instill curriculum that promotes comprehending concepts in their courses. Attending this session will enable interested and deprived faculty create a learning environment that uses concepts as a central theme within the framework of a traditional curriculum. Faculty will benefit from acquiring the balance between teaching essential skills using a variety of perspectives and communicating the importance of the concepts that lie within.

Darcel Ford (dford64@comcast.net) Member

A Comparative Study of Learning College Algebra in Non-Traditional Higher Education Programs

This research seeks to compare distance and classroom learning of college algebra in terms of efficiency and effectiveness. The study seeks to further understand the dichotomy (if any) between learning college algebra in the two learning modalities provided to non-traditional students in non-traditional higher education programs. The study also seeks to discover evidence based methods to maximize teaching and learning outcomes for all stakeholders. The current search results suggest a robust distance learning system for college algebra is a suitable learning medium for students that have routine access to a computer with an internet connection; possesses good basic study skills; are self disciplined; organized; motivated; and willing to adhere to policies on academic honesty. The traditional classroom learning of mathematics does not seem to require similar attributes. Observations from both settings will be presented along with literature that supports findings on the efficiency and effectiveness of learning under the conditions stated.

A. Dale Magoun (magoun@ulm.edu) University of Louisiana at Monroe
Charlotte Owens (owens@ulm.edu) University of Louisiana at Monroe
Azime S. Saydam (saydam@ulm.edu) University of Louisiana at Monroe

A Two Year Case Study of NCAT's Supplemental Model: Reversing the Pattern of Low Performance in College Algebra Using Modularity and Technology

Integrating technology into the college-level classroom has recently received much attention from the National Center for Academic Transformation (NCAT, 2009). Under their roadmap to redesign (R2R) criteria, mathematics classrooms at many universities have incorporated new approaches to student learning (Agenda-The Redesign Alliance Second annual Conference, 2008). This paper discusses each of the major components of designing a new modular, technology-based curriculum, the development of a mathematics resource center to support the curriculum, and the results of first two years of the implementation of R2R at our university

A. Dale Magoun (magoun@ulm.edu) University of Louisiana at Monroe
Azime S. Saydam (saydam@ulm.edu) University of Louisiana at Monroe
Charlotte Owens (owens@ulm.edu) University of Louisiana at Monroe

Methodical Intricacies in Designing Online Courses from Developmental Mathematics to Elementary Statistics

This talk will focus on some pedagogical strategies and the concomitant difficulties of designing online courses of developmental mathematics, college algebra, trigonometry, and elementary statistics. The presenters will discuss faculty training, student readiness, student-faculty interaction, and elements of successful teaching utilizing Moodle and Hawkes Learning System.

Monika Vo (monika.vo@saintleo.edu) Saint Leo University

How and Why a History of Mathematics Class Can Work Online

In this session we shall discuss how a traditional face to face History of Mathematics course was designed to be available online. At Saint Leo University, the History of Mathematics class is an upper division elective that Mathematics majors can elect to take. In addition, the course is also very popular among many of our pre-service teachers specializing in middle grades mathematics or high school mathematics students. Due to the nature of the population, we have been approached by the Education department to offer the course online as well as face to face. We shall discuss what changes were made to implement the course online, what benefits were seen as a result and what improvements would help make it a more successful experience for students.

Michael D. Miner (jcmhs77@aol.com) American Public University System

Utilizing Web-based Statistical Resources in Teaching Nontraditional Undergraduate Students in Online Learning Environments

The challenges facing delivery of a statistics class to nontraditional undergraduate students in nontraditional higher education programs are especially pronounced when the method of classroom delivery is online. However, a plethora of web-based statistical tools and concepts demonstrations are readily available to undergraduate students, additionally many of these statistical tools and demonstrations are also rich and powerful in scope. Web-based statistical tools and demonstrations range from the extremely simplistic (mundane) to the extremely complicated (theory driven using high level mathematics), however, the key for the instructor/facilitator is to not only balance the appropriate number and complexity of resources but also to insure that students effectively use them for understanding statistical concepts and methodologies. This research considers methods that online resources are introduced in the online learning environment and how each of these methods serves as enablers to learning statistical concepts and methodologies.

Donna Flint (donna.flint@sdstate.edu) South Dakota State University

Using Multimedia to Gauge Understanding in an On-Line Graduate Course

In a graduate mathematics course, students are required to read and understand the textbook. The traditional way of measuring a student's understanding of the material is through submitted homework and tests. In a face to face classroom, the professor also can gauge understanding by questions asked by students, class discussion, non-verbal feedback, and student board work. We will discuss how this is emulated for on-line students using multi-media student presentations. This presentation will include examples of student presentations and information about how to implement this activity into any on-line course.

Michael Johnson (johnsomi@meredith.edu) Meredith College

Why We Should Not Be Teaching One-Sided Hypothesis Tests in a First Statistics Course

One-sided or "one-tailed" statistical testing is often misapplied and a source of confusion to beginning students. I will briefly review conditions for validity that are often ignored, show examples of misapplication, and point out subtle detrimental consequences for students whose only statistics course emphasizes one-sided tests. I will then advocate for removing the topic completely from introductory statistics courses.

Marva Lucas (mlucas@mtsu.edu) Middle Tennessee State University
Nancy J. McCormick (nmccormi@mtsu.edu) Middle Tennessee State University

Research Results for Underprepared Students in a Redesigned College Algebra Course

While developing strategies to meet the needs of underprepared students, public colleges and universities across the nation are being faced with directives, often stemming from public viewpoint, state legislatures, and/or state governing bodies of higher education systems, to modify existing university programs. Efforts are being made to decrease the cost of serving incoming or first-year students, particularly those programs addressing developmental education. A comprehensive, coeducational, tax-supported university began a redesign of its developmental mathematics courses to satisfy strategic planning objectives of its state governing board. This session provides the results of implementing the redesign initiative that included transforming a former developmental mathematics course into "prescribed" sections of a general education mathematics course, College Algebra. Students enrolled in the prescribed sections of College Algebra were compared to students enrolled in the regular sections of the course. Comparisons were tested using a 2-proportion z test. The newly developed plan provides a more comprehensive approach that resulted in enhanced academic quality, flexible delivery options, greater uses of technology, and a reduction in the number of required courses. These results have strong implications for higher education institutions across the nation that are faced with meeting the needs of underprepared students in the area of mathematics. In addition to sharing the research results, the presenters in this session will provide the participants with the rationale for the redesign initiative, the structure of the former design, and the structure of the prescribed course and its features designed to enhance student success.

Robert F Fliess (fliessrf@westliberty.edu) West Liberty University

Making Connections Within Mathematics in Finite or Liberal Arts Mathematics Course

Many students feel that Mathematics is divided into separate subjects; i.e. Algebra, Discrete Mathematics. In an effort to dispel this notion, teachers should show different ways of arriving at the same conclusion. Two examples of making connections within Mathematics will be discussed. First, proving that the (positive) Rational Numbers are countably infinite using change of base, between base 10 and base 11. Second, doing Venn Diagrams by way of truth-tables.

