Uses of the WWW that Enrich and Promote Learning

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Math-ingo: Easy Creation of Math Bingo Cards

Games are often a useful and fun way to review material before a test, and encourage students to actively participate in review sessions. In fact, researchers have concluded that playing games can be an effective learning tool, especially in mathematics, in situations where the goal is to reinforce specific ideas or concepts. Two places this type of situation arises are in Calculus I and II, when students are learning rules to find derivatives and evaluate integrals.

A game of Bingo works well for reviewing this material. But putting together Bingo boards, and adding new derivative or integration techniques as they are covered, such as chain rule problems, trig problems, and so on, can be very time-consuming. This talk will describe an interactive web application we are creating to form custom-made Bingo boards. We will allow the user to specify either the specific functions he or she wishes to use, or specify the derivative or integral rules to be used and let the program choose the functions. (Note: The website is under construction and will be up by the time of Mathfest).

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WebCT Quizzes in Introductory Real Analysis

One of the difficulties in teaching upper-level mathematics is getting students to read the text carefully — in particular, to pay close attention to the "small words" (such as "if," "or," "some," "all," etc.) which are often superfluous in everyday conversation but vital in mathematics. In this paper, we discuss the effectiveness of frequent online quizzes (using the WebCT quiz feature) to assess students' understanding of assigned readings. In particular, we analyze these quizzes' effectiveness in motivating students to read carefully, their usefulness as an assessment tool, and students' (sometimes surprising) reactions to the quizzes.

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Learning What You Need to Teach — Just In Time

How do you know exactly what to teach? What parts of a concept do students understand and what parts are they struggling with? Modeled after the Just In Time Teaching (JITT) techniques popular in physics circles, a simple email feedback system can be used to help answer these questions *and* get students to actually read the textbook. With every reading assignment, students receive a question to answer before class. The instructor reads their responses, informing her/his class time interactions — just in time for class. Examples of questions and student responses will be given, along with self-reported student data indicating the effectiveness of this method in getting students to read the text and improving their overall learning experience.

Jon Breitenbucher (jbreitenbuch@wooster.edu) 1189 Beall Ave., Department of Mathematics and Computer Science, Wooster, OH, 44691

The AiM System

The Assessment in Mathematics (AiM) system is an open source effort to create an automatic homework/quiz grading system. I have used the system in my second semester calculus courses in Spring 2003 and Spring 2004 and am involved in the systems development. My talk will give an overview of the set up of the AiM system and try to address some questions that one might have about such a system. Attendees at this session will learn strategies for authoring questions and generating hints for the students. Student reaction to the system and student suggestions for improvement will also be shared.

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Analysis of On-line Gateway Testing Data

Gateway tests are a formative assessment tool that provide a means of assuring that students in reformed precalculus and calculus courses acquire the algebraic and computational skills needed in these and subsequent courses, while allowing the focus of the course to be on the conceptual understanding intrinsic to reformed mathematics courses. At The University of Michigan at Ann Arbor we have used gateway tests in our reformed precalculus, calculus I and calculus II courses since 1994, and have administered these tests on-line since Fall 2001. Our assessment of the efficacy of this testing program has found a strong correlation between completion of a gateway test and student acquisition of tested skills [LaRose and Megginson, Primus XIII(4), 2003]. In this paper we extend this assessment to a more general analysis of the gateway test data, including the numbers of gateway attempts made by students, the timing of these attempts, their eventual course grades, and their course and gateway test pass rates. Our data support many of the expected correlations between student success between student performance on the gateway and other metrics for learning and skill acquisition, while also revealing some less expected results. Finally, we discuss how the results presented corroborate our conclusions as to the effectiveness of the on-line gateway tests and inform structure and administration of our testing program.

Helmer Aslaksen (aslaksen@math.nus.edu.sg) Dept. of Mathematics, National Univ. of Singapore, Singapore, CA, 117543

Astronomical Java Applets on the Web

I teach a course called "Heavenly Mathematics: Cultural Astronomy" (www.math.nus.edu.sg/aslaksen/teaching/heavenly.html) at the National University of Singapore. It is easy to visualize the Earth revolving around the Sun, or the apparent motion of the Sun around the Earth, but what will the motion of the Sun and the Moon look like from different parts of the Earth? This requires good spatial visualizations skills. Most sources take a "high-northern-latitude centric" point of view, but I try to be "hemispherically-correct." I have created several interactive Java applets that allow the students to explore how the motion of the Sun and the shape of the Moon depend on the latitude and the time of the year. Many applets on the web demonstrate concepts that are easily visualized, but my applets help the students with complex three-dimensional problems. They have been essential for my course and are very popular with the students.

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OWL System with WebMathematica in Applied Calculus

For a large-lecture applied calculus course we adapted the locally developed OWL on-line web-based learning system to deliver, grade, and give immediate feedback on assignments drawn from pools of parameterized short-answer questions. We implemented a web*Mathematica* back-end for OWL to evaluate input, display symbolic expressions, and generate graphical displays that change as the student alters parameters. Such displays provide user tools for discovery modules on key ideas: e.g., one module guides discovery that an exponential growth population's doubling time

is independent of initial size; a parallel one concerns investment doubling time. As a result of using OWL, the proportions of students receiving high grades and failing tended to increase and decrease, respectively. Efficacy of the discovery modules is unclear, but students generally deemed them valuable. Many indicated that OWL was instrumental in keeping them on pace. (This paper is based upon work supported by the National Science Foundation under Grant No. DUE-0088758.)

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Preparing For Intelligent Internet Math Software

Mathematics lends itself to intelligent computer-based instruction and thus to Internet-based instruction more readily than most academic disciplines. Most presentations of new mathematical material could be conducted more effectively by computers than by instructors or textbooks. Interactive practice problem programs already exist that respond intelligently to each student step. Mathematics instruction worldwide costs many billions of dollars annually, mostly supplied by governments under constant pressure to control budgets. Although no existing instructional program exhibits all of the required features, such materials will inevitably appear due to the sheer size of the potential market. The source may be venture capital enterprises, existing commercial publishers or academic consortia with government funding. It may be in North America, in Europe or in India. The resulting materials will successfully emulate an omnipresent human tutor in a classroom or at home via the Internet. They will be marketed aggressively to legislators and to educational institutions that will find them irresistible. Mathematics educators will have to adapt to dramatic changes in their circumstances once their funding sources discover that improved results can be inexpensively obtained with far less human support than is currently required.

Doug Ensley (ensleyda@earthlink.net) 62 Diller Drive, Shippensburg, PA, 17257

What's New in DCR

The MAAs Mathematical Sciences Digital Library features high quality, peer-reviewed teaching material in its Digital Classroom Resources (DCR) section. In this presentation, we will highlight some of the new content in DCR, demonstrate the various platforms being used in the development of material for this site, and show off the features of the new-look MathDL!

David Smith (das@math.duke.edu) 1408 Shepherd St., Durham, NC, 27707

What's New at JOMA?

The Journal of Online Mathematics and its Applications (JOMA), a component of MAAs Mathematical Sciences Digital Library (MathDL), has an attractive new design that is also easier to navigate and search. After a half-year hiatus, it has reopened with exciting new content, as well as the archival content of the first three volumes. This presentation will give an overview of the new JOMA and will highlight some of the new content.

Lang Moore (lang@math.duke.edu) 2116 W. Club Blvd., Durham, NC, 27705

What's new at Math DL

MAA's Mathematical Sciences Digital Library (MathDL) has a new look and many new features. The collection has been completely redesigned to be easier to navigate and search. The original features of MathDL, the Journal of Online Mathematics and its Applications (JOMA) and Digital Classroom Resources (DCR), have added exciting new content. We have also added a new online magazine, Convergence, dedicated to the use of the history of mathematics in mathematics education. The presentation will give an overview of the new site and discuss the new features.

Kunio Mitsuma (mitsuma@kutztown.edu) 259 Lytle Hall, Department of Mathematics, Kutztown, PA, 19530

Creating Interactive Math Web Pages

Does the thought of HTML scare you, let alone creating interactive math web pages? Wouldn't it be great to be able to construct, with virtually no knowledge of HTML, web pages on which students can change values of variables within a standard web browser and see changes take place in *real time* with graphs and accompanying computations?

You can! Come see how easy it is for anyone to create such web pages. For instructors who teach secondary mathematics majors (= future high school math teachers), you can pass onto your students what they can do for their own students using WWW & interactive math web pages. For instructors in other areas of mathematics, applications of interactive math web pages are limited only by your imagination.

Thomas Hagedorn (hagedorn@tcnj.edu) P.O. Box 7718, Ewing, NJ, 08628

Using the Web in Linear Algebra

At the College of New Jersey, we have overhauled our introductory Linear Algebra course over the past two years. We have introduced the use of Matlab and ATLAST computer labs, computerized homework grading systems (Temple's Cow and the University of Rochester's WeBWorK), java applets, and online discussion groups. While the course continues to evolve, we have both anecdotal and quantitative evidence that such technological tools have greatly improved the learning environment. We have seen an ability to cover more topics, some at a higher level, and an increased understanding of linear algebra by the students, both during the course, and in subsequent courses. We will report on our experiences to date and what we have learned.

Thomas Banchoff (tfb@cs.brown.edu) Department of Mathematics, Providence, RI, 02912-9032

Internet-Based Multivariate Calculus and Geometry

This is a brief overview of an NSF-supported student project to develop materials for use in Multivariable Calculus and Elementary Differential Geometry. The materials include: a robust program for online communication, Java applets for classroom demonstrations, and online homework.

Extracurricular Mathematics

Jeremy Case (jrcase@tayloru.edu) 1012 W. Spencer Avenue, 236 W. Reade Ave., Upland, IN 46989, Marion, IN, 46952

What if No One Comes?

Despite the best intentions and efforts, students have trouble finding the time to participate in extracurricular mathematics activities. This talk will focus on our department's strategy to streamline the planning process and to ensure student participation in colloquiums, career nights, and math conferences. One particularly successful event involving math and music will illustrate our strategy.

Donna Beers (donna.beers@simmons.edu) Mathematics Department, Simmons College, Boston, MA, 02115

Integrating Learning, Assessment, and Extracurricular Activities

We experienced a record number of student requests to pursue independent studies during the past spring semester. In this talk, we will describe our efforts to build a learning community of students who shared the common desire to conduct independent studies in mathematics. We will show how we incorporated both a field trip to see Arcadia and participation in an undergraduate mathematics conference as integral learning activities. We will also describe tools for self-assessment that were developed to enhance and deepen student understanding of mathematics.

Joan DeBello (debelloj@sjtohns.edu) 8000 Utopia Parkway, Jamaica, NY, 11439

The Importance of Mentoring and the Influence on Women Mathematics Majors as Seen Through the Women in Science and Clare Booth Luce Scholarship Programs at St. John's University

This paper will discuss the importance and impact of the Women In Science Program at St. John's University. As a fellow graduate of the program and a current member, I have seen so many positive influences within an organization that pushes women to strive to be the best they can be in mathematics and the sciences where they have been normally under-represented. This paper will discuss the program and how it has shaped the lives of so many women who are graduates and current students at St. John's University. It will also discuss the different types of extracurricular activities and meetings that the program offers through the year.

Sarah Mabrouk (smabrouk@att.net) P.O. Box 2752, Framingham, MA, 01703-2752

Invite Them and They Will come!

Every instructor has the opportunity to encourage and to inspire his/her students to want to learn more mathematics and to want to explore the concepts and ideas about which (s)he teaches. For a course, the interest or the desire to want to learn more can be stirred through examining interesting questions, for example the what-if's for concepts, or by considering thought provoking problems or projects. However, courses can be limiting, not allowing the instructor to provide students with a glimpse or a taste of the different areas of mathematics that they may study or want to explore in the future. This glimpse into other areas of mathematics is important, possibly affecting the student's choice of major and/or career preference, and it could be, among other things, a student's first exposure to abstract mathematics, an introduction to research in mathematics, or a window into how mathematics can be used to examine/model a variety of interesting real problems. This exposure to new ideas and different areas of mathematics can be achieved through the student's attendance of special mathematics presentations/talks and local conferences. Attendance of local mathematics meetings provides students with an opportunity to meet and to attend presentations given by active researchers as well as to attend student paper presentations: both of these can add to the student's awe and wonder of mathematics as well as give the student the very important "I want to do that" and "I can do that too" sense that can spur him/her on to advanced study of mathematics.

In this presentation, I will discuss my efforts to expose my 43.222 Differential Equations students to different areas of mathematics as well as research in mathematics through their attendance of local mathematics conferences such as AFRAMATH, RUMBUS 2003, and RUMBUS 2004 and the NES/MAA Preskenis Dinner Meeting as well as special presentations given by Lucy Dechene, "Bell Ringing and Permutation Groups," Thomas Koshy, "The Ubiquitous Catalan Numbers," and Frank Morgan, "Soap Bubbles and Mathematics." I will discuss my students' attendance at these extra activities, how I motivated them to attend, our discussions of the presentations that they attended, student reaction to attending these conferences and presentations as well as how participating in these activities has affected their attitudes and interest in mathematics.

James Bruening (jbruening@semo.edu) Department of Mathematics, MS 6700, One University Plaza, Cape Girardeau, MO, 63701

Math Field Day

Can you imagine high school students cheering for winners in mathematics competitions? Every year that is one of the thrills that is a part of Math Field Day jointly sponsored by the Mathematics Department of Southeast Missouri State University and the Southeast Missouri Council of Teachers of Mathematics. Over 600 high school students and teachers annually attend this event. Individual and teams winners are selected for algebra, geometry, and trigonometry tests. Individuals also compete in varied events such as Matrices and Determinants, Elementary Data Analysis, Mental Arithmetic, History of Mathematics, Sets and Logic, to name a few. Several individual and cooperative problem-solving events are also part of the day's events.

Elizabeth Mauch (emauch@bloomu.edu) Department of Mathematics, Computer Science & Statistics, 1137 McCormick Hall, Bloomsburg, PA, 17815

Service Projects for Mathematics Honor Societies

Many mathematics honor societies for undergraduates (for example KME or PME) have projects for their members that involve mathematics. The students at Bloomsburg University wanted to not only learn new mathematics, but serve the community as well. This presentation will show how we did that by creating two activities for the Bloomsburg Children's Museum. A discussion of how to incorporate such activities will follow.

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Joel Foisy (foisyjs@potsdam.edu) 44 Pierrepont Av., Potsdam, NY, 13676

We Build Excitement!

Part of creating a successful mathematics program is establishing a presence on campus and in the local community. We establish and maintain this presence through a number of exciting activities that involve students as organizers and participants and are generally open to the campus and the community. Past events have included a Giant Human Knot, Baking Pretzels in the Shape of Knots, and Bowling for Primes. We will do a live demonstration with audience participation and give ideas for how you, too, can build excitement about mathematics in your community.

Ioana Mihaila (imihaila@csupomona.edu) 3801 W. Temple Ave., Dept. of Mathematics, Pomona, CA, 91768

Back by Popular Demand: The Problem Solving Group

This talk will bring together several activities that we organized at Cal Poly Pomona with the purpose of solving math problems and having fun. The activities varied with the calendar year (Putnam coaching sessions in the Fall, problem of the month ongoing, and problem solving class in the Spring). The presentation will focus on how to efficiently organize such activities so that they are exciting, productive and dont take an unreasonable amount of time. Several student solutions will be presented. Handouts with resources will be available.'

James Sellers (sellersj@math.psu.edu) Department of Mathematics, 107 Whitmore Laboratory, University Park, PA, 16802

Math Night!