Session 5: Friday, August 7, 1:00–6:30 PM

Timothy M Bergquist (tbergquist@northwestchristian.edu) Northwest Christian University

Using M&Ms and Excel to Have Fun with Statistics

Hands-on exercises are essential for students to understand statistics. Making them fun can be a big challenge for teachers. Of course, the more fun the exercises are, the more the material is retained and the less fear students have of statistics. Presented here is an exercise that teaches descriptive statistics, both graphical and numerical, in a basic college-level statistics class using plain M&M's in the fun size packs. Students count their candy (one pack per student) and report the results by color (blue, brown, green, orange, red, and yellow). The data is then entered into Microsoft Excel to demonstrate how this data can be analyzed using this widely available spreadsheet package. Pie charts along with bar charts are developed to display the data graphically. Averages and standard deviations are calculated to show central values and variation. Later in the course the same data is again used for chi-square goodness-of-fit tests, by comparing classroom results with the expected proportion from the manufacturer. That proportion for the milk chocolate candies used to be blue 10%, brown 30%, green 10%, orange 10%, red 20%, yellow 20%; it is now blue 24%, brown 13%, green 16%, orange 20%, red 13%, and yellow 14%. The color blends are selected by conducting consumer preference tests. Students thoroughly enjoy this exercise and literally "eat it up" when done.

Borbala (Bori) Mazzag (borim@humboldt.edu) Humboldt State University

Interdisciplinary Research With Undergraduates

During the summer of 2008, I participated in a research project funded by a Howard Hughes Medical Institute. There were approximately 15 undergraduate students participating in the project, the majority of whom were biology majors with minimal backgrounds in mathematics. My role in the project was to supervise the research of a mathematics student participant. In this talk, I will describe the format of the overall research program, the problem we worked on and, time permitting, give a summary of our results. My talk will highlight the advantages of such interdisciplinary work with undergraduate students as well as touch on some of the difficulties we encountered.

Magdalena Luca (magdalena.luca@mcphs.edu) Massachusetts College of Pharmacy and Health Sciences

Finances 101: Teaching Finances in Mathematics Courses

My presentation will discuss results from a research project I have developed with the intent to introduce an innovative practical issue in all my mathematics courses: teaching finances to undergraduate students enrolled in pharmacy and health care programs. The research project has two goals: first, to teach students basic financial concepts because, as college graduates, they should be able to understand, analyze and apply their knowledge to the many financial problems that arise in life, especially in the light of the present world financial crisis. Second, the project investigates assessment of student learning when civic issues are integrated in mathematics courses. To this end, incorporating financial literacy concepts into Calculus I and Calculus II courses proves to be very easy and valuable. All students immensely benefit from reading and learning about everyday financial issues.

Gayle M. Millsaps (millsaps@calumet.purdue.edu) Purdue University Calumet

Introducing Preservice Elementary Teachers to the Angles of Parallel Lines using Sketchpad

In this interactive lesson, the nomenclature for types of angles formed by intersecting and parallel lines is introduced using observed relationships. The students in a geometry course for preservice elementary teachers construct intersecting and parallel lines using Sketchpad. The students measure the resulting angles and observe the angle relationships that do not change when the relative positions of the constructed lines are changed. The class makes conjectures and explores the reasons for the observed relationships. In a follow-up lesson, students decide whether the measures of angles formed by the sides of a parallelogram and its diagonals can be predicted given the measure of two of the angles.

Michael Diehl (mdiehl@endicott.edu) Endicott College

Sexy Logic: Running a Math Learning Community

Learning communities focus on the tying together of interdisciplinary topics by enabling students to study it from different perspectives, and then bring them together in an integrated, team-taught seminar. This structure provides a unique opportunity for students to learn and faculty to teach in a very non-traditional environment. Last semester, I co-created a learning community entitled "Sexy Logic", where I taught a course in mathematical logic and a philosophy colleague taught a course in the philosophy of knowledge, then together we team-taught a seminar focusing on communicating information through effective visuals. Students who signed up for the learning community enrolled in all three courses, concurrently. While the individual courses were fairly traditional, the integrated seminar allowed students to explore how philosophy and science come together, and the important role that logic and data play in their everyday lives. The non-traditional atmosphere allowed us to examine the topic by viewing films, visiting a museum, watching a play, and diagramming puzzles. The seminar culminated in group poster sessions and research papers which gave students the chance to apply what they learned to a topic of their choice. The response from the twelve students who enrolled in our community has been tremendous and I will share the details about the design/structure of these courses, the format of the community and team-teaching, how it fits into a college's curriculum and the learning/social outcomes that our students have relayed to us.

Kazuko Ito West (westc-k@post.harvard.edu) Keio Academy of New York

Japan's Ten Year Experiment: What Can We Learn from It?

Japan implements a national curriculum for K–12 education. Since the Meiji Restoration in 1868 until its economic success by 1985, a major goal as a nation had been catching up to the West, and the national curriculum was to serve this purpose by educating the whole Japanese population. Around 1990, when Japan not only caught up to the West but also made its economy ranked first among major industrial nations in 1990 in per capita GNP, Japanese came to believe that "the East Asian cultural stress on the collectivity rather than the individual" (Huntington, 1996) must be a contributor to its success. After the period of euphoria, however, the Japanese have gotten increasingly concerned about relatively small numbers of internationally recognizable achievements by Japanese at an individual level, such as the number of Nobel Prize recipients compared to those of Western countries. This brought the issue of creativity to a center of the educational debate. In 1998, Japan revised its national curriculum and reduced the amount of materials that were to be taught assuming that creativity would somehow spontaneously occur in students who had more free time presumably to explore the world on their own. If Japanese think that more work should be done at school to foster students' creativity, a reform plan should not have been made based on a naive assumption such as this but should have been made based on a careful analysis to find out what was missing in students educated in Japanese schools.

Carol Vobach (vobachc@uhd.edu) University of Houston - Downtown
Linda Becerra (becerral@uhd.edu) University of Houston Downtown

Program Assessment in Mathematics and Statistics—Will My Institution Pass an Accreditation Review?

All accredited universities are reviewed for "institutional effectiveness" on a regular basis. The process determines whether or not they are performing as promised, based on elaborate plans submitted to national or regional accrediting boards every ten years. Until recently, many institutions submitted non-specific plans for compliance until a warning or unfavorable review occurs, with its projected threat of losing accreditation, students, government money, etc. Recent emphasis on a culture of evidence has mandated examination of all areas of university life, with carefully-defined assessment plans and resulting changes as a key factor. In 2007, our university administrators began to notify academic departments that assessment reports would be needed; and, in early 2008, deans notified faculty that each degree-granting program would have to submit annual reports to follow an unfamiliar outline containing perplexing information. There was no apparent model to follow; yet every submission was flawed in some unknown way. After many trials, e-mails and meetings over a period of 6 months, our mathematics and statistics faculty were finally able to generate acceptable assessment plans for review of our degrees and begin writing appropriate annual reports. This presentation will deal with our hectic history, eventual success in SACS (Southern Association of Colleges in the South) accreditation and plans for a placid future.

Ashley Marie Lorenz (alorenz1@kent.edu) Kent State University
Aloysius Bathi Kasturiarachi (akasturi@kent.edu) Kent State University

The Theory and Application of Fractional Derivatives

This paper studies the development of fractional derivatives from its origin to the present-day definition. This study includes an analysis of the theory of fractional calculus by examining the fractional integral and fractional derivative definitions. Historical notes are made as the theory is developed through time. Most noteworthy are new results with known functions using the fractional derivative definition. An application problem is also examined.

Roland Shen (rolandshen@yahoo.com) Olympia Institute and Gunn High School

Beyond Classic: Unified Law of Conservation of Kinetic Energy and Linear Momentum

We revisit the laws of conservation regarding the kinetic energy and linear momentum of elastic objects in a closed system, which are classic and fundamental in physics. Based on a novel complex quadratic function and its solutions for complex variable and velocity, we obtain a new law of conservation that unifies the two existing laws of conservation of kinetic energy and momentum for such systems. Under this new law of conservation, we discover a special constant that associates with the closed system and we describe its unique properties in both mathematics and physics. In this talk, we present the mathematical solutions along with explanation of their physical meanings, and we confirm our theoretical findings by visualizing the analytic results in computer simulation.