In this talk, I will discuss the metamorphosis of the extracurricular mathematics activity that affectionately became known at Cedarville University as Math Night. This once-a-month activity became a highlight for my students and family alike as we opened our home to a variety of students (including a number of non-mathematics majors) for the evening. I will discuss the logistics and the goals for Math Night with the hope of encouraging those at my talk to consider hosting similar events at their own institutions.

G. Daniel Callon (dcallon@franklincollege.edu) 101 Branigin Boulevard, Franklin, IN, 46131

Integrated Extracurricular Program and Its funding

Extracurricular programming should be designed with the same focus and care as the mathematics curriculum. Experiences and activities should provide breadth and depth and build on each other. I will describe the comprehensive array of extracurricular offerings in the Department of Mathematics and Computing at Franklin College and how we encourage students to take advantage of them. I will also highlight the various types of funding sources we have developed.

Zac Cropper-Alt (zmc1@geneseo.edu) 60 Woodhaven Apts., 80 Cisar Raod, Willington, CT, 06279

Planning and Implementing Workshops for Teenagers

The talk will tell the story of the planning phase and implementation of an annual math workshop event for high school girls. The workshop was intended for girls who were at risk for discontinuing their math education. The event included workshops on topology, statistics, and Frieze Groups, as well as a math games session and a careers session in which young women from outside of math spoke on how math was important to their fields of study and work.

Angelo Segalla (asegalla@csulb.edu) 1250 Bellflower Blvd, Long Beach, CA, 90840

"Math Day at the Beach," an Annual Festival

Five years ago, the CSULB Mathematics Department connected in a meaningful way with mathematically talented high school students in its geographical area by instituting an annual competition:"Math Day at the Beach," ("the Beach" is the university's nickname) a problem-solving festival intended for students who want to have fun with mathematics and enjoy a day of camaraderie with university mathematics students and professors.

Two serendipitous outcomes resulted: (1) A mentoring program, that paired a handful of high school students each with a different volunteer professor, and was designed to cultivate (mostly) email "apprentice-specialist" mathematics dialogue. (2) the formation, a first time ever and now very competitive, Southern California ARML team.

Putting Some Analysis Into Introductory Real Analysis

Robert Rogers (rogers@fredonia.edu) Department of Mathematics, SUNY Fredonia, Fredonia, NY, 14063

Providing a Course Theme in Introductory Analysis

Following the approach used in David Bressoud's "A Radical Approach to Real Analysis", the presenter has adapted the theme of comparing power series to Fourier series to motivate the precise definitions and theorems in introductory real analysis. Samples of assignments from Bressoud's book and others designed by the presenter will be provided along with classroom strategies used to develop students' ability to transform intuitive ideas into formal arguments.

Betty Mayfield (mayfield@hood.edu) Department of Mathematics, 401 Rosemont Ave, Frederick, MD, 21701

Computer Explorations in Real Analysis

Isaac Newton and Pierre Fermat had to do their calculations by hand, but present-day students can speed through computations, and go on to draw conclusions and ask new questions, by using a computer. We will discuss some learning modules we are developing for undergraduate students in a real analysis course, consisting of an explanation of a problem or concept in real analysis and a series of guided explorations.

Loren Pitt (ldp@virginia.edu) Department of Mathematics, Kerchof Hall, Charlottesville, VA, 22903

An Analysis Course Focused on Communication

In the spring of 2003 I taught an introductory junior level real analysis course at the University of Virginia. In this course I tried to emulate the learning experience that I recently had experienced at a *Developing Mathematical Ideas* leadership institute (for elementary school teachers) at Mount Holyoke. In this class I placed an unusually strong emphasis on using problems to teach mathematical concepts and on oral and written communication. The result was an exceptionally active and engaged class. I will describe how the class was organized and my perception of the results. Enrollments were up 70% a year later.

P. E. Fishback (fishbacp@gvsu.edu) Department of Mathematics, 1 Campus Drive, Allendale, MI, 49401

Projects in a Real Analysis Course

Group projects can play an important role in the introductory real analysis course in that they are one of several means of encouraging students' active participation in the learning process. This presentation will focus on two such projects, one dealing with the construction of the real numbers using Dedekind cuts, the other dealing with the construction of Lebesgue measure. The primary goal of these projects is to challenge students intuitions regarding difficult mathematical issues. A second goal is to develop further students proof-writing skills, especially insofar as set-related proofs are concerned. Because the nature of the mathematics in the projects differs significantly with the epsilon-delta proofs that many students equate with real analysis, students leave the course with a broader view of the subject.

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Kirk Weller (wellerk@unt.edu) University of North Texas, P.O. Box 311277, Denton, TX, 76203

An Exploratory Approach to Analysis

In our real analysis classes, we use a laboratory approach to help students become comfortable with the fundamental concepts of analysis: the limit, supremum and infimum, boundedness, limsup and liminf, continuity, and uniform convergence. Without a solid understanding of these fundamental concepts, it is hard for students to read text materials, to write proofs, or to develop solid understandings of difficult and important real analysis concepts. In the labs, we ask students to work with many examples as they formulate definitions, make connections between different concepts, derive conjectures, or complete a sequence of guided tasks designed to facilitate concept acquisition. Several of the labs can be done by hand or with a graphing calculator. For others, a computer algebra system such as *Maple* can be used. We have generally assigned the labs as a precursor to any class discussion or lecture on a given topic. We have found that students have become more adept at solving problems and in proving theorems that involve the application of fundamental course concepts. In this talk, we will describe how we use one of the labs in our courses.

Larry Knop (lknop@hamilton.edu) 198 College Hill Road, Clinton, NY, 13323

Tricks of the (Real Analysis) Trade

Real Analysis should generate mathematical wonder and a healthy sense of paranoia in students, and a feeling of satisfaction in the instructor. I have found no grand organization of ideas that is particularly effective; the devils are in the details. In this talk I will speak of some devils that I have banished, and of some devils that continue to torment. The single most important improvement I have made in Real Analysis is the extensive use of team assignments. I am not a team assignment type of person, but the benefits include greater involvement of students in the course, greatly increased interaction among students, a somewhat reduced grading load, and significantly higher student evaluations. In short, team assignments are as close to a win-win situation as I have encountered in teaching. I have also freed myself from the tyranny of the blackboard. One solution to the problem of students as biological copy machines is a computer, a computer projection system, and Microsoft Word in outline format. In outline format Word functions like an overhead projector, with the advantage that your lecture notes can be made accessible to students. The initial investment of time is horrendous, but the freedom is worth the price. The electronic gods willing, I will use this talk to demonstrate. A useful addition to Real Analysis is paper writing. For instance, students should be able to take a mathematical passage and decode it so why not ask them to do so? Take a short textbook section that you normally skip and ask students to read the text and write an expository paper on it. Students learn how to do everything from filling in mathematical gaps to learning how to type mathematics, and each year my students teach me something new. A demon that continues to torment is the illogic of the language of Real Analysis textbooks. One simple example: for the nth term test for series we distinguish between an implication and its converse, and we emphasize that for the nth term test the converse is false. Yet definitions are stated as implications (only!) and the truth of the converses are assumed.

Stephen Abbott (abbott@middlebury.edu) Department of Mathematics, Middlebury, VT, 05753

Integration: From Cauchy to Riemann to ...Riemann

One satisfying way to make the epsilons matter in an introductory analysis course is to look at the issues surrounding various historical definitions of $\int_a^b f$. Cauchy is usually given credit for divorcing the integral from the derivative, and Riemann then modified Cauchy's definition with the goal of trying to understand what types of functions could actually be integrated. A particular focus for this talk will be whether the class of integrable functions—as Riemann described it—includes every derivative. That is, having given the integral its own rigorous footing, we'll explore the terms under which the equation $\int_a^b f' = f(b) - f(a)$ holds. The investigation offers a preliminary glance at Lebesgue's contribution to integration and, surprisingly, leads back to a lesser known and recent modification of Riemann's original approach.

Robert Bradley (bradley@adelphi.edu) Adelphi University, Dept. of Mathematics & Computer Science, Garden City, NY, 11530

Putting History into Introductory Real Analysis

The original meaning of *analysis* was in reference to the analytic method (as opposed to the synthetic), itself closely related to algebraic methods. This is reflected in the early conception of analysis as an extension of coordinate geometry to infinitely small or infinitely large quantities, and to infinite series.

In this talk, I will discuss the use of historical sources to motivate an introductory Real Analysis course. By exploring how 18th century mathematicians manipulated suspect quantities, we can introduce results whose modern proofs might be beyond the scope of the course, and also motivate the need for a more rigorous approach to the subject.

This talk will be illustrated with episodes involving logarithms, cusps and series, based on the works of Euler, d'Alembert, Gua de Malves and Lagrange.

Cooperative Projects Between the Mathematical Sciences and the Life Sciences

Timothy Comar (tcomar@ben.edu) Department of Mathematics, 5700 College Road, Lisle, IL, 60532

Calculus Labs for Biology and Pre-med Students

In an effort to better prepare biology and pre-medical students for the increasing level of mathematical background needed for their future coursework, the Department of Mathematics at Benedictine University has begun to offer a two-semester calculus sequence for this audience. This sequence is the introductory mathematics sequence for students in the University's new biochemistry and molecular biology major. Although these two courses are offered at the same level of mathematical rigor as the traditional sequence for students majoring in mathematics, physics, and engineering, the content is specifically geared to meet the needs of the biology and pre-med students. In this talk, we present several computer algebra laboratory projects designed for this biology oriented calculus sequence.

One of the most valuable aspects of these activities is the integration of mathematical and biological reasoning, which better prepares the students for future course work in the biological sciences and which motivates the students to understand the importance of mathematics in the biological sciences and hopefully continue with future mathematics courses. Traditional problems in mathematical biology focus on differential equations. The examples presented in this talk will investigate models based on difference equations. Difference equations not only provide straightforward introduction to population models and differential equations but also provide nice examples for the study of other topics including limits and, in the multivariable setting, basic concepts in linear algebra.

Limits are introduced in this course via the study of discrete time population models (difference equations). One lab project compares and contrasts discrete population models including the exponential growth model and the discrete logistic equation. Through further long-run analysis of the discrete logistic equation, the activity provides both geometric (visual) and numerical intuition to help develop the formal notion of a limit at infinity. Another activity with the discrete logistic equation studies period-doubling and chaotic behavior. We also have a project that investigates local stability of population models defined by difference equations using cobwebbing, derivatives, and limits. These investigations of these discrete time models early in the course help set the stage and develop needed intuition for the subsequent study of differential equations, including the exponential and logistic differential equations.

Another way in which this calculus sequence differs from the traditional sequence is that the students are introduced to basic linear algebra and multivariable calculus during the second term. This change in content allows for activities that may not usually occur during a single year of calculus. An example of such a project investigates the Leslie matrix, which can be used to model the population dynamics in an age-structured population with discrete breeding seasons. This activity involves eigenvalues and eigenvectors of matrices, iterations of linear maps, and limits. Another activity addressing content from linear algebra investigates classical discrete generation, host-parasitoid models via systems of difference equations. An extension of this activity, which requires partial differentiation, addresses some simple nonlinear difference equations.

The projects are intentionally short and designed to be completed in two-hour lab sessions. As such, problems in these labs could also be assigned as extended homework problems or presented as in depth examples in class. Each lab contains additional questions for further investigation, which would allow for deeper investigation and possibilities for course projects.

Technology desired is a computer projection device to plug in a Windows laptop. If this is not possible, the standard overhead projector will suffice.

Laurie Heyer (laheyer@davidson.edu) PO Box 6959, Davidson, NC, 28035

Genomic and Bioinformatics at Davidson College

At Davidson College, we have been developing an education and research program in genomics and bioinformatics for several years. In this talk, I will describe how we developed our courses and a new interdisciplinary concentration that promises to draw students from many majors. I will describe our joint undergraduate-focused research program. Finally, I will provide some examples of how these experiences are changing students graduate school and career decisions.

Deborah Lurie (lurie@sju.edu) 5600 City Avenue, Department of Mathematics and Computer Science, Philadelphia, PA, 19131

IBASE: Integrating Biology and Statistics Education

Although statistical literacy has become an essential tool for researchers in the life sciences, most undergraduates majoring in biology receive no formal training in the science of statistics. Recognizing this gap in the education of their students, members of the mathematics and biology departments at Saint Joseph's University collaborated to design an introductory applied statistics course that would: (1) provide undergraduate biology majors with a sound foundation in statistical methodology and reasoning; (2) demonstrate to the students the importance and relevance of statistics to the field of biology; (3) provide experiences for teamwork and oral and written communication; and (4) reduce students' anxiety level with quantitative methods. An in depth description of this collaboration and the resulting course curriculum and impact on the students' education will be presented.

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"Meeting the Challenges"

Are the mathematics and biology communities "meeting the challenges"? That was the basic question for the conference of that title organized by the MAA, in conjunction with the American Association for the Advancement of Science and the American Society for Microbiology, and with support from the National Science Foundation and the National Institute of General Medical Sciences. One of the main purposes of the "Meeting the Challenges" conference was to bring together mathematicians, biologists, statisticians, and computer scientists to try to come up with models for the mathematical education of biologists for the twenty-first century. This talk will preview the report to come out of the conference, which is being edited by Lynn Steen.

Innovative Approaches in Mathematics Education

Katie Acker (kathleen_am@hotmail.com) 610 King of Prussia Road, Radnor, PA, 19087

Teaching With Technology

In recent years technology has become ubiquitous in the mathematics classroom in the form of the graphing calculator. Advanced capabilities coupled with portability make the graphing calculator an ideal tool for the math student on the go. The problem is that anecdotal evidence suggests that students sell their calculators upon completing required coursework. Later, employed graduates find themselves placed into problem solving situations, sans calculator, but with a computer. As members of the future workforce, students enrolled in Modern College Mathematics I at Cabrini College have begun learning how to use Microsoft Excel as a problem-solving tool. This talk will discuss the assignments given to the students, the challenges of using the computer as a teaching tool, and student reaction to what they learned.

Timothy Comar (tcomar@ben.edu) Department of Mathematics, 5700 College Road, Lisle, IL, 60532

A Sketchpad Project for Elementary School Teachers

Many of the students in the mathematics for future elementary school teachers course come into this course with a weak background in geometry and little understanding of connections between geometry and other mathematical topics. Moreover few of the students, if any, have experience with dynamic geometry software. The culminating activity in the course is a project using The Geometers Sketchpad to create several exploratory activities designed for use in elementary school classrooms. We will describe the nuts and bolts of the assignment as well as how this assignment allows for the students to take ownership of the mathematical material and begin to shift roles from that of the student-learner to that of the teacher-learner.

Sketchpad is introduced to the students near the beginning of the geometry unit in this course. This is often the first time students have seen this software or any dynamic geometry package, one of which is likely to be utilized at future places of employment. Students learn the basics of using Sketchpad to make geometric constructions, make measurements, and perform transformations. The students are then given an assignment: create four complete Sketchpad activities that can be used in an elementary classroom.

Specifically, the students work in teams of four to create their exploratory activities. The teams choose which geometric topics from elementary geometry they would like to illustrate. For each of the exploratory activities, the students are required to include an appropriate introduction, directions, and complete solution. At least activity must address notions of transformational geometry, and another activity must address the concepts of direct and indirect measurement. Moreover, the activities are designed to make connections between geometry and other topics such as (numerical) pattern recognition and arithmetic. Chosen topics include symmetries of (regular polygons), illustrations of rigid motions in the plane, relationships between rigid motions and congruence, tessellations, similarity, relationships between perimeter and area, area calculations, understanding, the Pythagorean Theorem, and basic trigonometry.

Each student is required to present orally one of their team's activities to the class. Moreover, each student is required to address several issues about each of the team's creations. These issues include an accurate description of the mathematics of the illustration or activity, a description of how the activity would be incorporated into the classroom environment (including the appropriate grade-level audience), and how the illustration or activity meets each of the NCTM standards. The students are also required to critique how well Sketchpad would aid the learning process for each activity.

This assignment not only creates an environment for students to actively learn Sketchpad and deepen their geometric skills but also provides the opportunity to address and organize this mathematical content from the role of an instructor rather than from that of a student preparing for a pencil and paper exam, thus enabling the students to take ownership of the content and skills that they will use in their future profession.

For future implementations of this project, we are planning to add a clinical component by requiring the future teachers implement their activities for actual elementary school students in an elementary school classroom. Examples of student work will be presented.

Robert Rogers (Robert.Rogers@fredonia.edu) Department of Mathematics, SUNY Fredonia, Fredonia, NY, 14063

"Real World" Math for Pre-service Teachers

This talk describes how an upper level requirement in mathematics literacy is used to provide "real world" relevancy of mathematics for pre-service adolescence and middle childhood mathematics teachers. Samples of assignments combining searches related to specific applications and problems explaining the underlying mathematics will be given.

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Sr. Barbara Reynolds (breynolds@stritch.edu) Cardinal Stritch University, 6801 N. Yates Rd., Milwaukee, WI, 53217-3985

Teaching College Geometry With Computer Activities

The presenters are nearing completion of a textbook and CD for college geometry, an important course for future educators. Topics are introduced by exploratory activities using The Geometer's Sketchpad. These activities are conducive to working in small groups. After students have this foundation, readings and class discussions clarify and extend the concepts, as well as providing context and appropriate notation. Then students are expected to demonstrate their understanding through follow-up exercises. Developing skill with proof is a theme throughout the course.

We have been class-testing these materials since Fall 2002. We will report on our experiences and on student reactions.

George Ashline (gashline@smcvt.edu) 11 Pine Grove Terrace, Winooski, VT, 05404

A Mathematics Education Seminar for K–12 Teachers

Preparing students to teach mathematics at various levels is of critical importance given current national pressures and trends in education. In this talk, I will describe a seminar that I have created to enhance the knowledge and preparation for teaching mathematics of pre-service and in-service teachers. In the seminar, students have opportunities to discuss important issues in mathematics education and to create and present lessons on mathematical topics of their own choosing.

Sarah Greenwald (greenwaldsj@appstate.edu) 121 Bodenheimer, MATH, ASU, 326 Walker Hall, Boone, NC, 28608

NCM^2 Class in Geometry and Measurement

The North Carolina Middle Math Project (NCM^2) is a collaboration between the Mathematics and Science Education Network (MSEN) of the University of North Carolina and the North Carolina Department of Public Instruction (NCDPI). The goals are to improve mathematics education in grades 6–8 across the entire state of North Carolina, support teachers in their professional development, and provide academic renewal and financial recognition to support their retention. Nine universities in the North Carolina university system participated in the NCM^2 project by sending faculty members to serve on the writing teams. In 2002, the NCM^2 Leadership Network developed and taught a series of three graduate-level courses for middle school mathematics teachers, one each in the content areas of statistics and data analysis, geometry and measurement, and number and algebra. Teachers use the course work as the foundation in obtaining National Board Certification in Early Adolescence/Mathematics and a master's degree in middle school mathematics education. Many of the universities are now offering the NCM^2 courses as a part of their master's program for middle school teachers. We will discuss the development and implementation of the NCM^2 geometry and measurement course over the last few summers.

Michael Scott (mbscott@math.ksu.edu) Kansas State University, Department of Mathematics, Manhattan, KS, 66506

On Web-Based Writing Assignments

Many students in a mathematics content course for pre-service elementary school teachers believe that a procedural knowledge of the course material is enough, and seem unwilling to accept that any deeper understanding is required. As an instructor how do you motivate such students to make an effort to understand the concepts and reasoning that goes beyond such procedural knowledge? How do you convince pre-service teachers that the aspects of mathematics you are requiring them to learn is what they need to know?

One way might be writing assignments pertaining to readings such as Liping Ma's "Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States." In addition to motivating students, writing assignments give an opportunity for students to reflect on the their knowledge, and the instructor an opportunity to obtain feedback on students' understanding. Recently, I've created a web-based writing assignment system that has some advantages over standard paper writing assignments. For students, the advantages include the ability to view and comment on other students work. For instructors, the advantages include the option of commenting privately and/or publicly on students' work, and the ability to submit grades electronically. The system is similar to an online message board, but with built-in features especially made for implementing writing assignments. In my presentation I will show how I have implemented this system in a mathematics content course for pre-service teachers and give examples of students' work.

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Fractals in the Classroom

We will discuss a summer workshop for in-service middle and high school teachers on fractal geometry. The workshop is intended to demonstrate the wide scope and appeal of this field. We emphasize ways in which fractal geometry may be incorporated into the current curriculum. The material is presented in lab activities ranging from the computer based to ruler and compass constructions. The ideas brought out in fractal geometry address a wide range of the NCTM Standards.

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Presenting Arithmetic as a Science of Machines

To the extent that elementary arithmetic is presented as science, that is, as a theory to account for some range of experience, it is generally taken to be a science of "quantity." Quantity is at best a vague concept and, as Piaget discovered, our intuitions regarding quantity develop slowly. Worse yet, quantity only directly accounts for a small subset of arithmetic and that not very well. As bizarre as it may sound I will try to make a case for presenting arithmetic to elementary school teachers and their students as a science of "machines."

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Projects for Pre-service Teachers

In a Math Concepts course, aligned to state and national standards, individual and group projects were assigned. The merits of various assessment rubrics used for the projects will be illustrated. A Constuctivist approach will be utilized to discuss difficulties encountered in assigning and grading projects by pre-service teachers.

Julie Belock (julie.belock@salemstate.edu) 352 Lafayette St., Salem, MA, 02144

Developing an MAT in Middle School Mathematics

In January of 2004, Salem State College received a grant from the Massachusetts Department of Education to develop a Master of Arts in Teaching program for middle school mathematics teachers. Salem State partnered with two cities, Salem and Lynn, MA, whose middle school math teachers would comprise the first cohort of students in the program. The grant requires that some of the courses being developed for the program be offered as soon as possible (as workshops convertible to courses in the MAT program, assuming it is approved), even before the program and the new courses are officially approved by the College.

In this paper I report on the process of developing the program as well as the creation of a particular course, History of Mathematics for Middle School Teachers. I will discuss the concerns of the department, including the feasibility of offering a master's degree from a mathematics department to students who in many cases lack a Bachelor's degree in mathematics, as well as how these concerns were ultimately addressed. The overall approach taken to the program has been fostering a deep understanding of the mathematics that is taught in middle schools, in line with what is advocated by the NCTM and the mathematics curriculum framework of Massachusetts.

I will use the history of math course as an example of how these ideas were implemented. By the time of the conference I will have taught the history of math course (albeit as a workshop) one time and will share the results of that experience.

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Pre-service Elementary Teachers Reaction to PFUM

In her book, Teaching and Knowing Elementary Mathematics (KTEM), Liping Ma compares mathematical understanding between U.S. and Chinese elementary school teachers as it relates to classroom teaching practices. Both the mathematical and mathematical education communities agree the most important audience for this book is college and university faculty members who teach mathematics to future teachers. With this in mind, the researchers believe that KTEM must also convey an important message for pre-service elementary teachers.

Students enrolled in the mathematics content courses for pre-service elementary teachers during Fall, 2003 and Spring, 2004 at Colorado State University-Pueblo, Illinois College, Plymouth State University, University of Memphis and the University of North Dakota read KTEM and wrote 4 reaction papers. We will present

1. pre-service elementary teachers' reactions to selected excerpts from KTEM;
2. how reading and reflecting on excerpts from Ma's book influences pre-service elementary teachers attitudes towards mathematics and the teaching of mathematics; and
3. how KTEM promotes the development of profound understanding of elementary mathematics.

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Pre-service Teachers and Effective Problem-Solving

This study investigates performance relative to reasoning and mathematical problem-solving skills and processes among three populations of pre-service teachers studying at the elementary school, middle school, and high school levels. A two page structured mathematics quiz was given to 89 pre-service teachers at the three aforementioned levels of teacher preparation. The first page of the quiz involved finding solutions to two dissimilar types of problems that required different reasoning processes and abilities. The second page of the quiz asked each student to reflect on the difficulty he/she had solving the two problems and analyze why these difficulties occurred. Quantitative and qualitative analyses were conducted with significant results that suggested that the preparation of future teachers needs to contain more opportunities for solving difficult problems of dissimilar types with both mathematics and real life applications.

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Algebra/Discrete Structures Online for Teachers?

Can pre-service and inservice teachers really learn abstract algebra and discrete structures in a course with absolutely no face-to-face interaction? This experimental summer course at James Madison University brought undergraduate and graduate teachers together in a online community to tackle the concepts of functions, groups, rings, integral domains, and fields. Find out what happened!

Martha Waggoner (waggoner@simpson.edu) 701 North C Street, Indianola, IA, 50125

Mathematics With and Without Words

I asked my elementary education students to participate in a project where they investigated a specific concept from the curriculum, write a concept paper, use the ideas from their topic to design a quilt block and, finally, to cut the fabric and sew the block. The semester-long project allowed them to explore a single topic in depth, to make connections between the analytical, visual and verbal representations of mathematics and gave them experiences that they could adapt to their own classrooms.

The students took great pride in the work they did, became more confident in their ability to understand mathematics and learned that there is room for creativity in mathematics. The quilt project allowed for opportunities to introduce scale drawing, the geometry of borders, map coloring, the importance of precision and the effects of error propagation into the course because they naturally fit into the design and sewing process.

I plan to give an overview of the project, describe a few specific designs and the underlying mathematics and summarize the impact of the project on me and my students.

Advances in Recreational Mathematics

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Fibonacci's Triangle and Other Abominations

There are many ways to generalize the Arithmetic Triangle studied by Pascal, and this is certainly another one. We will discuss the generalization of replacing the initial conditions (of "all 1s" down the outer edges) with arbitrary values. Of particular interest will be cases where the arbitrary values come from well-understood sequences such as the Fibonacci numbers. In these cases, combinatorial identities can be incorporated to give elegant descriptions of these generalized Arithmetic Triangles.

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Coloring Origami: Adjacent Edge Graphs

The author will report the results of an undergraduate research project he supervised involving the coloring of origami models constructed from several paper units. Given a planar graph representation for an origami model, a type of dual graph is constructed and analyzed. The construction is partially invertible and relates strongly to the standard dual graph. The process also generalizes to nonplanar graphs, and this will be discussed.

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Sequences of P/N Positions in NIM without Preperiods

Optimal play for single-pile NIM under a finite subtraction set yields a periodic sequence of P (pervious player wins) and N (next player wins) positions. Past research has started with the subtraction set and examined properties of the resulting P/N sequence. I solve an inverse problem: Given a P/N sequence without a preperiod (*i.e.* it exhibits no transient behavior before entering a cycle), I determine whether the sequence can be generated by a subtraction set. I view the result as an algorithm and construct subtraction sets for all non preperiod P/N sequences for given period lengths less than or equal to twenty. Data suggests that the number of P/N sequences without preperiods grows exponentially in period length. The algorithm can also be used to prove the behavior of P/N sequences for classes of subtraction sets thereby solving special cases of the forward problem.

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Knight Covering Solutions for 50×50 Chessboard

New computer algorithms have allowed us to find efficient, possibly optimal, knight covers for chessboards as large as 35×35. In this paper we take a divide-and-conquer approach to find efficient covers for the 50×50 chessboard. A known efficient pattern is placed in the center of the board, and then the edges and corners are solved separately.

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Surprising Probabilities and Simulations

Roll a single six-sided die twice. It is easy to see that the probability of getting a sum of seven is one sixth. What if the game is to keep rolling until the sum of all rolls either equals or exceeds seven? The resulting probability is a little harder to compute, and is surprisingly high. Similarly, it is easy to show that the four corners game always results in four zeros, but it is surprising how quickly one gets all zeros. This paper explains how simulations of these two games and others, using simple computer programs, demonstrate for students that the correct yet surprising probabilities they compute by hand can be supported with these simulations.

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Prisoners Dilemma Applied: "Friend or Foe"

Prisoner's Dilemma is a classic game theory problem involving two criminals detained separately for committing a crime. Each is offered a reduced sentence for describing the other criminal's involvement. John Nash's seminal insight into equilibrium in conflict situations can be used to evaluate the most strategic behavior for each of the criminals. The TV game show "Friend or Foe" on the Game Show Network creates a very similar dilemma for its contestants with a very attractive equilibrium point, at least for the producers of the show. In this paper I will describe the competitive situation presented by the game show, describe its equilibrium point, and relate it to Prisoners' Dilemma. I will also discuss the actual behavior of contestants on the show.

Hossein Behforooz (hbehforooz@utica.edu) 1600 Burrstone Rd, Mathematics Department, Utica, NY, 13502

A Unique 4x4 Magic Square for any Integer M

Archimedes claimed that "Give me a place and I move the earth." In this talk I will show that for any given integer M, there exists a unique magic square of order four such that its sum is equal to M. This magic square is unique with so many interesting properties.

Greg Frederickson (gnf@cs.purdue.edu) Dept. of Computer Science, West Lafayette, IN, 47907

Geometric Dissections Now Swing and Twist

A geometric dissection is a cutting of a geometric figure into pieces that can be rearranged to form another figure. Some dissections can be connected with hinges so that the pieces form one figure when swung one way, and form the other figure when swung another way. These dissections have remained as magical as when the English puzzlist Henry Dudeney first exhibited a hinged dissection of an equilateral triangle to a square almost a century ago. Based on my recently published book, "Hinged Dissections," the talk will explore two fundamental ways to hinge dissections of 2-dimensional figures such as regular polygons and stars. The first way uses "swing hinges," which allow rotation in the plane. The second way relies on "twist hinges," which allow one piece to be turned over relative to another, using rotation by 180 degrees through the third dimension. I will present several techniques for designing both types of dissections and demonstrate a variety of physical models.

Model Lessons from First-Year Calculus

Cheri Boyd (clboyd@naz.edu) Mathematics Department, 4245 East Avenue, Rochester, NY, 14618

Making Friends with Epsilon and Delta

A goal of our calculus course is its introduction to mathematical rigor. Helping first year students learn the epsilon-delta definition of limit is one opportunity to meet that goal. Mathematics, chemistry, biology, economics and physical therapy majors become conversant and increasingly comfortable with absolute value notation, the purpose of two unfamiliar Greek letters, two different quantifiers, and the entire notion of limits. A sequence of questions utilize the graphing calculator, informal language (verbal and written), hand-drawn graphs, carefully explained algebraic notation, a proof (!), and numerical examples for verification. In one seventy minute lesson, our students have made significant mathematical progress. Our math majors tutoring in the Math Center benefit that week as well!

Howard Penn (hbpenn@erols.com) 160 Tall Tree Trail, Arnold, MD, 21012

Making Faces and Other Parametric Equations Art

This paper will present the results of an assignment given to the author's calculus I students the last two years. The assignment was to use parametric equations to draw a face. After the results are shown, the author will present other parametric equation artwork that he has collected over the years.