John L Simons (j.l.simons@rug.nl) University of Groningen

Cycles of the Generalized Collatz Problem

The generalized Collatz problem (also called the $px + q$ problem with p and q an odd prime) is defined by a sequence of natural numbers, generated conditionally by $x \to x/2$ if x is even and by $x \to (px + q)/2$ if x is odd. Empirically $px + q$ problems show for $p \geq 5$ mostly divergent trajectories and exceptionally cyclic behavior. An m-cycle of the $px + q$ problem has m local minima x_i and if $(x_i, q) = 1$ we call such a cycle primitive. Let $C(m, p, q)$ be the number of primitive m-cycles and let $C(p, q) = \sum_m C(m, p, q)$ be the total number of primitive cycles. We prove for each m, p, q that $C(m, p, q) < \infty$ and for each p : (i) there exist infinitely many $px + q$ problems with $C(1, p, q) \geq 1$, (ii) there exist for every $k > 0$ infinitely many $px + q$ problems with $C(p, q) \geq k$.

Micol Hammack (mhammack@vcu.edu) Virginia Commonwealth University

Using Mathematics to Teach the Art of Argument and Rhetoric

In a freshman humanities seminar, I use a “proof” that $1 = 2$ in order to introduce concepts in argument and rhetoric. The primary lesson illustrated by this faulty proof is the idea that disagreeing with a conclusion, or pointing out its implausibility, does not disprove that conclusion. Instead, we have to look at the steps taken in the proof and question their validity and relevance. When we find the invalid step in our proof, we can then argue that the conclusion is invalid. Likewise, when we are trying to disprove a verbal argument, we have to look for invalid, irrelevant or weakly supported steps taken by the speaker or writer that lead to his/her conclusion. Only in this way can we successfully refute the conclusion. Use of this faulty mathematical proof further allows us to look at how an audience is swayed, and how we can enter into productive, intelligent discourse in response to a variety of arguments, either academic or informal. (Note: The course that I use this lesson in is a year-long critical thinking, writing-intensive seminar taken by all Virginia Commonwealth University freshmen. The lesson could easily be adapted to other courses, as well.)

Bryan Nankervis (bn10@txstate.edu) Texas State University-San Marcos

Gender Inequities in University Admissions Due to the Differential Validity of the SAT

Previous research has documented sex differences in mathematical skills and abilities across various content areas and suggests these differences are a result of a complex mix of biological, sociological, and psychological factors. Consequently, males significantly outscore females on the SAT I quantitative section, which is designed to predict first-year college success in mathematics. This paper, however, demonstrates that gender gaps in performance on the SAT I have little to do with college readiness, but rather are due to the misaligned content of the instrument as well as the environment in which the exam is administered. In particular, the SAT quantitative section contains a large percentage of content that not only significantly favors males, but is also not representative of entry-level college mathematics courses. Further, the conditions under which the SAT is administered have been shown to be highly conducive to stereotype threat and responsible for more than half the gender gap on the quantitative section. A statistical analysis provides specific examples of gender inequity resulting from admissions criteria based on SAT scores at four-year institutions. This study informs research on access to post-secondary education and has far-reaching implications for the design and administration of standardized mathematics tests utilized in the admissions process at most universities.

Marvin Quenten Jones Jr. (marvinqjones@gmail.com) North Carolina A&T State University

Stability Analysis of Gross Domestic Product

Gross Domestic Product is a standard measure used to calculate a nations wealth. Currently the United States is experiencing a recession, which by definition is two consecutive quarterly declines in GDP. Furthermore upon investigation and analysis of Federal Reserve data it is found that the GDP is not very stable nor is economic growth from preliminary calculations. Analyzing percent changes in data from 1929 to present about GDP reveals these fluctuations that have ranged from high positive to high negative. We seek to investigate the problem of stability in GDP using econometric techniques as well as difference equation techniques by running analysis on the stability of the multiplier coefficient of GDP. As seen below we have the definition of GDP using the expenditure approach:

$$Y \equiv C + I + G + NX$$

In this equation C is actually a function of GDP defined by the consumption function as seen below.

$$C = C_0 + C_1 Y$$

where C_1 is derived by $\frac{dC}{dY} = \frac{\frac{\partial C}{\partial t}}{\frac{\partial Y}{\partial t}}$ and is known as the Marginal Propensity to Consume, which yields:

$$\frac{1}{1 - C_1} = \frac{1}{1 - MPC}$$

which is the factor controlling the stability of the GDP model. When applied back into the expenditure equation we obtain:

$$Y = \frac{1}{1 - C_1}[C_o + I + G + NX]$$

We seek to develop the functional form of this equation and test the stability of the growth of GDP to determine policy decisions that will serve to be useful in bettering the economy.

Orlin Stoytchev (ostoytchev@aubg.bg) American University in Bulgaria

Three-Dimensional Rotations—a Mathematical Playground

The space of all three-dimensional rotations—the group $SO(3)$—provides a wonderful toy on which many geometric, topological, algebraic and group-theoretic ideas can be illustrated. We give a few examples which are not very familiar to a broad audience. Traditionally rotations in three-dimensional space are parametrized by the Euler angles. An alternative parametrization allows us to understand easily the topology of $SO(3)$. Any rotated frame can be obtained from the initial one by first moving the initial z-axis to the final one along a geodesic on a sphere S^2, while parallel-transporting the other two axes. Then we do a rotation around the final z-axis to align the other two axes. Of course we have to take care of the ambiguity at the point opposite to the initial z-axis, for which there is more than one geodesic. When we do this we conclude that $SO(3)$ is a solid torus with points on the boundary identified along lines that wind half turn around the small circle as you go once around the large one. With some cutting and gluing this is reduced to a solid ball in R^3 with each point on its surface identified with its antipodal one. This is exactly S^3 with antipodal points identified. The fact that $\pi_1(SO(3)) = Z_2$ can be understood easily on this model. Three-dimensional rotations can be linked to braids, as shown in [1]. We can attach the ends of three strands to a table and the other ends to a ball. By rotating the ball an odd number of times (around different axes) we will get a braid that can never be untangled if we keep further the orientation of the ball fixed. On the other hand every braid obtained from an even number of rotations can be untangled. By making rigorous the connection between rotations and braids we reduce the calculation of $\pi_1(SO(3))$ to a calculation of a certain factor group of a braid group. If instead of three strands in the above example we use n strands, the braids that can be obtained are a subgroup of the braid group of n strands. Characterizing this subgroup is a classical problem solved algebraically in [2]. Using our treatment linking rotations to braids we are able to give a much simpler and quite visual characterization of this group. References: 1. Vesna Stojanoska and Orlin Stoytchev, "Touching the Z_2 in Three-Dimensional Rotations", *Mathematics Magazine* 81 (2008) No.5, 345–357. 2. Shepperd, "Braids Which Can be Plaited with Their Threads Tied Together at Each End", *Proceedings of the Royal Society of London. Series A,* Mathematical and Physical Sciences, Volume 265, Issue 1321, pp. 229–244

Yun Myung Oh (ohy@andrews.edu) Andrews University

Lagrangian Submanifolds in N-Dimensional Complex Euclidean Spaces

Lagrangian submanifolds have been studied in many areas of mathematics because of the applications into physics. I have constructed several kinds of Lagrangian submanifolds and this time I will present one example of minimal Lagrangian Submanifolds in n-dimensional Complex Euclidean spaces using Legendre curves.