Sarah Mabrouk (smabrouk@att.net) P.O. Box 2752, Framingham, MA, 01703-2752

Slope Graph Exploration

When many students examine a graph, they have difficulty using that graph to construct the corresponding slope graph. The main problems appear to be that they do not truly understand the relationship between the rate of change of the function and the sign on the slope and the relationship between curvature of the graph of the function and how the slope is changing. I have addressed these two problems using by-hand graphical analysis and using interactive workbooks that I created using MS Excel. In our initial by-hand graphical analysis, we explore the four main pieces of any graph — increasing-concave up, increasing-concave down, decreasing-concave up, and decreasing-concave down — using tangent lines to determine the relationship between how the function is changing and the sign on the slope of the function as well as the relationship between the curvature of the graph of the function and how the slope of the graph is changing. Then, we connect this to the mathematics that we have explored during class, considering the first and second derivative of the function, being careful to make the necessary connections between the symbolic mathematics and the concepts that we have examined. I use interactive workbooks that I created using MS Excel to help the students to make sense of the concepts that they have examined. Using these workbooks, the students can examine the slope graphically with user-controlled scrollbars, examine the associated tangent line, and consider and compare slopes of tangent lines.

In this presentation, I will present and discuss both the by-hand graphical analysis and the interactive workbooks. I will discuss the students' initial approach to slope graph analysis as well as how the by-hand analysis and the interactive workbooks help students to make sense of and to analyze graphs of functions. I will discuss student reaction to both approaches and how these approaches have helped students to analyze graphs of functions and to understand how to generate a slope graph using the original function graph.

Sheldon Gordon (gordonsp@farmingdale.edu) 61 Cedar Road, E. Northport, NY, 11731

Discovering the Chain Rule Graphically

The chain rule is one of the hardest ideas to convey to students in Calculus I. It is difficult to motivate, so that most students do not really see where it comes from; it is difficult to express in symbols even after it is developed; and it is awkward to put it into words, so that many students can't remember it or apply it correctly. In this talk, we present a way to introduce the chain rule graphically that both motivates it and gives the students a much better understanding of what it means and how to use it.

Michael Rogers (mroge02@emory.edu) 100 Hamill St, Oxford, GA, 30054

G.H. Hardy and the Limit of a Sequence

Based on a lesson from G.H. Hardy's *A Course of Pure Mathematics,* the author develops a way to motivate the definition of a limit of a sequence not found in most current textbooks. The method is fun and interactive, and the students learn a new way of thinking, always one of the best real-world applications of mathematics.

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Teruyoshi Yoshida (yoshida@math.harvard.edu) Department of Mathematics, Harvard University, Cambridge, MA, 02138

What Graduate Students Can Learn From Lesson Study

We will present experiences from a lesson on related rates that we (three graduate students and an experienced faculty member) developed and taught to two freshman calculus courses. After each lesson, we held a debriefing session with a small group of students. This enabled us to explore a student's perception of the lesson, evaluate the lesson's effectiveness, and to engage in an exchange of ideas about teaching and learning among faculty, graduate students, and undergraduate students in a non-evaluative setting. Focusing on a single lesson allowed us to explore teaching and learning issues in depth and realize the connections of related rates to the rest of the curriculum.

Alan Knoerr (knoerr@oxy.edu) Department of Mathematics, 1600 Campus Road, Los Angeles, CA, 90041

Rediscovering Richardson Extrapolation

Richardson extrapolation is a simple technique that can be applied to successive Euler's Method approximations, for example, to obtain a more accurate numerical estimate of the solution of an initial value problem. We discuss the process of rediscovering this method in the context of a number of years of work developing a calculus sequence that emphasizes modeling with first-order differential equations. This talk will focus on the interaction between teaching goals and methods, and student work and responses, that led us to recreate this technique and an effective strategy for teaching it.

Susan Wildstrom (susan@wildstrom.com) 7100 Whittier Blvd., Bethesda, MD, 20817

Using a Hand-held Graphing Calculator to Discover C

High school calculus students need to know how to differentiate transcendental functions, but the rules can become just so many memorized formulas. By using an exploration technique, students can actually see the graphs of those derivatives and they can discover the patterns that constitute those formulas. The presenter has developed four such explorations and will share one of them: the explorations that enable students to discover the derivatives of functions of the form $y = b^x$ and $y = \ln x$. Handouts of the student and of teacher notes for this exploration will be available as well as those for the others.

Mathematical Modeling Modules and Materials

Bart Stewart (bart.stewart@usma.edu) Department of Mathematical Sciences, West Point, NY, 10996

Heat Flow in a Finite Rod

One-dimensional heat flow from a partial differential equations perspective can be challenging to many students, but building the model from a few fundamental assumptions and observing solutions using discrete difference equations, rather than the traditional separation of variables approach, provides students an opportunity to explore heat phenomena across various boundary conditions. The application of constructing a root cellar solidifies the modeling effort.

Daniel Birmajer (abirmaj6@naz.edu) 4245 East Avenue, Rochester, NY, 14618

A Model for the Price of a European Stock Option

In this talk we present a Stock Market Model module based on the binomial method for pricing a stock option. The Binomial Method is a widely used technique, in which a binomial tree is used to represent the possible path that might be followed by the stock price over the life of the option. The implementation of such model depend on the assumption that the continuous random walk followed by the stock price can be modelled by a discrete random walk with the following properties:

- The asset price S changes only at discrete times
- If the asset price is S at a given time n then at the next step it may take one of only two possible values uS or dS with $d < 1 + r < u$. (Here, r denotes the risk-free interest rate).

The Binomial method rely indirectly on the celebrated Black-Scholes analysis through the assumption of risk neutrality, that is, the value of the option today is its expected future value discounted at the risk-free rate. Under this circumstances we will derive the risk-neutral probability measure and the risk-neutral valuation of the derivative security.

No previous knowledge of math finance is assumed, and the basic terminology will be discussed during the talk.

Bruce Torrence (btorrenc@rmc.edu) Randolph-Macon College, P.O. Box 5005, Ashland, VA, 23005-5505

Investigations in Critical Path Scheduling

In this presentation I will demonstrate tools that I have developed for investigations into a rich and exciting mathematical topic that is easily accessible to students in a 100-level mathematics course: list scheduling. Over the last 40 years this topic has developed a vast literature, and is a central part of chapter 3 in the popular liberal arts mathematics text *For All Practical Purposes* (published by COMAP). That text, however, only touches on the more interesting scheduling investigations that become accessible to students with the aid of a computer algebra system. Before discussing these investigations, some background is in order.

Background. We assume that there are a finite number of *tasks* to be completed, $T_1, T_2, \ldots, T_n$, and that each will take a finite amount of time $t_1, t_2, \ldots, t_n$. We assume that there are finite number of *processors* to carry out the tasks, numbered 1 through m. A processor works on only one task T_k at a time, and is occupied for precisely t_k time units while working on this task. In addition, some tasks cannot be started until others have been completed (in turning a commercial airliner around, for example, the passengers must be unloaded before the cabin can be cleaned). These dependencies are specified with an *order-requirement digraph*. An edge $T_i \to T_j$ in this digraph indicates that task T_i must be completed before T_j is begun. The goal is to generate a *schedule* (an ordered assignment of tasks to the various processors) that both respects the order dependencies and is optimal in the sense that the total time required to complete all tasks is minimal. This problem is known to be NP-complete.

A common practice for dealing with the scheduling problem is *list-scheduling*. In addition to the information given above, suppose that we create a *priority list*, an ordering of the tasks from most important to least important. Given all this, list scheduling works as follows: At a given time, the highest priority task that is ready is assigned to the lowest numbered free processor. *Critical path scheduling* is a heuristic algorithm which uses paths of maximal length in the order-requirement digraph to generate a priority list which often (but not always) will generate an optimal schedule when it is used in list-scheduling. Various bounds have been established that show (very roughly) that critical path scheduling will return an optimal or nearly optimal schedule almost all the time.

A Module in Critical Path Scheduling. The main topic of investigation is to determine the likelihood that critical path scheduling will produce an optimal schedule. Several innovations combine to make such an investigation possible. First, I have developed a comprehensive *Mathematica* package for generating schedules via list processing, and for using brute force to generate all feasible schedules. It (and its documentation) is available under "List Scheduling Package" at: http://faculty.rmc.edu/btorrenc/mma/packages.html. The package can create graphical renditions of schedules (Gantt diagrams) that match the look of those appearing in the COMAP text. Ultimately, the students work in small groups, and each group is given a unique scheduling problem. This not only eliminates any concerns about blind cheating, but it allows for a compilation of student results that is illuminating. Students apply the list scheduling algorithm to all possible priority lists, eventually finding one that produces an optimal schedule. A variety of counter-intuitive examples can be explored where the student's intuition is challenged — such settings in my experience provide students with great motivation. For instance, a famous example of Ronald L. Graham's shows a scheduling problem where if an entire time unit is subtracted from each individual task time, the optimal schedule for the entire job takes *more* time to complete.

Thus, this talk will demonstrate an approach to introducing a mathematical topic whose relevance is obvious, that is easily accessible to first year university students, and which lends itself naturally to significant investigation with a computer algebra system. I have used these materials successfully for two years, and am happy to provide both the software and/or the written materials for others to consider or to use freely.

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Predicting Pumpkin Weights

A typical contest in autumn is to guess the weight of a pumpkin without a scale. How would one model the weight of a pumpkin? What measurements and variables should be considered? How would the model be implemented in various situations? This presentation will describe an activity that allows students to practice modeling techniques as well as analyze each step of the modeling process. The activity has been used as an individual project primarily done outside of class. However, it could be adapted to be done in class and/or as a group activity.

Erica Johnson (erica.johnson@arapahoe.edu) 5900 S. Santa Fe Drive, Littleton, CO, 80160

Incorporating ILAPs in the Classroom

As part of a grant designed to incorporate engineering applications into math and physical sciences courses, ILAPs (Interdisciplinary Lively Application Projects) were developed and implemented in math courses at ACC, RRCC, and CU-Denver. We will discuss the use of ILAPs in the classroom as (1) an introduction of applications of mathematics to students, (2) part of the math department assessment program, and (3) reinforcement of mathematical concepts learned in math courses. We will discuss the problems, successes, and strategies we have faced and developed during the process.

Yaping Liu (yliu@pittstate.edu) Department of Mathematics, Yates Hall, Pittsburg, KS, 66762

Mathematical Modeling of a Tennis Match

Sports events provide a rich source for mathematical modeling problems. The modeling of a singles tennis match exposes students to a variety of basic ideas and methods in mathematics, including directed graphs, Pascal triangle, Markov chains, random walks, and diagonalization of a matrix. While the project requires minimal mathematical background, students will have to acquire the basic knowledge in linear algebra and probability theory during the modeling process. Upon finishing this project, students will be able to employ these techniques to investigate related problems, such as the gambler's ruin and the Hardy-Weinberg Law in population genetics.

Charlotte Knotts-Zides (knottszideca@wofford.edu) 429 North Church St., Spartanburg, SC, 29303

Determining the Best Seat in a Movie Theater

This talk discusses the results of a math modeling problem which I assigned my students, namely the problem of determining the optimal choice of seats in a movie theater. Initially, the students attempted to determine the choice of seat in the theater which maximizes the viewing angle of the screen, taking into account the slope of the theater floor, the average height of a viewer's eyes from the floor, and the height of the screen on the wall. However, the students quickly realized that, for most theaters, this choice of seats results in the viewer sitting near the front of the theater, with his or her neck bent backwards in an uncomfortable angle. The students devised a means to determine what they considered to be a range of angles which would be comfortable to the viewer. Incorporating this new value into their problem, they were able to predict a choice of seat in the theater which gave them the largest feasible viewing angle while still requiring that their necks only be bent at a comfortable angle backwards. (Portions of this project were based on ideas from UMAP Module 729.)

Theory and Applications of Graph Theory

James Benedict (jbenedic@aug.edu) Augusta State University, 2500 Walton Way, Augusta, GA, 30904-2200

Discovery Method Undergraduate Graph Theory

Mathematicians meet once a year in Austin, Texas to discuss the methods of discovery learning, especially as espoused by R. L. Moore. Attendees learn how to conduct discovery method classes, examining the best techniques used by the more experienced conference professors who employ the method.

The intention of this talk is to encourage professors to present research oriented discovery method Graph Theory courses to undergraduates. This talk will:

Detail the half-dozen or so management techniques experienced professors use to run a discovery method course;

Introduce the freeware text *Introduction to Graph Theory: A Discovery Course for Undergraduates*;

Present case-histories from a course in which the text was the only book allowed.

These examples tend to show that, for undergraduates, Graph Theory may be an area of mathematics that is especially appropriate for the usage of the discovery method.

Chris Wu (ccw3@psu.edu) Math Department, Penn State U, Beaver Campus, Monaca, PA, 15061

On the Number of Self-Avoiding Walks on Hyberbolic

Imagine that you are standing at an intersection in a city where the street system is like a square grid. You choose a street at random and begin walking away. At each intersection you reach, you choose either to continue straight ahead or to turn left or right. There is only one rule: you must not return to any intersection which you have visited. In other words, your path should be self-avoiding. One of the fundamental questions is: if you walk n blocks, how many possible paths could you have followed?

What we have described above is the self-avoiding walk on the square lattice. In this talk we will first give an intuitive definition on hyperbolic graphs and then introduce self-avoiding walks on these graphs.

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A Complement Criterion for Cyclicity

The *cyclidity* of a graph is the largest integer n for which the graph can be contracted to the cycle on n vertices. Although the problem of computing the cyclicity of an arbitrary graph is known to be NP-complete, I describe an algorithm that solves this problem in polynomial time provided the graph's complement has radius unequal to 2.

Pallavi Jayawant (jayawant@math.arizona.edu) Department of Mathematics, Bates College, Lewiston, ME, 04240

Graphs and Orthogonal Polynomials

Certain kinds of graphs have been associated with some of the classical orthogonal polynomials such as Hermite, Laguerre, Charlier, etc. This has lead to new identities and generating functions involving these polynomials. In this talk, I will discuss the graphs associated with the Hermite polynomials and the Charlier polynomials. I will show the use of these graphs in the proof of the Mehler formula for Hermite polynomials. I will end with new generating functions for Hermite polynomials and Charlier polynomials.

Benjamin Woodruff (bmw@math.byu.edu) 772 Wymount 15A, Provo, UT, 84604

Random Pseudo-Manifolds and Regular Multi-graphs

Triangulable manifolds can be realized as the quotient of simplexes by a face pairing. However, in dimensions larger than 2, an arbitrary face pairing does not result in a manifold, but rather a pseudo-manifold. If the number k of n-simplexes is large, on average how many connected components does the resulting pseudo-manifold contain? This calculation is equivalent to the following graph theory question. For fixed n and k, what is the expected number of components in an n-regular multi-graph containing k vertices? I will show that as k tends to ∞, this expected number approaches 1 when $n \geq 3$. The same result is obtained after replacing the regularity condition with the condition that the degree of each vertex is at least 3. Essentially this says that a random multi-graph will almost always be connected as long as every vertex has at least degree 3 and the number of vertices is large. Further applications to triangulations of manifolds are explored, including open problems in this area.

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Cartesian Products of $K3$ as Unit-Distance Graphs

The Cartesian product of n triangles is a unit distance graph of diameter n. It is difficult to produce a drawing of such a graph in the plane such that adjacent vertices are unit distance apart. The difficulty arises because the number of vertices increases exponentially while the diameter increases linearly in n. Moreover, the graph realized as a mechanism has $n-1$ degrees of freedom. We analyzed the motion of these graphs and examined their graph theoretic and combinatorial properties.

Nicholas Horne (nickhorne@cox.net) 606 Mowry Street, Harrisville, RI, 02830

Analysis of Viable Network Configurations

The purpose of this research is to explore properties of the sequence Q_n generated by the number of viable permutations of active pathways between n computers on a network, given that each computer can sustain a maximum of two possible pathways. The first six terms of the sequence are systematically generated before these results are generalized to a known sequence. Connections to graph theory, the combinatorial nature of the sequence and a brief discussion of the parallels between this sequence and adjacency matrices are discussed. The discussion then leads into the construction of a triangular array where the sum of the nth row is Q_n and the elements $q_{n,k}$ in each row correspond to the number of viable network permutations that sustain k active pathways, where $0 \leq k \leq n$. Properties of this triangle are explored, with importance placed on systematically generating formulae for successive diagonals of the triangle, which contain all networks that sustain k active pathways.

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Knot colorings using knot graphs

At the 2003 joint winter meeting, Louis H. Kauffman presented the Kauffman-Harary conjecture for alternating knot colorings, with a proof for the case of rational knots, and a challenge to further the result. Using a non-standard coloring technique for the knot graph, the author has given an alternate proof for rational knots, and a proof for pretzel knots concurrent with the work of Przytycki et al. The goal is a constructive algorithm for all alternating knots.

Getting Students to Explore Concepts Through Writing in Mathematics

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Integration: Exploration and Writing

This is a laboratory, experimentation, and writing project designed to introduce and reinforce the concept of integration. The technology used in this project is a sonic distance probe, a Texas

Instruments CBL, and a TI-83 calculator equipped with a data collection program suitable to the sonic probe and CBL. The student rolls a ball down a wooden ramp, using the probe-CBL-Calculator combination to collect data on the position and velocity of the ball as functions of time.

The central question asked in the project is: How can we construct the position graph from the velocity graph? The students are guided in seeing that this would be easy to do if the velocity were constant—an example of constant velocity motion is given to them, and they work with it sufficiently to remind themselves of how constant velocity problems work.

The objective then is to see how to use these same concepts to deal with motion the velocity of which is not constant. They have data for the velocity of their ball as a function of time; it will be a linear function, more or less, and the experiments I and my students have done successfully produce graphs that are convincingly linear. A TI-83 calculator can fit a linear equation to such data. Using Maple, I instruct my students to print out a copy of this graph; one may also use the Texas Instruments Graphlink.

At this point, the student is introduced to the idea of breaking the time interval into sections and approximating the distance travelled by using the velocity of the ball at the beginning and the end of each section. Thus we introduce left hand sums and right hand sums. The student is guiding in thinking about geometrical interpretations of these sums, and I found that with a few hints, they think of the area of rectangles. They are instructed to draw in these rectangles. Thus they are guided to see that distance travelled can be interpreted as area under a velocity curve. Since the velocity curve, in this case, is very simple, and in fact just a straight line, the student is asked to find the area under the curve geometrically, as a function of time. All this amounts to is using the formula for the area of a trapezoid; however, I find it is tremendously convincing for the students to see for themselves that the graph of the function thus produced does in fact match the original graph of the data of distance as a function of time.

I have students do this project in pairs. They are expected to produce a three to four page paper in which they explain carefully the problem they are working to solve, the methods that they used, and their results. I employed this project this past January to introduce the concept of integration to my Calculus II students and I found it quite successful; I will have several examples of work produced by students to present.

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Mathematics Journals: Writing to Learn

Studies show that writing can be a powerful way to learn. Journals create opportunities for students to write about mathematics. In doing so, they clarify their own conceptual understanding, learn to identify important ideas, and communicate using appropriate vocabulary and notation. I will share some of my calculus students' journal writings, including their final reflections on the merits of keeping a journal, and describe the instructions they receive about the process.

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Problem Zero

Students have trouble reading mathematics, and worse, they often refuse to. When working from a textbook, many students will attempt the exercises before reading the section, and then only refer to the reading to look up examples that mimic the homework problems they are working on. "Problem Zero" is a simple way to encourage students to read the material and organize it into information that makes sense to them. A tiny idea, but one that works, and is easy to grade!

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Reality Calculus: Critical Thinking and Organized Writing

Writing assignments, when combined with traditional problem sets, help students develop important academic and professional skills. In particular, we will discuss engaging elementary calculus students with exercises posed through context-rich business letters. While helping to develop their mathematical abilities, the writing assignments also actively interest students, who appreciate the practical nature of the problems and enjoy the creative aspect of the solutions.

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Writing Assignments Using Mathematical Quotations

How does one engage the typical student found in a Liberal Arts mathematics course? Last fall I decided to develop a sequence of short writing assignments based on mathematical quotations found on Furman University's Mathematical Quotation Server (math.furman.edu/∼mwoodard/mquot.html). As a final writing assignment, the students were required to write a scene from a play which involved the mathematicians whose quotes we had discussed. My talk will present the assignments and some improvements that could be made. I will also share student reactions and some examples of the scenes created by the students.

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Using Writing to Get Students to Read Mathematics

For years it has been apparent that many students in required general mathematics courses either choose not to or simply cannot read the textbook. The ability to read and understand mathematics text should be a primary goal of general mathematics courses. I have assigned writing assignments in my classes in which the students are responding to a reading assignments. The results are mixed. For the past two summers I have taught a class for pre-service high school teachers. One of the assignments has been to write a review of an algebra textbook. I would like to share with you the results from this assignment. Though the members of the class make some valid criticisms, the view of most of the class is that mathematics textbooks are simply impossible to read. It is my intention to do research on why this attitude is so prevalent and how to overcome this attitude.

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Integrable Writings in Calculus

The idea of integrating writing in a mathematics course was very sudden. Most of the non-science students miss the heart of calculus, though some do pick up a few rules of computation. I used a psychological approach; if you love or hate some thing, then you want some one to know about it. In the present mathematical culture, the epsilon-delta definition of limit being marginalized, drilling a distinction between the value and limit of a function becomes challenging. Being an avid letter writer myself, at the end of three weeks, I assigned a letter to your dear one about whatever has been grasped about the Limit. The papers were very revealing since students tried to put their entire understanding together. That encouraged me to assign letters on Differentiation after 8 weeks, and Integration after 15 weeks. More thoughts go into a black and white narrative writing, and hence a better and lasting understanding of a topic ensues. The ideas of the paper can be incorporated in other math courses.

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Reading and Writing in Mathematics? The Math Book

Efforts have been made recently to incorporate writing in the mathematics classroom. Coupled with this challenge is another one common within mathematics departments: how to be effective instructors of liberal arts mathematics courses. This session will describe an assignment that addresses both of these challenges. The objective of the assignment, called the Math Book Club, is to expose students to books about mathematics with the hope that they would come to appreciate some of the significant events that have taken place in the history of mathematics. For the Math

Book Club, students in groups of four read and discuss one of six books related to mathematics intended for the general audience. Students take turns developing discussion questions and summarizing each meeting's main themes. This assignment not only exposes students to books that they do not typically peruse at the bookstore, but also encourages them to reflect upon and share their thoughts about major ideas that have shaped the history of mathematics.

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Research Projects as a Means of Student Engagement

The culmination of my liberal arts general education mathematics course at Drake University is a research paper. This writing project typically generates more engagement than any other assignment in the course since the students get to choose the topic themselves.

Most students in this class do not have a positive view of mathematics. As a means to help them see how mathematics relates to their lives, students are strongly encouraged to base their paper on a personal interest. For example, art students might examine the role of mathematics in painting, sports fans might write about statistics and educators might create lesson plans.

This presentation will briefly describe the course, but the focus will be on the writing project. I will discuss the preparation process for the project, which is modeled on what's typically done in a writing course and is designed to help students sharpen their writing, as well as ensure sufficient mathematical content.

I will also present examples of student projects—both successful and otherwise—as well as discuss student reactions.

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The Write Stuff: Using Labs in Precalculus

Precalculus need not be a spectator sport when you include labs as an integral part of the course. Students, working in groups on real-world problems, become engaged in talking and writing about mathematics. In the process of negotiating agreements about how to interpret results, of constructing meaning from the work of the laboratory, and of organizing their conclusions for presentation in a written report, group members develop for themselves a solid understanding of the fundamental mathematical ideas. Sample student work and ideas on grading labs will be included as part of the presentation.

Lisa Rome (lisa_rome@mail.msj.edu) Department of Mathematics and Computer Science, 5701 Delhi Road, Cincinnati, OH, 45233-1670

Writing in a Sophomore-Level Proofs Class

This talk will discuss writing assignments used in teaching Foundations of Advanced Mathematics (our "proofs" course), in Fall 2004. Along with the mathematics textbook, selected chapters in a book on scientific writing were used. The writing emphasis was on providing good transitions between sections, providing the appropriate level of detail, being precise, and being clear. The goal was to have the students pay attention to high quality scientific writing as they wrote about mathematical concepts and situations. Students were given three writing assignments, each one asking them to focus on a specific mathematical topic as well as on certain qualities of scientific writing. Assignments were designed with the hope that they would be varied and interesting enough to force the students to learn some new mathematics, without being so challenging as to prohibit students from focusing on the quality of their writing. Two of the assignments successfully achieved this. One of the most successful assignments even involved the freshmen Calculus I class.

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What is the MEANING?

At the University of Michigan, we have been exploring students' understanding of concepts by asking questions on final exams that involve explanations to "a friend in high school" or similar scenarios. The answers to these questions (on uniform exams) have been compared across sections, sometimes across courses (i.e., honors vs. regular, Calculus I vs. Calculus II). In this talk we will look at some of the rubrics for evaluating these questions, some typical answers, and what we hope to gain from this type of evaluation.

Susan Wildstrom (susan@wildstrom.com) 7100 Whittier Blvd., Bethesda, MD, 20817

Reading and Journals and Websites, Oh My!

Students in pre-college math courses often think of mathematics as something to "get through." Even the best students don't often think of mathematics as something that can be enjoyable and engaging. The presentation will acquaint participants with this teachers use of a website and email journals as a means of direct one-to-one student/teacher communications. Also an innovative reading-writing-technology assignment will be described and student reactions shared.

For a number of years now, the website has provided both general and specific information for students. Each course has its own pages which include general information about the course as well as up-to-date assignment sheets, worksheets, and handouts. Students (high school) and their parents are able to visit the site and see what is scheduled as well as being able to print out copies of materials. The materials on each course page are left to accrete throughout the year giving a clear picture of the flow of the course.

Email journals are required at approximately two-week intervals. Students may discuss anything mathematical or pedagogical in them and this encourages an individual dialogue between student and teacher which frequently results in a more personalized and productive educational experience for the students.

A reading and technology assignment is a third component of this plan designed to broaden the mathematical experience. Students read an excerpt (although many end up reading the whole work) of about 20 pages from a book with a mathematical theme about which they then write a short exposition. The technology component of the assignment involves locating a website that is pertinent to something being studied in their course at the time. Students do this assignment once each quarter.

Informal assessment of these resources suggests that students come away from the courses with more enthusiasm for mathematics, a broader understanding that mathematics encompasses much more than "calculus," and a willingness (even an interest) in continuing to study mathematics. Email from around the world suggests that teachers in other communities have discovered these materials and found them useful as well.

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Dora Ahmadi (d.ahmadi@moreheadstate.edu) Dept of Mathematics and Computer Science, Morehead State University, Morehead, KY, 40351

Enhancing Mathematics Learning Through Writing

One of the topics in this paper involves journal assignments in a general mathematics course. The course is composed mostly of students who are not mathematics majors, and their purpose for enrolling in the course is usually to satisfy the core mathematics component for general education requirements. The journal entries sometimes provide an opportunity for a student's misconception about the class material to be corrected before an exam. The journal entries also give insight into student attitudes toward the subjects covered in class and toward mathematics in general.

This paper will also describe a course experience pairing statistics and English. This pairing provided an interdisciplinary context for addressing two major challenges facing today's educators: innumeracy and illiteracy. Students registered in the English course were registered in the mathematics course but not all students in the mathematics course were taking the English course. This situation lent itself to performance comparisons of the statistics students taking English versus those not taking the English course. This paper will give comparison results on test performance, course performance, and conceptual understanding of selected statistical concepts.

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Creating and Using Effective Writing Prompts

In order to use a significant number of short writing assignments in the calculus sequence there are several issues to be resolved: there has to be a ready supply of writing prompts, the students need to clearly understand the objectives and expectations, and the grading of the writing must be effective and efficient. I think I have made significant progress toward the resolution of these issues.

In Calculus I, II and III during the 2003–2004 academic year, I required the students to respond to writing prompts once or twice a week. The students were given 4 to 5 writing prompts from which to choose, and the responses to these prompts were hand-written ranging from 3/4 to 2 pages in length. The prompts directly addressed the concepts of the course and gave the students sufficient opportunity to integrate numerical and graphical representations of mathematics into the writing.

In order to have a comprehensive list of questions, during sabbatical leave in the spring of 2003 I gathered and wrote 700 writing prompts that span the entire calculus sequence. These prompts are available on-line and the list continues to grow (http://www.simpson.edu/∼waggoner/). I would like to help others begin to develop writing prompts that fit well with their own teaching styles by explaining how I develop new questions.

The second issue was to help the students understand, from the very beginning of the semester, the expectations of the assignments. I used several activities that combined the learning of calculus with learning to write mathematics to help the students understand the objectives and assessment of the writing.

Finally, I needed to be able to read many writing responses in a short time and still give the students effective feedback, and to do so I needed a rubric that adequately assessed the expectations of the assignments and the required mathematical rigor. In cooperation with a writing instructor, I created a rubric that has satisfied my needs.

Strategies for Teaching Multiple Audiences in One Class

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The Challenges of Chance: Mathematics and Writing

Incoming students at Albion College are required to complete an interdisciplinary writing-intensive First-Year Seminar (FYS). The department of mathematics and computer science has for several years offered a FYS called "Chance", derived from the course originally developed at Dartmouth. As the college allows only minimal prerequisites for a FYS, Chance students arrive with a wide range of backgrounds in both mathematics and writing—although one common quality they bring to the course is a lack of experience with writing in mathematics classes. This talk will describe a varied collection of writing assignments that are used in Chance to reach all students, regardless of their writing proficiency.

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The Interactive Classroom: A Dynamic Environment

The wide range of student math backgrounds encountered in the developmental math classroom demand innovative approaches in both presentation and assessment. Recent advances in educational software have made it possible to transform the traditional lecture class into a dynamic learning environment. This talk will focus on the potential of the Interactive Classroom as a multidimensional learning experience. Students have access to short presentations from the instructor, video presentations associated with the textbook, interactive exercises, small group activities, etc. The effectiveness of this system depends on accurate assessment tools which identify student strengths and weaknesses. Reinforcement through continuous assessment is now a reasonable option using software grading capabilities. Two software packages will be discussed: MyMathLab and ALEKS.