Cynthia L McCabe (cmccabe@uwsp.edu) University of Wisconsin - Stevens Point

Introducing Clickers into Mathematics Classes: A step toward more active learning

Clickers may be used for interactive polling in any class, from precalculus to proof courses. Instead of asking your class a question and hearing responses from only a small number of students, you can hear from the whole class and they can hear from each other. Using clicker questions in your classes can be a positive way to take a step toward a more active learning environment. Clickers can be used to check homework results or concept comprehension quickly, to facilitate group work, or to review topics before exams. This talk will present types of questions used in a precalculus trigonometry course and in an introduction to proofs course. Grading methods and ideas for future uses will also be addressed.

Jeff Hildebrand (jhildebr@ggc.usg.edu) Georgia Gwinnett College

A Mathematics Program Without a Mathematics Department

Many proposals for revamping higher education include a call to abolish departments. Georgia Gwinnett College, founded in 2005, is structured without any departments and has no plans to create them in the foreseeable future. Four years into this experiment, how is it working? The pros and cons of this approach are examined, both looking back at the development of a general education mathematics program and looking forward to the implementation of a mathematics major in the near future.

Gulden Karakok (gulden.karakok@math.umu.se) Umea University

Eigenvalues and Eigenvectors: What Do Students Say About Them?

Most of the topics covered in a typical undergraduate linear algebra course- matrix algebra, determinants, systems of linear equations, linear transformations, eigenvalues and eigenvectors, and diagonalization, are the prerequisite topics for many client disciplines- from physics, economics, statistics, computer science, and engineering (Carlson et al., 1997). Eigenvalues and eigenvectors are revisited during the study of quantum mechanics in physics. In this talk we will present the results of a study conducted with third-year college students who were enrolled in quantum physics courses. The purpose of this study was to investigate students' "transfer of learning" of the concept of eigenvalues and eigenvectors from the physics courses to interviews in which seven students participated during and after these courses. The presentation of the results will be followed with implications of the study for teaching and learning of linear algebra topics.

Elizabeth Mathai (emathai@norwich.edu) Norwich University

Number Theory in Sound Diffusion

Music directors are often faced with the problem of singers or instrumentalists not hearing each other well. How can they sing or play together properly, if they cannot hear each other? Acoustic gratings come to the rescue here. Specially designed and strategically placed, they help conserve music energy and also help spread it around to achieve ensemble between musicians. Here we take a quick look at the role that number theory plays in the design of some of these acoustic gratings.

John Doty (john.doty@notes.udayton.edu) University of Dayton

Power Plant System Identification Using Inferential Statistics: Towards Feasible and Efficient Optimization

The purpose of this paper is to apply inferential statistics using Design of Experiments (DOE) to identify important system-level characteristics that may be used to design and/or optimize a steam power plant. Energy- and exergy-based methodologies presented in previous work is extended to statistical surrogate models that are used in lieu of the power plan model and simulation. This surrogacy is then used to simulate the power plant a posteriori in an extremely efficient manner. As such, the preliminary design and optimization are approximately 1000 times faster than the original models enabling "what if" scenarios to be captured independently of the original model and simulations. A multi-component system (energy source, energy transfer devices, energy conversion device) was modeled in which realistic physical losses were considered (e.g., heat transfer, fluid head losses, etc.). Multivariate analyses results (three degrees of freedom) were performed in an ad-hoc, parametric fashion as a baseline for comparison to the Designed Experiments approach. Bounded and constrained optimization techniques were utilized in all analyses. Both results yielded the same values for system performance, indicating that the DOE approach using inferential surrogates is a valid system identification pedagogy.

Herman Rubin (hrubin@stat.purdue.edu) Purdue University

Teach Concepts First, Then How to Use Them

The general method of teaching now is to teach facts and manipulations, without teaching the general concepts. I claim that the general concepts are more easily learned if taught early, and then the facts and manipulations become easier to understand and more meaningful. I believe that this can be done quite rigorously (rigor does not require completeness) starting with the first grade, and that most of the concepts in the current strong college curriculum can even be done in elementary school, or at the worst, high school. This does not mean slighting the manipulations, but putting them in their proper place. Children can understand general concepts directly; it is harder after manipulations are learned.

Charlie Smith (charlie.smith@park.edu) Park University

The Linda Hall Library of Science, Engineering, and Technology: Kansas City's Best Kept Secret

The talk will explain why the Linda Hall Library is an indispensable resource for a History of Mathematics class, and why a field trip to the library is a mandatory activity. It houses thousands of excellent sources for students who need to write a term paper pertaining to any topic in mathematics. The History of Science Collection contains rare books, including classics such as the 1482 first printed edition of Euclid's *Elements*. The Rare Book Room is ideal for historical research. Many facts, figures, and photographs will be included.

Session 6: Saturday, August 8, 8:30 AM–12:00 PM

Muhammad Usman (Muhammad.Usman@notes.udayton.edu) University of Dayton

Teaching Numerical Methods as Interdisciplinary Subject

Theory and experiment are traditionally considered as tools for scientific discovery. Recently Scientific Computation has been added as a new synergy for discovery. In this talk I will share some examples that I use in teaching numerical methods to create the interest in the subject.

Thomas Q Sibley (tsibley@csbsju.edu) St. John's University

Puzzling Groups

Many puzzles are permutation groups in disguise. We investigate the groups connected with a family of puzzles developed by Bob Earles and generalize to related puzzles. Earles' puzzles consist of slotted wheels and pieces fitting into the slots. The pieces are permuted when the wheels are turned. These puzzles are easy enough to be useful teaching tools in an abstract algebra course, but present some interesting mathematics as well.

Philip Kneil Hotchkiss (photchkiss@wsc.ma.edu) Westfield State College
Christine von Renesse (cvonrenesse@wsc.ma.edu) Westfield State College

Discovering the Art of Mathematics

As part of a proposed NSF CCLI grant we are working on developing books to support inquiry-based learning. In our talk we will illustrate the topics we have already started developing and using in our classes. Our target audience is liberal arts students who are taking mathematics courses as a general core requirement. We are passionate about using inquiry-based techniques in our classes and we are interested in connecting and sharing with others.

David Richeson (richesod@dickinson.edu) Dickinson College

A Japanese Temple Problem

For over 200 years, during a period of strict national isolation, Japanese mathematics developed independent of western influences (such as the discovery of Calculus). During this time mathematical problems were drawn on wooden tablets (sangaku) and placed in Buddhist temples and Shinto shrines throughout Japan as an offering to the gods and as a challenge to the visitors. These sangaku featured beautiful geometric problems. In this talk we showcase one particularly beautiful Sangaku problem and discuss ways that it can be generalized.