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Alternate Instructional Methods for Introduction to Linear Algebra

Introduction to Linear Algebra (MATH171) course at IUP serves students with diverse majors, such as mathematics, physics, chemistry, mathematics education, undeclared-science, economics, and pre-engineering. Moreover, at IUP, MATH 171 has no prerequisites. Since spring 2003 the author has taught this course for 3 semesters. The strategies which were used in the last three semesters were 1) exploration of new concepts through interactive modules; 2) using computer projects to connect students' prior experience and applications; 3) teaching the writing of mathematics through opportunities of revising certain problems in homework. This paper will present course material related to those strategies and compare effectiveness between semesters through students' semester grades.

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Allegorizing Algebra: Reaching a Diverse Classroom

County colleges often find themselves between a rock and a hard place when it comes to first year math courses. The course I have in mind is the first course for credit in most traditional math sequences, College Algebra. For a host of reasons, such classes typically contain students ranging in ability and degrees of math-fitness from college students who have just completed the last of the non-credit math courses – those specifically developed to prepare students for College Algebra – to students who have not taken a math class in a number of years and who happen, by hook or by crook, to squeak by the institutions placement test. One strategy that I have employed with some measure of success has been the allegorization of mathematics. In other words, when dealing with diverse student populations taking College Algebra, I aim to present multiple metaphors for algebraic concepts in an effort to connect with whatever cultural or interpersonal experiences a student may be bringing to the classroom. I find these to be especially fruitful in small tutoring sessions. My presentation will focus only upon a handful of algebraic allegories that I have communicated with apparent success. For example, when presenting one girl from Ghana with the simplification of rational exponents, I explained to her that she should not think of the denominator of the exponent as part of a fraction but rather as the number of little boys who were at the market that day asking her for fruit. The numerator, too, should not be thought of as part of a fraction but rather as the number of fruit that she had that day to distribute to the little boys. I would then ask her, given a certain rational exponent, how many fruit could she give each boy so that each one got the same amount of fruit (if she did not give each boy the same amount, they would start fighting)? She naturally divided her number of fruit by the number of little boys at the market. I instructed her to write the variable with that quotient (I didn't call it the quotient) on top of the variable outside the market roof (the radical sign) and then write how many she would have left over inside the market roof. It is difficult to convey accurately the measure of amazement she expressed to me at the facility with which she was soon manipulating rational exponents. During the presentation, I will also mention a handful of other allegories that have driven home otherwise jumbled algebraic mishmash to students. I will also provide suggestions for multiple allegories that refer to the same algebraic concepts in the case that a single allegory fails to resonate with particular students. I will close with the suggestion that automated multiple choice tests as presently administered at many community colleges may not accurately assess students' level of comprehension if students are indeed relying upon such allegories for their apprehension of algebraic concepts. These tests, perhaps, should be supplemented by algorithmic exams that require that a student describe their algorithm for simplification of rational exponents, for example, allegorically.

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Keeping a Small Math Program Going

With four or fewer graduating math majors per year, we at St. Joe's have been faced with many challenges because of our inability to run post-calculus level courses every year. In particular we are unable to teach either a "bridge" course for younger students or a "senior seminar" course in

which seniors write their thesis. Our attempted solution to these problems is a math seminar course taken every semester by every math major regardless of class level. This course has individualized work tailored to the students' experience level. I will discuss what we have learned in four years of running the course.

Joan DeBello (debelloj@sjtohns.edu) 8000 Utopia Parkway, Jamaica, NY, 11439

Coping With Multiple Audiences in A Beginning Level College Algebra Course

College Algebra is usually a prerequisite for almost any other College Mathematics course, and is usually a core requirement for almost any degree offered at a university or college. By teaching a College Algebra course with multiple audiences involving many different types of backgrounds and knowledge of mathematics content, I have been able to help those who needed extra help and challenge those who had a stronger knowledge and grasp of the material. I will discuss some of the methods I have used to teach this course and the experience from it.

Marc Laforest (marc.laforest@polymtl.ca) cole Polytechnique, C.P. 6079, succ. Centre-ville, Montral, QC, H3C 3A7

Uneven Programming Skills in Numerical Analysis

Numerical Analysis is traditionally taught to mathematicians, computer scientists, and engineers and a good understanding of the topic requires a synthesis of mathematical and programming skills. Despite the fact that a programming course is usually a prerequisite for this course, programming is often a significant hurdle for half the students while for the other half mathematics represents an unwelcome level of abstraction. Moreover for the instructor, teaching programming can be a time consuming distraction from teaching numerical analysis. We present a series of exercises, to be done in a computer laboratory, that teach basic programming skills using algorithms from numerical analysis. The progression of exercises parallels the topics usually seen in a numerical analysis course. We discuss how levelling the playing field in programming influences the learning process for the different groups in the class. We also suggest strategies to keep the different groups challenged and rewarded.

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Projects: Helping Students Customize Their Education

The research project can be used in several mathematics classes in a variety of ways. Because of its adaptability, the research project provides one technique to help different groups of students design their own unique educational experience within a standardized curriculum.

For instance, students taking Calculus could complete a research assignment that highlights how Calculus is used in their major. Biology students might study how Calculus can help them find solutions to certain population models, whereas mathematics majors might come up with a proof of an important calculus theorem.

Linear Algebra also offers many project topics. Engineering students might research a specific application to solving differential equations, while computer science majors might study various discrete applications.

I will describe ways in which my Calculus II and Linear Algebra students have used their research projects to help tailor their own educational experience. Sample assignments and copies of student work will be shown.

General Contributed Paper Session

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Timothy Povich (Timothy.Povich@usma.edu) MADN-MATH, Attn: Tim Povich, 646 Swift Road, West Point, NY, 10996-1905
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Airfoil Design: Tying it All Together

Near the conclusion of their final term in the calculus sequence at The United States Military Academy cadets are given a week to complete a group project. At the end of the week, the project is briefed to their instructors, classmates and superior officers. From a teaching perspective, the goal is to encapsulate as much of the course as possible in one coherent student endeavor. We (the authors) developed a project that asked cadets to redesign the shape of a wing to improve performance, yet maintain costs. What we learned while helping cadets complete the project, during their briefings and from a post-project survey provided insights to help improve the project and the entire course. Cadet performance clearly revealed what course topics need more time and which were learned thoroughly. For the first time, some students were forced to integrate the calculus concepts and technology rather than rely only on the technology. This presentation will give an overview of the project and its administration, how the project was seen from both the instructor and student viewpoints and provide lessons we learned to improve the project and integration of technology into calculus courses.

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Enumerating Thompsons Group F

In May 2003, Professor Jim Cannon, three undergraduates, and I began a project to count the number of elements of Thompson's group F that are represented by a reduced pair of n-caret trees. The formulas we found required the solution of a number of intermediate counting problems. The tools we used to solve these problems were generating functions and the inclusion-exclusion principle. The Catalan numbers (in particular their difference triangle), and the Riordan numbers play a central role in our study. We were able to prove a main result only after finding an Euler transformation, which compares generating functions of a sequence and the diagonal of the difference triangle of the sequence. I will begin with a description of Thompson's group, present some of our results, and then show various ways that this Euler transformation might be used in future research. This paper is accessible to an undergraduate who has had at least one semester of abstract algebra.

Haohao Wang (hwang@semo.edu) Math Department, MS6700, Cape Girardeau, MO, 63701

A Little Beyond Calculus and Abstract Algebra

In order to offer an opportunity to students and promote their interest in advanced mathematics, this talk introduces the concepts of cohomology and homology to students who have a little background on calculus and abstract algebra.

Rob Harger (rharger@highpoint.edu) 5809 Beckenham Way, Oak Ridge, NC, 27310

A Look at the Ratio of x to $\pi(x)$

In this talk we will consider an interesting property of the ratio of x to $\pi(x)$.

Ilhan Izmirli (ixi@strayer.edu) 4400 Massachusetts Avenue, NW, Washington, DC, 20016-8068

Generalization of Pascal and Fibonacci Triangles

In this paper I will develop an algorithm based on some simple recursion formulas involving coefficients of some rational functions, which, as a special case, will yield Pascal and Fibonacci triangles.

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Linda Becerra (BecerraL@uhd.edu) University of Houston-Downtown, One Main Street, Houston, TX, 77002

The Shifting Axiomatic Foundations of Mathematics

This talk will demonstrate how the evolution of mathematics has periodically forced re-evaluations of some of its basic underlying assumptions.

The evolution of the idea of mathematical certainty — that in any axiomatic system the truth or falsity of every statement is decidable within the system — touches on many interesting and intriguing incidents in the development of the discipline.

This presentation will trace how the belief in the efficacy of reason in explaining everything from Nature to our present day society has developed from the times of the Greeks to our present era. Along the way, we make excursions into non-Euclidean geometries, the problems of consistency and completeness and comments on Riemann, Hilbert and Godel's contributions.

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The Holor Foundation of Vector Calculus

Holors have been introduced (P. Moon and D.E. Spencer, Theory of Holors, Cambridge University Press, 1986) as a necessary generalization of tensors because not all quantities needed to describe physical concepts can be expressed in terms of tensors. The paper shows how the roots of many of the concepts of vector analysis are already present in arithmetic space. Vector analysis can be formulated in Riemannian space by introducing the metric tensor. The vector analysis of Euclidean space follows when the metric is Euclidean. This formulation is valid in oblique coordinates and in all curvilinear coordinate systems. It gives results that differ from the erroneous del operator formulation in all curvilinear coordinate systems.

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A Problem of Riemann

Let $\sigma(x)$ be defined as

$$\sigma(x) = \sum_{1}^{\infty} \frac{\sin(\pi n^2 x)}{n^2}.$$

$\sigma(x)$ is originally attributed to Riemann who thought it was nowhere differentiable. In 1970, J. Gerver, using tedious elementary methods, showed that $\sigma(x)$ is differentiable at x_0 if and only if

$$x_0 = \frac{2p+1}{2q+1},$$

where p and q are natural numbers. The proof of Gerver is the point of departure for a number of works, among which the most remarkable is the use of analysis in wavelets by Holschneider and Tchamichian. Here we give a novel proof of the differentiability of $\sigma(x)$ at $x_0 = 1$, using complex analysis techniques.

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An ESPeriment With Cards

You casually shuffle a packet of cards overhand while talking about an ESP experiment you are about to perform. Then you deal cards into a pile on the table until somebody calls stop, whereupon you riffle shuffle that pile into the remaining cards. The packet is fanned to demonstrate how mixed up it is, and the audience is given a choice of using either the top five or the bottom five cards. These five cards are placed in a face-down row from left to right.

"It would be an absolute miracle if anybody outside this room could identify all five of those cards correctly," you proclaim. "Let's try something a little less ambitious, though still impressive in its own way." Picking up two of the cards, you replace them in the packet and shuffle further.

You leave the packet face-down on the table, having first closed up any remaining gaps between the three face-down cards—without disturbing their order—and then turn away.

A person who claims to have special ESP talents now enters the room for the first time, having seen nothing so far, and surveys the scene. Acting on her never-failing ESP instincts, she concentrates hard, and quickly identifies the three face-down cards correctly, in order.

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Benjamin Holt (bvh6@humboldt.edu) Humboldt State University, Department of Mathematics, Arcata, CA, 95521

Group Actions in Number Theory

Students having had a semester course in abstract algebra are exposed to the elegant way in which finite group theory leads to proofs of familiar facts in elementary number theory. In this note we offer two such examples of group theoretical proofs suitable for presentation in a second semester algebra course. The first is Fermat's Little Theorem and the second concerns a well known identity involving the famous Euler phi function. Our methods use the class equation of a group action and Burnside's theorem, both standard topics in a second semester algebra course. These two notions are somewhat sophisticated, and the results we derive from them are, comparatively, small. Nevertheless, both examples provide attractive classroom examples and exercises that naturally continue a classic theme.

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Outreach — Planting the Seeds for the Future

Women and disadvantaged minorities lag behind in the technology sector. The migration away from the science, technology, engineering and mathematics (STEM) fields starts in middle school and continues until the undergraduate years. We risk our nations leadership role in the high technology society we have developed if the declining college enrollments in STEM subjects remain unchecked.

To arrest this trend and inculcate interest in STEM fields, University of Houston-Downtown offers two mathematics based enrichment programs for pre-college students, Houston PREP and Saturday Academy, starting as early as 7th grade.

Houston PREP is an academically intense, seven week mathematics related summer program, which stresses the development of abstract reasoning, problem solving skills, and their application. To maintain a current perspective on emerging areas in computational science, projects such as 'Introduction to Neural Networks, System Dynamics, Modeling using STELA' are added.

Houston PREP is producing results. Since 1989, more than 1,200 students have completed at least one summer of PREP, and 99.9% of these students have graduated from high school. Approximately 89% go to college. Roughly 55% of the PREP graduates who are still in college are majoring in STEM subjects.

The Saturday Academy also introduces advanced topics to students. Since this program is conducted during the school year, advanced topics are tied to junior and high school coursework. Unlike the PREP program, the target is not only the student, but the parents as well. To help build the necessary support structure for our students, we have extended the remedial computer science and mathematics courses to the parents. Parents gain a clearer understanding of the students' coursework, and are able to support their child's pursuit of STEM careers.

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Julie Jones (julie@shsu.edu) Dept. of Math. and Stat., Box 2206, Huntsville, TX, 77341-2206

Starting an Undergraduate Research Program

This talk will discuss the resurrection of undergraduate research at Sam Houston State University. We have effectively used topics in the history of mathematics to introduce our students in lower level courses to mathematical concepts that they might not have access to until substantially later

in their careers, and have used interest in particular mathematicians to springboard undergraduate mathematics research. The enthusiasm of our students has been infectious, and has produced a community of mathematics students on our campus. This talk will describe the trials, tribulations, and joy of beginning this sort of program at our institution.

Kari Lock (Kari.F.Lock@williams.edu) 2 Haley Dr., Canton, NY, 13617

Identifying Best Rational Approximations

Using the theory of continued fractions, we produce a new sharp Diophantine inequality involving an irrational number and a rational approximation to that number such that the only solutions are precisely all the "best rational approximates" to the given irrational number; that is, the complete list of its convergents. This work generalizes and extends previously known results appearing in the literature. This work is part of a Senior Honors Thesis.

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Partnership to Increase STEM Enrollment and Student Success

Funded by the National Science Foundation, Oakton Community College is implementing its project, Partnership to Increase STEM Enrollment and Student Success. In this presentation, the principal investigators will share with the audience the activities of the project and what works, what does't work, and what the PIs learn.

Randy Maddox (randall.maddox@pepperdine.edu) 24255 Pacific Coast Hwy, Malibu, CA, 90263

Placing Points in a Polygon

Lucács and András posed in the *Monthly*: "Prove that there exists a set S of $n-2$ points in the interior of a convex n-gon such that for any three vertices of the n-gon, the interior of the triangle determined by the three vertices contains exactly one element of S."

A solution by Cahill was published in the June-July 2003 edition of the *Monthly*. We investigate and characterize solutions to this question, then present some generalizations and open questions. First we exploit the power set of $\{1, 2, \ldots, n-4\}$ to calculate the number of solutions with all points lying in the perimeter regions of the polygon, where the topology of the triangles is invariant under distortions of the n-gon. Then we characterize all solutions in terms of a construction from the (topologically) unique solution for $n = 5$.

Secondly, we generalize the question to the following: Given a convex n-gon and integer k, determine all assignments of integers whose sum is $k(n-2)$ to the regions in the interior of the n-gon such that for any three vertices of the n-gon, the sum of the integers in the interior of the triangle is k. For $k \geq 1$, we are particularly interested in solutions where all assignments are non-negative. Solutions where the non-zero assignments lie solely in the perimeter regions are precisely sums of the $k = 1$ solutions described above, and decompositions into these sums are not unique. If non-zero integer assignments are allowed in the more interior regions of the polygon, there exist solutions that are not sums of solutions for $k = 1$.