Janet Nichols (janet.nichols@colostate-pueblo.edu) Colorado State University - Pueblo
Janet Heine Barnett (janet.barnett@colostate-pueblo.edu) Colorado State University - Pueblo

The ABC's of Problem Solving: A Capstone Course for Pre-Service Elementary Teachers

This talk describes the course "Problem Solving for K–6 Teachers" at Colorado State University-Pueblo. Modeled in part on courses offered at Metropolitan State College of Denver, this course was first offered at CSU- Pueblo in Spring 2007, and serves as a capstone experience in our required sequence of five mathematics courses (four content, one methods). Designed to improve student's problem-solving skills and to deepen the student's mathematical content knowledge, the course also serves as a model of problem-solving instruction appropriate for the elementary school classroom. Throughout the semester, students are provided with opportunities to actively explore lots of problems and, in the process, to learn useful strategies for solving them. Problem sets related to NCTM content standards are regularly assigned from a specially developed Course Problem Solving Booklet. Class time is spent discussing strategies, investigating problems in small groups, and presenting problem solutions both individually and as groups. Presentations of different solutions for the same problem are encouraged as a means to deepen students' mathematical content knowledge, and to provide important experience in analyzing and evaluating the mathematical thinking and strategies of others. Students also complete individual formal write-ups of designated problems and submit weekly journal entries designed to promote reflection on mathematics and problem solving. An overview of the course rationale, instructional and assessment design, and the Course Problem Solving Booklet (developed at CSU-Pueblo with support from MAA-PMET) will be shared, along with the joys, frustrations, breakthroughs and growth which the course has brought to students and instructors alike.

Jim Fulmer (jrfulmer@ualr.edu) University of Arkansas at Little Rock
Thomas McMillan (tcmcmillan@ualr.edu) University of Arkansas at Little Rock

Using Proofs without Words to Explore the Pythagorean Theorem

This talk will describe our experience using *Proofs without Words* for the Pythagorean Theorem (published by the MAA) as a means for getting students to think about the Pythagorean Theorem and for coming up with a proof for it in their own words. We distributed to our students a variety of picture proofs without words and let them work with partners in developing a written proof that explained the picture proof. Our talk will describe the insights and innovations that students came up with as they used the pictures as a guide for a proof of the Pythagorean Theorem. This was a required activity of the course, and students presented their results as part of the course portfolio. We felt that this experience helped students develop the confidence to create their own proofs.

Robert Ely (ely@uidaho.edu) University of Idaho
Jodi Frost (jfrost@uidaho.edu) University of Idaho

Infinite Processes and Sets in the Learner's Mind

The Tennis Ball (or Ping-Pong Ball) Problem has proven to be a rich context for students to display their conceptions about infinite sets and processes; we used this problem to interview students from a wide variety of mathematical backgrounds, including graduate students and even mathematics professors. In order to project a final resultant state for any infinite process, a student must determine which feature(s) of the finite states will be preserved by the limiting process. Most students attended to counting, preserving the sizes of the sets, and thus consistently produced one answer. But many students focused instead on indexing, attending to the labels of the individual elements, and thus produced a dramatically different answer. In several remarkable cases students generated and produced coherent arguments for both answers, and were unable to decide between them. These two different answers reflect precisely the crucial idea involved in Cantor's theory of sets, which entails the shift from using numbers for counting to using numbers for indexing. Unlike prior interpretations of the Tennis Ball Problem, ours suggests that students' (and professors') difficulty with the problem is not due to their inability to envision final resultant states of infinite processes, but rather is due to the unavoidable tension between the natural usage of numbers for counting and the Cantorian usage of numbers for indexing. This suggests also that students will learn about infinite processes most effectively if they are asked to deliberately specify what features of the finite situations are preserved by a limit process.

Jack G Mealy (jmealy@austincollege.edu) Austin College
Shannon Mulligan (smulligan@austincollege.edu) Austin College

Area Estimates for Paracycles in Snell Geometries

Further results in the category of Snell Geometries are discussed. (See the abstract, "Paracycles in Snell Geometries", MathFest 2008.) Recall that a Snell Geometry is a system consisting entirely of regions of locally constant curvature, wherein Snell's Law (of optics) is in play across the boundaries between these regions of constant curvature. In this talk, we first focus on the subcategory wherein the regions are 2-dimensional and all have curvature zero, but have different "indices of refraction"; the boundaries are smooth curves. Previously, the existence of paracycles (certain limits of circles) was established; these cycles have infinite circumference, but everywhere constant (and finite) radius. In this presentation, we are led to define slightly modified objects, para-ellipses (generalizing paracycles), which also have infinite circumferences; however, the areas for a natural subclass of these para-ellipses are shown to be finite. The construction may be modified in ways which ensure appropriate symmetries; the set of these objects is seen to be large. Second, we take up the extension of these ideas to regions of constant positive curvature, but with different values in different regions. The Snell dynamic is developed in this setting. Then the extension of the paracycle construction to this category is discussed.

John Starrett (jstarret@nmt.edu) NMT

A Suspension of the Hénon Map by Periodic Orbits

We create polynomial differential equations for a suspension of the Hénon map. By globalizing the local tangent vectors to suspended periodic orbits of the Hénon map, we are able to find approximate autonomous differential equations for that geometric suspension. Using as few as two suspended periodic orbits, we are able to generate a robust three dimensional attractor whose Poincaré map has very nearly the dynamics of the original Hénon map.

James Thomas Smith (smith@math.sfsu.edu) San Francisco State University

Supplement to Tarski's Collected Papers

With Andrew and Joanna McFarland, the presenter is preparing for publication a supplement to Alfred Tarski's "Collected Papers" (Birkhäuser, 1986). This will include (1) English translations of Tarski's articles, posed problems, and conference discussions that until now have appeared only in Polish; (2) new translations of two Polish papers on equidecomposibility of polygons; (3) information about and translated excerpts from Tarski's 1935–1944 Polish school geometry text; and (4) information about Tarski's works, major surveys of them, and biographical materials about him, that have been published since 1986. The translations will include Tarski's first (1921) paper on well-ordered sets and his first (1930–1931) abstract on the concept of truth. Many of the other items stem at least partially from his service as high-school teacher in Warsaw. The 1924 and 1930–1932 papers on polygons with equal areas present very interesting results about their degree of equidecomposibility, which Tarski connected with logical theory in later years and related most vividly in lectures to lay audiences. The three editors are attempting to discover the circumstances of publication and use of Tarski's text, "Geometrja dla trzeciej klasy gimnazjalnej," coauthored with Zygmunt Chwialkowski and Waclaw Schayer. Any information about that would be most appreciated.

Terry Jo Leiterman (terryjo.leiterman@snc.edu) St. Norbert College

Trajectory and Flow Properties: Spheroids in Stokes Flow

An exact mathematical solution for the low Reynolds number motion induced by a rod in the form of a prolate spheroid sweeping a symmetric double cone is developed, and the influence of the ensuing fluid motion upon passive particles is studied. The advected particles are observed to admit slow orbits around the rotating rods and a fast epicyclic motion roughly commensurate with the rod rotation rate. The epicycle amplitudes, vertical fluctuations, arclengths and angle traveled per rotation are mapped as functions of their initial coordinates and rod geometry. These trajectories exhibit a rich spatial structure with greatly varying trajectory properties. The Eulerian and Lagrangian flow properties of the fluid flow are also studied and shown to exhibit complex structures in both space and time. We establish the origin of these complexities via an auxiliary flow in a rotating frame, which provides a generator that defines the epicycles. Finally, an additional spin around the major spheroidal axis is included in the exact hydrodynamic solution resulting in enhanced vertical spatial fluctuation as compared to the spinless counterpart. The present study is of direct use to nano-scale,actuated fluidics.