Thirdly, we allow negative integer assignments to the regions. Using Euler's formula for planar graphs, duals, 2-degenerate graphs, and other basic techniques from graph theory and combinatorics, we can determine the number of (linearly independent) solutions and construct a convenient basis using only $\{0, \pm 1\}$.

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Exploring Number Theory With a CAS

The Computer Algebra Systems MATHEMATICA 5.0 and the TI-89 aid in furnishing dynamic investigations in the theory of numbers, especially the prime numbers. Explore such famous open problems as Goldbach's Conjecture, Twin Primes, Sophie Germain Primes, Home Primes, Prime

Decades, and The Collatz Problem. A short program accessible from the TI-89 manual entitled Next Prime enables one to secure the next prime after a given integer. Examples of arbitrarily long sequences of composite length, so-called prime generating formulas as well as primes in arithmetic progression and consecutive primes in arithmetic progression will be discussed. Handouts will be provided and demonstrate results utilizing the more powerful package MATHEMATICA 5.0. Please join us to witness the interface of mathematics and technology leading to potentially stimulating research.

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Using Maple to Help Teach Calculus III

Learning calculus and especially polar coordinates and three dimensional graphs can sometimes be a boring and hard concept for students to graph. I have used Maple to help teach these concepts by including it in my lessons and by assigning homework using the program. I have seen that the students were able to get a better understanding of the content when they used Maple. I will discuss the benefits of using Maple to aid in the teaching of a Calculus III course or any course.

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An Undergraduate Statistical Consulting Laboratory

The University of Redlands has developed an interdisciplinary statistical consulting laboratory experience for undergraduates. The Lab is modeled along the line of graduate consulting laboratories found in most graduate statistics curricula. Students with prerequisites of at least one statistics course and instructor permission, are available to the campus community at large for statistical consulting purposes. Faculty, staff, and administration are free to present problems of academic interest or operational need to the consulting lab. Off campus organizations also are a source for projects done in the lab. Students are assigned or select projects that interest them. We have involved various faculty on campus who deal with projects of a statistical nature, yet may lack any formal training in statistics or mathematics.

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The First and Second Editions of Flatland

Flatland first appeared in November 1884, and when the first printing sold out, a second edition was published one month later. We compare the two editions, with particular attention to the new introduction written by Edwin Abbott in reaction to early reviews, especially the one by Arthur John Butler in *The Athenaeum* of 15 November, as answered in the same journal by A Square on 6 December. We also consider the extent to which the changes from the first to the second edition recast the fundamental questions in the book.

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Observations on Fundamental Mathematics Skills

For the past ten years, the United States Military Academy (USMA) has administered Fundamental Skills Examinations (FSE) to its freshmen mathematics students. FSEs are designed to measure individual capabilities and assist in student placement. They also provide a general indicator of the state of calculus and pre-calculus preparation in America's high schools and a unique viewpoint on the state of fundamental high school mathematical knowledge in the United States. Each year, more than 1100 new students from all fifty states and several foreign countries join the Corps of Cadets.

As incoming students enter the Military Academy, they are given the FSE almost immediately to evaluate their mathematical backgrounds as well as periodically during their freshman year to evaluate their fundamental math skills. The observations gained from the FSE data are especially valuable because the students at the Military Academy represent a true cross-section of the country's students. This presentation will give a short overview of the USMA Mathematics Core Curriculum and discuss the general makeup of the FSE. We will provide observations from these exams and discuss trends from students over the semesters and we will end with a discussion of the future direction of the FSE.

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Cooperative Learning and Technology in Calculus

As part of a scholarship of teaching and learning project, I investigated the question "What is the effect of small groups and a technology-enhanced curriculum on students learning in a calculus course?" Students were placed into groups that met weekly to discuss homework problems. Graphing calculators and Maple were also used regularly throughout the course. The project centered on how these groups and technology affected students' understanding of concepts and problem-solving skills. Results of this two-semester project will be presented, including student feedback. Issues on shifting the focus of the classroom to student learning, rather than teaching, will also be discussed.

Jason Molitierno (molitiernoj@sacredheart.edu) Department of Mathematics, 5151 Park Avenue, Fairfield, CT, 06825

Visualizing Families of Differential Equations

In this talk, I discuss out-of-class group projects that enabled my students to visualize families of differential equations. Three projects in particular will be discussed. The first of which concerns first order differential equations. Students use Maple to plot the slope fields of such equations. They are asked to find solutions using the algebraic methods taught in class and then by using their Maple-generated slope fields. Students are also asked to analyze these slope fields and determine quantitatively and qualitatively how the solutions relate to each other.

The second project concerns higher order linear equations. When an nth-order linear equation with constant coefficients has n initial conditions, there exists a unique solution. However, if there are only $n-1$ initial conditions, then there are a family of solutions. Students are asked to use Maple, graph the families of solutions, and then determine which graph corresponds to the unique solution for a given nth initial condition.

The third project concerns systems of differential equations. Students are asked to use Maple to sketch the phase portrait of various systems and then highlight particular solutions corresponding to various initial conditions. The purpose of these projects is to help students use technology in order to visualize the solutions that they learn how to compute in class.

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A 300-Level Prob/Stat Course Without Exams

We will discuss the perceived benefits and advantages as well as the potential pitfalls and problems of structuring a course without traditional closed-book tests. Our yearlong calculus-based sequence in mathematical probability and statistics is taught every other year to an audience of mathematics majors and minors. In 2003–04 I taught the course for the first time in four years. In an effort to stress the applied aspects of the subject we made two substantial changes from the typical style of upper level mathematics courses. The first was a significant increase in extended homework assignments and take-home testing. These typically involved large data sets, lengthy calculations, extensive numerical experimentation and/or the use of software to explore data sets. The second change was an extensive small group project near the end of each semester. Students worked in groups of two or three. Each group was required to find an appropriate data set, propose tools to be used for their investigation, and secure the approval of the instructor. Rather than schedule a final examination, students presented their project results in a poster session during final exam week.

In the first semester of the sequence, there were two in-class, closed-book tests in addition to the graded homeworks, take-home tests and the poster session final exam. The second semester of the course (with a smaller enrollment) had no in-class tests. Final grades for this course were based on extensive take-home assignments and the poster final exam. Student response to the second term was particularly good. However, this may be idiosyncratic to the particular group of students and the rapport established between them and the instructor in the first semester.

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Palm Pipes and the Math of Music

Palm pipes are short tubes made from PVC pipe. A classroom set is cheap and easy to produce. They provide a practical example of rational functions, and can be used to demonstrate a variety of mathematical relationships. Both the theory and the construction will be discussed.

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A Course Module on the Probabilistic Abacus

We have developed a course module that introduces students to chip firing games and their use as a “probabilistic abacus” for solving probability questions. A *chip firing game* is a game played on a graph. Each vertex is assigned a nonnegative integer *weight* representing the number of chips (counters, markers, poker chips) that have been placed on the vertex. Chips may be added to vertices subject to the *firing rule*: If the number of chips on a vertex exceeds the degree of that vertex, then the vertex “fires,” sending one chip to each neighboring vertex. Each firing of a vertex may cause neighboring vertices to fire in turn, possibly leading to a cascade, or avalanche, of firings. Chip firing games and related models have been defined for finite and infinite graphs, directed and undirected graphs, multigraphs, and even mutating graphs. The chip firing games that we use in our study are played on finite directed graphs that allow loops. We present an example of a probability question that seems to call for a solution involving infinite geometric series. After giving the geometric series solution we compare it to solutions using the probabilistic abacus. In addition to introducing the probabilistic abacus, our module includes a section introducing Markov analysis as a natural extension of the chip firing methods used to analyze probabilistic questions. The module can be used as a two-week session introducing these concepts in a course on probability or discrete mathematics, or as the foundation for an independent study project.

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How to Enhance Learning Through Outreach

The United States Military Academy’s primary mission is to educate cadets to lead tomorrow’s Army. To accomplish this goal, the faculty must maintain a focus on teaching, but also conduct research to maintain relevance to the Army. To that effect, USMA and the Department of Mathematical Sciences encourages outreach to the Army and the Department of Defense. The faculty, which consists of both military and civilian instructors, uses these experiences to develop college senior level projects, enhancing the educational experience of the student. This presentation will highlight recent research projects that allowed the student to apply mathematical and operations research skills to solve real-world problems faced by the United States military. Additionally, we will discuss several exciting research opportunities available to both faculty and students at any U.S. college or university.

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Richard Brazier (rab27@psu.edu) College Place, DuBois, PA, 15801

Partial Fraction Decomposition in One Easy Lesson

To thoroughly describe the usual methods of computing the Partial Fraction Decomposition (PFD) without the aid of software takes too much time out of an already overcrowded Calculus syllabus. We present a method for computing the PFD which is (1) Easily learned, in the sense that there are no special cases and (2) Easily taught in a single fifty minute class.

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Alternative Multi-Restricted Numbers

The multi-restricted numbers of the first kind is obtained by the inverse matrix of the multi-restricted numbers of the second kind. However, the multi-restricted numbers of the first kind and Stirling numbers of the first kind do not share similar combinatorial interpretation, while the multi-restricted numbers of the second kind and Stirling numbers of the second kind do. In this paper, the alternative multi-restricted numbers of the first kind is defined to satisfy the regular sign behavior and relate to the Stirling numbers of the first kind.

Bart Stewart (bart.stewart@usma.edu) Department of Mathematical Sciences, West Point, NY, 10996

Techniques in Voice Recognition/Analysis

While voice recognition/analysis can have many real-world applications, we will examine the how-to of performing voice recognition. Template matching and feature analysis will be our primary methods, with an emphasis on employing Fourier tools and minor statistical analysis to validate the process.

Peter Ross (pross@scu.edu) Santa Clara University, Santa Clara, CA, 95053

Experiential Worksheets for Sequences and Series

I will describe a simple method that I have successfully used in calculus courses for introducing the two difficult, but related, topics of infinite sequences and infinite series. Both topics are conceptual hurdles for many calculus students, in part because standard texts provide a minimal experiential base, emphasizing instead even in their introductory sections exact limits of sequences and closed formulas for convergent infinite series.

Before my classes read about sequences and series I ask them to hand in as part of their homework two separate calculator worksheets, one on sequences and one on simple series. These worksheets require students to tabulate numerical data, and then to make educated guesses based on the data for the limits of the sequences and for the sums to which the series appear to converge. The worksheet examples have been carefully chosen to illustrate the following:

a) key limits for convergent sequences, such as the six Limits That Arise Frequently in Table 8.1 on p. 616 of the Thomas Calculus, Tenth Edition, 2001;

b) prototypes of important classes of series like p-series and alternating series. I will provide copies of both worksheets and discuss their evolution, including why in using good texts like Stewart and Thomas I have found it important to supplement them in introducing sequences and series. I will discuss some advantages of doing this, such as being able to refer in class to the key numerical examples on the worksheets when later generalizing, thus keeping students grounded in reality.

Another, not-so-obvious advantage for students of the worksheets is that using numerical experimentation suggests to them that they can later sometimes test conclusions about more complicated sequences and series; students often do not think of doing this. Use of such experiential worksheets helps students clear up misconceptions and avoid mistakes with the important, but intellectually-demanding, topics of infinite sequences and series. Audience members who like the idea of such worksheets can design their own that are tailored for their situation, perhaps using some of the examples that I will provide.

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$T_{i,j}$ Separation in Topological Spaces

This paper introduces three separation axioms for topological spaces, called $T_{0,1}, T_{0,2}$, and $T_{1,2}$. The $T_{0,1}$ and $T_{0,2}$("pre-Hausdorff") axioms generalize the classical T_1 and T_2 axioms (resp.), and they have advantages over them topologically which we discuss. We establish several different characterizations of $T_{0,2}$ spaces, and a characterization of Hausdorff spaces in terms of $T_{0,2}$ spaces. We also discuss some classical Theorems of general topology which can or cannot be generalized by replacing the T_2 axiom by the $T_{0,2}$ axiom.

Mohammad Salmassi (msalmas@frc.mass.edu) 100 state street, Framingham, MA, 01701

Role of Inequalities in College Geometry

We will look at exterior angle inequality and inscribed angle theorem in neutral geometry. Students do not quite appreciate the importance of inequalities in algebra or calculus. But in geometry, changing an equation to an inequality or changing the direction of an inequality makes a whole lot of difference and puts one from one universe (Euclidean geometry) into other universes (hyperbolic or spherical.). In fact, the whole story of Euclidean versus Non-Euclidean geometry can be looked from viewpoint of inequalities.

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Centers of Ellipses inscribed in Quadrilaterals

Let R be a four–sided convex polygon in the xy plane and let $M1$ and $M2$ be the midpoints of the diagonals of R. It is well-known that if E is an ellipse inscribed in R, then the center of E must lie on Z, the open line segment connecting $M1$ and $M2$. We use a theorem of Marden relating the foci of an ellipse tangent to the lines thru the sides of a triangle and the zeros of a partial fraction expansion to prove the converse: if (h, k) lies on Z, then there is a unique ellipse with center (h, k) inscribed in R. This completely characterizes the locus of centers of ellipses inscribed in R. We also show that there is a unique ellipse of maximal area inscribed in R.

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Pure Mathemetics, A True Liberal Arts Course

The course consists mainly of seminars and a few lectures. We read primary sources in geometry and arithmetic, Aristotle, Euclid, Galileo, Lobachevsky, Hilbert, as well as use secondary sources for Cantor and Peano. Students write papers and proofs. The goal is the development of the imagination, the use of precision and logic in language, and the ability to spot hidden assumptions (or biases) and to consider alternatives, as well as a kind of perspective of the evolution of mathematical thought.

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The Humble Sum of Remainders Function

The sum of divisors function is one of the fundamental functions in elementary number theory. In this talk I shine a little light on one of its lesser-known relatives, the sum of remainders function.

I do this by illustrating how straightforward variations of the sum of remainders can 1) provide an alternative characterization for perfect numbers, and 2) help provide a formula for sums of powers of the first n positive integers.

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PDAs in the Classroom

This past year, Elmhurst College incorporated the Palm-OS based Personal Digital Assistant (PDA) into our Calculus I and II classes. In addition to using the PDAs as a replacement for the traditional graphing calculator, we also used the freeware program SmallBasic to create programs (similar to Java applets) the students can use away from the classroom.

In this talk, we: (1) demonstrate the technology and the applicable software; (2) explain how we plan to incorporate these hand-held computers into our technology course for pre-service high school mathematics teachers, MTH 199: Mathematical Software Tools; and (3) relay the responses of high school and college faculty to our presentations on the use of the PDAs. All of the programs that we currently use in Calculus I & II (including the ones we have written ourselves) can be downloaded for free from our project website www.elmhurst.edu/~abigailh/pdaproject.html.

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Creating a Poor Person's CAD Program with Excel

The relationships between a Bezier curve and its control points can be explored and understood by students familiar with calculus concepts and parametric curves. Neither Geometer's SketchPad nor Mathematica make it easy to create interactive parametric plots. Learn how to use Excel to help your students explore Bezier curves and to understand the mathematics behind drawing software.