Kendra Killpatrick (Kendra.Killpatrick@pepperdine.edu) Pepperdine University
Kristina Garrett (garrettk@stolaf.edu) St. Olaf College

Balanced Modular Tableaux

Given a $k, a, b \in \mathbb{Z}$, we define a modular tableau to be a tableau such that the element in position (i, j) (with rows labeled from the top down and columns labeled from left to right) is labeled with $aj + bi$ mod k. Let n_l be the number of l's in a modular tableau T. Then we will say that a modular tableau T is balanced if $n_0 = n_1 = \cdots = n_{k-1}$. In addition, we will say that the partition is balanced with respect to k, a and b. The main question we will address in this talk is the following: For a given k, a, and b which partitions give rise to balanced modular tableaux?

Jeff Johannes (johannes@member.ams.org) SUNY Geneseo

The Mathematical Fiction and Exposition of Rudy Rucker

Rudy Rucker began his career as a logician, but it was even apparent in his earliest papers that he had a strong interest in writing. Since 1976 he has written 30 books of fiction and nonfiction. These books have discussed such mathematically diverse topics as the fourth dimension, infinite cardinalities, cellular automata, and topology. In this talk we will overview Rudy's contributions to making mathematics entertaining and accessible.

Gene Klotz (eklotz1@swarthmore.edu) Swarthmore College/The Math Forum

Wiki Technology Facilitates Math Writing Course

I've been teaching a very elementary mathematics writing course and the wiki tools and approach I've developed may be of wider interest to the college mathematics community The students are writing their papers on a wiki based on MediaWiki—the same software that powers Wikipedia. This seems to intrigue the students enough that they overcome any technophobic inclinations (with a little bit of help), and actually make good use of this attractive new venue. Some of their papers are real delights, incorporating beautiful images and organizing their work so that it is both attractive and easily accessed. More importantly it appears that presenting their work in this context and available for public scrutiny encourages students to write with more care and effort than for papers printed on dead trees. Many of these students are scarred from their school experiences in mathematics. The phrases "math quiz" and "math exam" still strike fear into many hearts. Since this is a writing course I decided that having them discuss questions about their readings and work would be much better than standard exams, so there are none - the students make summaries and answer questions about their readings and post them on the wiki, too. The wiki is very good for organizing student work so that it's easy to find, it's easy to see where they are, and easy to see what improvements they've made. I post assignments on the wiki, along with objectives and grading policy, and engage students in modest dialog right on the wiki. They also use it for further interaction with their student "writing associates" who look over and comment on the first drafts of their papers. I post my comments on their work, as well. This somewhat radical approach to a math writing course seems to be working well and I will have the results from two such courses upon which to report how well. In the talk I'll show the site and give its address so that interested parties can examine it at leisure.

Session 7: Saturday, August 8, 1:00–5:45 PM

Virginia (Ginny) L. Keen (keenvirl@notes.udayton.edu) University of Dayton

Strengthening Mathematics Knowledge with Streaming Video

In order to more actively engage students with the content of the foundational mathematics courses for early childhood and intervention specialist education majors, in the Fall of 2008, I assigned students the task of preparing video presentations/explanations of concepts included in the course curriculum. The videos (2–5 minutes in length) were put on the course website as streaming video for all students to view. Videos served as devices for expanding the resources available to students to strengthen their understanding of the content through both creation and examination of the video presentations. These same videos are used as a resource in support of the professional development of preschool teachers preparing to take Praxis 1, prior to entering our degree program. By viewing video showing how preservice teachers make sense of concepts, it is hoped that mathematics-anxious preschool teachers will gain competence and confidence in their own ability to make sense of the mathematics. With the increased availability of less expensive digital video recording equipment, more faculty will be able to take advantage of this type of assignment. There are some not-insignificant issues to be worked through in order to ensure a satisfactory outcome in terms of completed videos, quality control, and other aspects of content and presentation. We will discuss ways of dealing with task creation, equipment access, video production, logistics, quality control, and other topics that colleagues find useful. Sample videos will be available.

Michael Townsend (met@u.washington.edu) University of Washington

A Law-School Quantitative Methods Course

I describe a quantitative methods course offered as an elective to 2nd and 3rd year law students at the University of Washington School of Law. This is a capstone-type course, whose main goal is to get students to see how another discipline can help them take a fresh look at the law. I use mathematics drawn from statistics, game theory, and social-choice theory, together with legal examples taken from discrimination law, tort law, and the philosophy of law to provide external vantage points for rethinking some traditional legal issues.

Fumiko Futamura (futamurf@southwestern.edu) Southwestern University

Creating a Culture of Pedagogical Learning

We may strive to create a culture of learning in our classrooms, but often neglect to create a culture of pedagogical learning in our departments. We may sometimes read up on the latest pedagogical trends, we may come to MathFest for new ideas, but we're often isolated from each other, not really knowing how others around us teach. We discuss a variety of ways in which we can promote the sharing of ideas, discussions, feedback and support to keep all of us fresh and inspired.

Gabriela Schwab (gschwab@epcc.edu) El Paso Community College
Helmut Knaust (hknaust@utep.edu) University of Texas at El Paso
Emil Daniel Schwab (eschwab@utep.edu) University of Texas at El Paso

Cross-Institutional Implementation of Supplemental Instruction (preliminary report)

El Paso Community College (EPCC) is the largest Hispanic-serving community college in the United States. At the same time, EPCC and the University of Texas at El Paso (UTEP) are de facto the only institutions of higher education within reach for a growing population of local students, the overwhelming majority of whom are Hispanic. Due to this constellation, the Mathematics departments of both institutions have a long tradition of close cooperation. We will report on a new joined initiative, supported by the Department of Education, to integrate mandatory Supplemental Instruction workshops into EPCC's Precalculus courses. Graduate students from the Department of Mathematical Sciences at UTEP are serving as workshop leaders for this Supplemental Instruction component. A large portion of these graduate students started their academic career at EPCC and will therefore be able to act as role models for their EPCC peers. We expect the project to have a significant positive impact on student transfer rates between the two institutions in the STEM disciplines. Our presentation will introduce in detail the concept and practice of mandatory Supplemental Instruction in beginning Mathematics courses. We will also discuss some of the challenges we are facing at the institutional level in implementing such a strategy.

Camillia Smith Barnes (cammie@math.harvard.edu) Sweet Briar College

Enumeration of the Distinct Shuffles of Permutations

A shuffle of two words is a word obtained by concatenating the two original words in either order and then sliding any letters from the second word back past letters of the first word, in such a way that the letters of each original word remain spelled out in their original relative order. Examples of shuffles of the words 1234 and 5678 are, for instance, 15236784 and 51236748. In this paper, I enumerate the distinct shuffles of two permutations of any two lengths, where the permutations are written as words in the letters $1, 2, 3, \ldots, m$ and $1, 2, 3, \ldots, n$, respectively. I will also discuss generalizations and related problems.

Sean Cox (scox@math.uci.edu) UC-Irvine

The Mathematical Uses and Philosophical Aspects of Downward Reflection

The simple notion of closing a set under a function appears throughout mathematics. The typical math major's first explicit encounter with the notion is when they find the span of a subset of a vector space. The Downward Lowenheim-Skolem Theorem is a tool which is closely tied to this notion of closure, and is especially useful in set theory and model theory. For example, it was used in Godel's proof of the consistency of the Continuum Hypothesis, and also to show that countability is not an absolute property (the latter phenomenon is known as the "Lowenheim-Skolem Paradox"). The Downward Lowenheim-Skolem Theorem is an example of "downward reflection"; informally, downward reflection means that for any structure S there are small approximations of S. The reflection guaranteed by the L-S Theorem is closely linked to the notion of closing a set under a function; but there are interesting variations of the L-S Theorem which partly remove this link, and provide more powerful downward reflection.

Mohammad Salmassi (msalmassi@framingham.edu) Framingham State College

A Generating Function for Lobb's Double Sequence

Lobb's sequence includes Catalan number as a special case. Tom Koshy has studied this sequence extensively. In this talk I will answer one of his questions. He asked if a generating function can be determined for this sequence. Such a generating function will be presented and its consequences will be discussed.

Xiao-Xiong Gan (xiao-xiong.gan@morgan.edu) Morgan State University

Generalized Composition of Formal Power Series and Its Applications

Both the theory and applications of formal power series have been developed rapidly these past years. The composition of formal power series is a very interesting and useful part in this field. A necessary and sufficient condition for the existence of such composition was established several years ago which took away the restriction of the nonunitness for the composed formal power series. We will introduce some of these results as well as some further developments about this generalized composition. The application of those results could be expected in many fields including the solutions of differential equations.

Robert W McGrail (mcgrail@bard.edu) Bard College

Knots, Quandles, and the Constraint Satisfaction Problem

Quandles form a class of algebras that arise naturally in many settings, such as knot theory and group theory. Moreover, the class of quandles has an interesting computational structure through its relationship with the constraint satisfaction problem. This talk describes a general undergraduate-centered research program on the computational structure of finite quandles that includes research projects in the following fields: Algorithmic complexity, universal algebra, knot theory, group theory, symbolic computing, graphical user interface design, and mathematical knowledge management.

William Randolph Oscarson (Bill2math@yahoo.com) Aerospace Engineering

Method for Determining the Minimum number of Primes Between the Perfect Squares of Consecutive Integers

We investigate the pattern of primes between the perfect squares of consecutive integers by permuting the sequence of primes through all possible arrangements within carefully defined constraints and find a pattern emerges that seems likely to lead to a proof that there are at least two primes between all sets of perfect squares. The computer program is written in C++ and runs on a dual core laptop running at 1.9 Ghz. Results are discussed in relationship to how they may be used to generate a proof of Legendres classic unsolved problem.

Anjan Biswas (biswas.anjan@gmail.com) Delaware State University

Optical Solitons in a Non-Kerr Law Media with Inter-Modal Dispersion

This talk is going to present the dynamics of optical solitons in non-Kerr law media in presence of inter-modal dispersion. The time-dependent coefficients of group velocity dispersion, nonlinearity and inter-modal dispersion are considered. Besides the Kerr law nonlinearity, the power law, parabolic law and the dual-power law nonlinearity are considered. Both bright and dark (topological) solitons are studied in this case. An exact soliton solution is obtained in all cases. The only requirement, as will be seen, for the solitons to exist is that these time-dependent coefficients must be Riemann integrable. Finally, numerical simulations will be given.

Douglas Magomo (doug.magomo@gmail.com) Northland College

Parameter Regions of Interacting Species using Resultant Theory

Researchers are more concerned about the dynamics of the interacting species, that is, population sizes and regions of existence of equilibria and even chaotic dynamics. They worry more about that parameter that influence the most these changes through sensitivity analysis. Most parametric conditions are those conditions that define the existence or not of equilibrium solutions. We study outcome variability of interacting species by incorporating the theory of resultants and explain the ecological settings that are determined by variation of more than one parameter.

Joon H. Kang (kang@andrews.edu) Andrews University

Positive Steady State Solutions of General Cooperation or Predator-Prey Biological Model

The non-existence and existence of positive solutions for the generalized cooperation or predator-prey biological model for two species of animals is investigated in this paper. The techniques used in this paper are from elliptic theory, the upper-lower solution method, the maximum principles and spectrum estimates. The arguments also rely on detailed properties of solutions to logistic equations.

Kawa Mustafa Aziz (kawa@uni-sci.org) University of Salahaddin

Solution of the System of Linear Fredholm Integral Equations of the Second Kind Using Modified Homotopy Perturbation Method

This paper is about the applied of a simple method which is modified to homotopy perturbation method to approximate the solution of system of linear Fredholm integral equations of the second kind (SLFIESK). Solved problems reveal that the proposed method is very effective and simple and in many cases it gives the exact solution rather than the approximate one.

Abdulmajeed M. Abdurrahman (ababdu@ship.edu) Shippensburg University
Alan Cresswell (alcres@ship.edu) Shippensburg University

The Operator Connecting the SCSV 3-Vertex and the Comma 3-Vertex

The comma interacting 3-vertex of the open bosonic string is constructed in the full string basis of the open bosonic string. This form of the vertex is then used to construct the conformal operator connecting the comma interacting 3-vertex and the Sciuto-Caneschi-Schwimmer-Veneziano 3-vertex (SCVS-Vertex).

Robert Travis Kowalski (travis.kowalski@sdsmt.edu) South Dakota School of Mines and Technology

The Sine of a Single Degree

Much like the number π, the sine of 1 degree – the vertical altitude of a slice of the unit circle whose angle is 1/360th of a full revolution – has played an important role in understanding the measurements of circles. Determining a –simple" formula for the exact value of the sine of a single degree has long been an important problem in the history of mathematics. Though it is not difficult to get accurate approximations of it, determining the exact value of $\sin(1°)$ defied first a geometric solution and then an algebraic one for centuries. In this talk, we will look at the cultural significance of the sine of 1 degree, along with some of the historic near misses in attempting to compute it. Along the way, we'll find the golden ratio in 18 degrees, learn to help out a depressed cubic equation, and build up a theory of imaginary numbers... and in the end, we'll not only find a formula for the $\sin(1°)$ using nothing but integers and radicals, we'll actually find infinitely many!

Ilhan Michael Izmirli (izmirli@american.edu) American University

What is the Mathematically Correct Interpretation of Probability in Quantum Mechanics?

As is well known the concept of probability has deep implications in quantum mechanics. We claim that a philosophical and mathematical analysis and clarification of the exact meaning of "the probabilistic nature" of quantum mechanics (namely, interpreting probability as relative frequency) would not only disperse some of the perplexity and mystification surrounding the idea of measurement in quantum mechanics but would also show its mathematically more intuitive character.

Ching-Chia Ko (koch@onid.orst.edu) Oregon State University
Gulden Karakok (gulden.karakok@math.umu.se) Umea University

The Longitudinal Evaluation Study of National Research Experience for Undergraduates Program (NREUP) and Preliminary Results

In this talk the preliminary results of a longitudinal evaluation study of National Research Experience for Undergraduates Program (NREUP) will be discussed. NREUP is sponsored by the Mathematical Association of America (MAA) and its Strengthening Unrepresented Minority Mathematics Achievement (SUMMA) program and funded by NSF and NSA. NREUP is structured to reach underrepresented groups majoring in mathematics or a closely related field who are at the transition point between lower division and upper division studies and to provide them a challenging research experience. The goal of the NREUP is to increase the interest of underrepresented groups majoring in mathematics or a closely related field in obtaining advanced degrees and career in mathematics or a closely related field. Each year approximately 12 universities or colleges are selected to host a MAA student research program at their institutions. The evaluation study was designed to answer the following question: Is there preliminary evidence that the NREUP increases the interest of underrepresented groups in pursuing advanced degrees or/and career in mathematics or a closely related field? To address the question, each participating student was invited to complete pre- and post-surveys at the beginning and at the end of the summer program. Students were later also invited to participate in an interview. The design of the longitudinal study will be described further and some of the results from the surveys and interviews will be shared at the presentation.

Poster Sessions

First Day of Class Activities

Friday, August 7, 10:30 AM – noon

Mike Pinter (pinterm@mail.belmont.edu) Belmont University

Encouraging Problem-Solving the First Day of Class

I will describe a specific problem that I've been able to use very effectively the first day of class for both an introductory quantitative reasoning course and for an Honors Program course. The problem is simple for the students to understand but eludes a quick solution; for those reasons, students of varying ability levels find something both understandable and challenging about the problem. As the students work in small groups on the problem, I guide them through several aspects of problem solving, such as: be clear about the "rules" of the problem; start with a smaller version of the given problem in order to gain insight and to generate some data; look for patterns in data that's been generated, conjecturing and testing along the way. So, in addition to ideas of problem solving, I also establish a collaborative approach to the course where students help each other and I guide their efforts. Students leave class the first day with a true taste of the problem-solving they will encounter many places in the course.

Susan Lea Beane (beanes@uhd.edu) University of Houston-Downtown

Life's Expectations And Requirements

My first class day stresses that the class is an important *life* event, and in order to be successful in life, we need some necessary skills. Those skills are mathematics and reading and writing. Math comes first in life skills. Everything in life is built around the sciences and science is build around math. The expectations are the course goals and objectives. The requirements are the necessary study skills needed to successfully carry out the course expectations.

Alice Kaseberg (kaseberg˙alicemsn.com)

Gather Specific Student Information on an Index Card

Ask students to record on index cards: their major, personal goals, number of hours employed each week, number of course credits, and major personal responsibilities. In addition to taking roll, this lets you personalize to the students your comments about goal setting, persistence, and time management. For courses from algebra to calculus, discuss an equation for a realistic load. Pass out cards as students enter. Have all questions displayed before class so students are involved in class from the first minute they arrive.

Stacey Allyn Cederbloom (cederbsa@muc.edu) Mount Union College

I Don't Teach Math. I Teach Students Math.

"You are the first math teacher who ever cared whether I understood or not." I have heard these words again and again out of the mouths of students who work hard in my classes. As math teachers, we have the unfortunate reputation of loving our content first, and our students a far second, if at all. So, my first key to motivating students is to convince them, in both words and actions, that I care about them and their success in math.

My second key to motivating students is to spend a good part of the first day doing math with them and introducing them to my teaching style. I choose material which is relevant to them, challenges them to think, but does not completely intimidate them. More importantly, I create an environment in which participation is not only "safe," but is also crucial in the learning process.

Because I have found that students "will not care how much I know until they know how much I care," I carefully weave this theme throughout everything I do on the first day—from necessary administrative tasks, to my choice of content, to the way in which I teach that content. For me, designing such a lesson on the first day reaps huge rewards for the rest of the semester: it encourages students who "hate math" to persevere, and it inspires many others to achieve success in math that they did not think was possible.

Caren Diefenderfer (cdiefenderfer@hollins.edu) Hollins University

Creating a Rubric for Graphing

I start my "Introduction to Quantitative Reasoning" class by giving students some frequency data on a variety of topics. We form groups of 3–4 people and each group receives a data set. The data sets come from our text book and consider the total sales (in billions of dollars) of the top eight retail companies in the US, the number of platinum albums of the top ten musical groups (with the most platinum albums) in the US, the areas (in millions of square miles) of the world's major land masses or the religious preferences (measured as a percent of the sample) of first-year college students. I ask each group to create a graph of the data set and draw it on an overhead transparency. Together we look at the graphs and critique them. As a class we develop a list of guidelines, or a rubric, for graphing data. Creating this rubric allows us to establish some shared terminology and we use the group rubric for grading graphs throughout the entire semester. This activity is a good way to start a class and emphasizes that good written communication skills are crucial for success in this class.

Research by Early Career Mathematicians

Friday, August 7, 3:00–5:00 PM

Chris Camfield (camfieldc@kenyon.edu) Kenyon College

Notions of Bounded Variation and Perimeter in Metric Measure Spaces.

The study of functions of bounded variation and sets of finite perimeter has been an important tool in analysis and partial differential equations for many years. This theory also provides a nice link between analysis and geometry via measure theory. It generalizes the notion of perimeter measures and outward normal vectors to a large class of sets, many without smooth boundaries. In recent years, much progress has been made in analysis on metric spaces. Tools are now available to define and study functions of bounded variation and sets of finite perimeter in abstract metric measure spaces. In particular, we will present different proposed definitions in the setting of a weighted Euclidean space. We will give conditions when these definitions are equivalent and cases when they are not even comparable.

Oscar Macedo (odmacedo@miners.utep.edu)
Melissa Anna Maria Pugh (map172004@gmail.com)
Jessica Ione Reyes (jireyes@miners.utep.edu)
University of Texas at El Paso

Supplemental Instruction for Calculus

The Department of Mathematical at the University of Texas at El Paso has implemented a unique delivery mode for all its Calculus I courses, adding to the existing modular design a mandatory peer-led supplemental instruction component. We will present a three year report on our experience with and outcomes of implementing modular course delivery along with mandatory peer-led supplemental instruction, the impact it has had on student success rate, as well as our own experience as supplemental instructors.

Pratik Talati (pratik@uab.edu) The University of Alabama at Birmingham

Exploring Nearly Planar Graphs

In graph theory, a graph is nonplanar if it cannot be embedded in the plane without crossings of edges. A graph G is nearly planar if G is nonplanar but $G - v$ is planar for every vertex v of G. Using Kuratowski's characterization of nonplanar graphs, this research is aimed at generating the connected nearly planar graphs by giving explicit constructions for all of them.

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Peter Kosek (no1piman55@yahoo.com) SUNY Brockport

Sums of Consecutive Integers that make Perfect Squares

We will look at values of n and k such that $n + (n + 1) + \cdots + (n + k)$ is a perfect square.

Jason Molitierno (molitiernoj@sacredheart.edu) Sacred Heart University

Applications of Linear Algebra to Graph Theory

A graph on n vertices, labeled $1, \ldots, n$, can be represented by a Laplacian matrix L. The Laplacian matrix is an $n \times n$ matrix where each diagonal entry $\ell_{i,i}$ is the degree of vertex i, and the off-diagonal entries $\ell_{i,j}$ are -1 if vertices i and j are adjacent, and 0 if vertices i and j are not adjacent. Clearly L is a symmetric matrix, hence all eigenvalues are real. By the Gersgorin disc theorem, all eigenvalues are nonnegative. Since the row sums of L are all zero, L is singular as the vector of all one's is an eigenvector corresponding the the eigenvalue zero. My research focuses on the second smallest eigenvalue of L. This eigenvalue is known as the algebraic connectivity of a graph as it measures how connected the corresponding graph is. For example, the algebraic connectivity is zero if and only if the graph is disconnected. Moreover, adding edges to nonadjacent vertices of an existing graph causes the algebraic connectivity to monotonically increase. For a fixed n, the connected graph on n vertices with the smallest algebraic connectivity is the path while the complete graph is the connected graph on n vertices with the largest algebraic connectivity. My poster will display many other interesting results concerning the algebraic connectivity, the other eigenvalues of L, and how the structure of L can give us insight into the structure of the corresponding graph.

Index of Speakers

CALENDAR

of

Upcoming MAA Meetings

2010

AMS-MAA Joint Mathematics Meeting

San Francisco, CA
January 13–16, 2010

MathFest

Pittsburgh, PA
August 5–7, 2010

2011

AMS-MAA Joint Mathematics Meeting

New Orleans, LA
January 5–8, 2011

MathFest

Lexington, KY
August 4–6, 2011