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Analysis of Proteins Using Statistical Geometry

This talk begins with a brief overview describing proteins and their amino acid (aa) residue building blocks. Next, the Voronoi and Delaunay tessellation constructs from graph theory are illustrated, as well as a description of how this is performed on a protein structure by representing the structure using its set of alpha-carbons (1-1 correspondence between the aa's and the alpha-carbons in a protein). Sets of four aa's in a protein are defined to be nearest neighbors if they form the vertices of one of the tetrahedra obtained after performing Delaunay tessellation on the protein structure. A nice counting argument shows that there are 8855 possible quadruplets of aa's, after which a log-likelihood score is assigned to each quadruplet. This score is based on information from a training set of roughly 1000 protein structures that form a basis for the space of over 25,000 structures currently deposited in the Protein Data Bank (http://www.rcsb.org/pdb/). Finally, these scores are used to analyze the enzyme HIV-1 protease and its various mutations. By comparing the computational data of the mutants with wet-lab experimental results in which mutant enzymes were synthesized and their activity was measured, a correlation between the structure and function of these mutants is illustrated. The material as presented is accessible to undergraduates and provides fresh examples for teachers of geometry, finite math, statistics, and math modeling / mathematical biology.

William Wardlaw (wpw@usna.edu) Mathematics Department, 798 MacSherry Drive, Arnold, MD 21012, Annapolis, MD, 21402

For Which C and D Does $AB = C$ imply $BA = D$?

The 1969 Putnam Exam problem B6 motivated Adv. Prob. 6251 in the January 1979 American Mathematical Monthly quoted here: Let n and m be positive integers. What pairs of matrices C and D, over any field K, have the property that if A is an $m \times n$ matrix over K and B is an $n \times m$ matrix over K such that $AB = C$, then $BA = D$. This talk solves the problem and gives a simple procedure for producing such matrix pairs C and D.

Jaehoon Seol (jseol@valdosta.edu) 1500 N. Patternson, Valdosta, GA, 31698

Three-Dimensional Vector Normalization for Gaussian Elimination

Gaussian elimination is generally considered the most efficient means of solving small to medium size linear systems. It is also one of the first algorithms along with the LU factorization being taught to starting students on scientific computation.

Some of the early efforts to visualize this algorithm such as LAVA and other linear algebra related web sites are based on two-dimensional computer graphics and use a pair of lines to find the intersection point. This approach is sufficient to teach students what is the goal of the algorithm, but does not show how to generalize to a linear system of order $n \geq 4$.

In this paper, we present a three-dimensional visualization toolkit that can be used to visualize the Gaussian elimination in two different modes. The first mode is the intersection mode and the second one is the normalization mode. The intersection mode takes the same approach as the one taken by the traditional visualization toolkit. The difference is we present a set of planes with the goal of finding the intersection point instead of a set of lines. The goal of the normalization mode is to transform the coefficient vectors into the standard basis using elementary operations.

We believe the combination of these two approaches will enable students to have a scalable understanding of the algorithm.This toolkit is implemented using three-dimensional API such as OpenGL and Open Inventor.

We also provide a brief discussion of comparison between the implementation based on Java 3D and the implementation based on C/C++.

Todd Lee (tlee@elon.edu) 5013 Carolwood LN, Durham, NC, 27713

Visualizing the Pains of a Random Search

An undergraduate investigation in seriating tribal tales led to the problem of comparing cost functions on various permutations of tales, namely, minimizing dissimilarities versus gaps in tale components. This talk is on some interesting visualizations of the total number of possible pairs of costs. These pictures not only helped the student see what was going on, but also clearly illustrate the futility of non-clever approaches to optimizing over sets of permutations.

David Finn (david.finn@rose-hulman.edu) Department of Mathematics, CM 140, 5500 Wabash Avenue, Terre Haute, IN, 47803

Geometric Modeling: An Applied Geometry Course

Over the past few years, the speaker has been teaching a course on geometric modeling as the geometry course at Rose-Hulman Institute of Technology. The course covers some of the mathematical methods for describing physical and virtual objects used in computer-aided geometric design, CAD/CAM systems and computer graphics. The course has for prerequisites only multivariable calculus. This course has generated interest among students from various majors (computer science, mathematics and engineers) to pursue additional studies in projective geometry, differential geometry, and computational geometry. This talk describes the course, the motivation used to generate interest in additional courses in geometry, and some of the course materials.

Michael Livshits (mlivshits@hbs.edu) 36 Linnaean Street #14, Cambridge, MA, 02138

Simplifying Calculus by Using Uniform Estimates

Calculus of polynomials can be developed without Analysis because $x - a$ divides $p(x) - p(a)$. As Hermann Weyl pointed out on page 4 of his 1938 treatise on classical groups, a continuous f is differentiable at a if $x - a$ divides $f(x) - f(a)$ in the ring of continuous functions. This point of view is close to the Weierstrass' definition of the derivative.

Now assume that we have an algebra A of "nice" functions. We call a nice function f nicely differentiable if $x - a$ divides $f(x) - f(a)$ in A, i.e. $f(x) - f(a) = q_a(x)(x - a)$ for some q_a in A and the derivative $f'(x) = q_x(x)$ is also nice. Taking uniformly Lipschitz functions as nice leads to a dramatic simplification of Calculus and makes it more accessible to HS and College students.

Visit world.std.com/~michaell/calculus.html for more details.

Elizabeth (Betty) Rogers (b.rogers@prodigy.net) P.O. Box 907311, Gainesville, GA, 30501

Online Mathematics for MAT Candidates

Mathematics content courses for candidates in the Master of Arts in Teaching program in cohorts and on-campus locations require innovative strategies. A combination of traditional online methods, utilization of other technology and cooperative learning provides a unique learning experience. The teacher candidates take an active role in their own learning. A variety of activities are provided to meet individual learning styles and needs as a future teacher. Sample syllabi will be available for courses in combinatorics, cryptology and math history.

Thomas Hagedorn (hagedorn@tcnj.edu) P.O. Box 7718, Ewing, NJ, 08628

Zeros of Fourth Degree Recurrence Relations

It has been shown by several authors that for a cubic linear recurrence relation of the form $a(n+3) = ba(n+2)+ca(n+1)+da(n)$, the value of term $a(n)$ can be zero at most three times if the associated polynomial $x^3 - bx^2 - cx - d = 0$ has three real roots with distinct absolute values. We report on our extension of this result to recurrence relations of fourth degree.

Mohammed Tesemma (tesemma@math.temple.edu) Department of Mathematics, 350 Spelman Lane, S.W. Box 953, Atlanta, GA, 30314

Multiplicative Invariant Theory

Let G be a finite group acting by automorphism on a lattice A and hence on the group algebra $K[A]$ over a field K. The algebra $K[A]^G$ of G-invariants of $K[A]$ is called *an algebra of multiplicative invariants*. A result of Lorenz (2001) [M. Lorenz, *Multiplicative invariants and semigroup algebras*, Alg. and Rep. Theory **4** (2001), 293–304.] states that if G is generated by elements that act as reflections on A then $K[A]^G$ is a semigroup algebra over K. This is a multiplicative analogue of the classical Shephard-Todd-Chevalley theorem for polynomial invariants. But the converse of Lorenz's theorem is open. We will give an extended version of Lorenz's theorem which does indeed have a converse. Our approach uses SAGBI (The term SAGBI from computational algebra stands for *Sublgebra Analogue to Gröbner Bases for Ideals. Z. Reichstein systematically applies SAGBI basis in multiplicative invariant theory in one of his recent papers.*) bases and special type of polyhedral cones called *simplicial cones*.

In the talk I will give statement of the theorem and a brief outline of the proof. Our approach is accessible and with very minimal requirement compare to most results in multiplicative invariant theory that uses Lie algebra (in particular the theory of root systems).

Zikica Perovic (zikica.perovic@normandale.edu) Normandale College, 9700 France Ave S, Bloomington, MN, 55431

Trees — key to substitution, composition, chain rule

Teaching introductory Math courses we identified many situations where the main obstacle that students face is inadequate understanding of the recursive structure of expressions.

We use boxes to emphasize the last node of the expression tree while hiding the rest. This technique helped our students to perform much better in:

A) Solving equations (biquadratic, log, trig) using substitution

B) Construct compositions of functions

C) Perform the Chain rule

This technique could be used even by students that have very limited experience in manipulating variables. It is a proper way to teach order of operations and the meaning of the notion "solve the equation." It also comes natural to the students that already had some computer programming experience.

Combinatorics (Invited Paper Session)

Arthur Benjamin (benjamin@hmc.edu) 1250 N. Dartmouth Avenue, Claremont, CA, 91711
Naiomi Cameron (ncameron@oxy.edu) Occidental College, 1600 Campus Road, Los Angeles, CA, 90041

Counting on Determinants (Part I)

How many ways can n determined ants reach their destinations so that their paths never cross? The answer is given by the determinant of an n by n matrix. Applications to Fibonacci and Catalan numbers will also be given.

Zsuzsanna Szaniszlo (zsuzsanna.szaniszlo@valpo.edu) Gellersen Hall, Valparaiso, IN, 46383

An Overview of Recent Results on Pebbling

A pebbling move on a graph consists of lifting two pebbles from a vertex, throwing one away and moving the other to a neighboring vertex. The pebbling number of a graph is the minimum number of pebbles needed so that regardless of the distribution of the pebbles any target vertex can be reached by a sequence of pebbling moves. The area is attracting more and more interest, for example the SIAM Conference on Discrete Mathematics had a minisymposium on pebbling, and the invited talk of Aparna Higgins at this MathFest is on this subject. We will survey recent results on pebbling and its variations; optimal pebbling, cover pebbling, and pegging.

Jennifer Quinn (jquinn@oxy.edu) 1600 Campus Road, Los Angeles, CA, 90041
Arthur T Benjamin (benjamin@hmc.edu) Harvey Mudd College, Claremont, CA, 91711

The Politics of Exclusion: Doing Away with P.I.E.

The Principle of Inclusion-Exclusion is a favorite mathematical method to attack identities where consecutive terms have alternating positive and negative signs. While a powerful tool, it has a tendency to obscure any relationship among the sets being considered. In this talk, bijections between odd sets and even sets are used to prove the same identities. The approach is simpler, more direct, and more concrete. Is it better? You decide.

Naiomi Cameron (ncameron@oxy.edu) 1600 Campus Road, Los Angeles, CA, 90041

Counting on Determinats (Part II)

How many spanning trees does a graph have? It turns out that the answer to this question is a matter of computing a determinant. In this talk, we present a combinatorial proof of this result, known as the Matrix-Tree Theorem.

Applications of Topology (Invited Paper Session)

D. Sumners (sumners@math.fsu.edu) Department of Mathematics, Tallahassee, FL, 32306-4510

Knots in DNA

Cellular DNA is a long, thread-like molecule with remarkably complex topology. Enzymes which manipulate the geometry and topology of cellular DNA perform many important cellular processes (including segregation of daughter chromosomes, gene regulation, DNA repair, and generation of antibody diversity). Some enzymes pass DNA through itself via enzyme-bridged transient breaks in the DNA; other enzymes break the DNA apart and reconnect it to different ends. In the topological approach to enzymology, circular DNA is incubated with an enzyme, producing an enzyme signature in the form of DNA knots and links. By observing the changes in DNA geometry (supercoiling) and topology (knotting and linking) due to enzyme action, the enzyme binding and mechanism can often be characterized. This expository lecture will discuss topological models for DNA strand passage and exchange, and using the spectrum of DNA knots to infer bacteriophage DNA packing in viral capsids.

Robert Ghrist (ghrist@math.uiuc.edu) 1409 W. Green St., Urbana, IL, 61801

Topological Methods in Robotics

Given a collection of robots which have to cooperate to accomplish a task, how do you program them to automatically work together? One method is to build an abstract 'configuration space' and exchange physical planning problems for topology or geometry problems (which, hopefully, will be easier to solve). This talk will demonstrate the use of topological methods in the context of 'metamorphic' or 'shape-changing' robots.

Robert Franzosa (franzosa@math.umaine.edu) 417 Neville Hall, Orono, ME, 04469

Applications of Topology to GIS

A geographic information system (GIS) is a computer system capable of assembling, storing, manipulating, and displaying geographically referenced data. A fundamental mathematical modeling issue in GIS is to establish a framework for distinguishing relationships between sets representing geographic regions. We present a simple topological model that is based on intersections of boundaries and interiors of the sets involved. The model has been adopted as a GIS-industry standard for describing the relationships it addresses.

Gregory Buck (gbuck@anselm.edu) Box 1641, Dept of Mathematics, Manchester, NH, 03102

Tangling and Untangling in Biology and Physics

We will discuss applications of physical knot theory in biology and physics, including perhaps the actions of enzymes that detangle DNA, and a phase transition in entanglement rates.

Environmental Mathematics SIGMAA Invited Paper Session

Charles Hadlock (chadlock@bentley.edu) Bentley College, Waltham, MA, 02452

The Challenges of Environmental Consulting

Environmental consulting, including the use of modeling, encompasses a considerable range of activities depending on both the nature of the client organization and the objective of the investigation. For example, regulatory and legal cases can be conducted very differently from scientific and engineering investigations. The speaker will discuss his experience in a wide range of consulting assignments and will also suggest ways that others may involve themselves in this kind of work.

"SMALL" Mathematics (Invited Paper Session)

Thomas Colthurst (thomasc@alum.mit.edu) BNN Technologies, 8 Spencer Ave. #2, Somerville, MA 02144

A Gray Path on Binary Partitions

A binary partition of a positive integer n is a partition of n in which each part has size a power of two. A gray path on binary partitions is an ordering such that adjacent partitions differ only by replacing $2^k + 2^k$ with 2^{k+1} or vice versa. Donald Knuth asked if such gray paths existed for all n; we prove constructively that they do. This is joint work with Michael Kleber.

Frank Morgan (frank.morgan@williams.edu) Williams College, Williamstown, MA 01267

Sixteen years of the SMALL Undergraduate Research Project: 1988–2004

Observations on the first sixteen years of the SMALL Undergraduate Research Project, which now has to its name over 200 students, 20 faculty, and 50 publications.

Timothy D. Comar (tcomar@ben.edu) Department of Mathematics, Benedictine University, 5700 College Road, Lisle, IL 60532

Regular and Almost Regular Stick Numbers

For $0 < a < 180$, we define an a-regular stick knot to be an embedding of a knot in space formed out of equal length sticks such that adjacent sticks meet at an angle of a degrees. An a-almost regular stick knot is an embedding of a knot in space formed out of sticks such that small deviations from regularity are permitted in the lengths and angles. The a-(almost) regular stick number of a knot K is the minimal number of sticks required to construct K as a a-(almost) regular stick knot. We will discuss several problems about such knot conformations including the existence of such conformations for given topological knot types and methods for determining bounds for regular stick numbers.

Brian A Wecht (bwecht@physics.ucsd.edu) Center for Theoretical Physics, MIT, 77 Massachusetts Avenue Building 6, 3rd Floor, Cambridge, MA 02139

Algebraic Geometry, String Theory, and Baryon Charges

I will describe a prescription for computing certain quantities in quantum field theory from a string theory perspective. This is done by assigning curves in the string theory geometry to particular fields, and then computing intersection numbers. This procedure will be illustrated in several examples, and will also reveal interesting (heretofore unknown) geometric identities.

Joseph Corneli (jcorneli@math.utexas.edu) University of Texas at Austin, Department of Mathematics, RLM 8.100, Austin, TX, 78712

Math on Computers: Current Capabilities and Visions for the Future

We will briefly review the contemporary relationship between computers and mathematics, with an emphasis on pure mathematics research. We will then discuss possible future applications, together with concrete steps that may be needed to realize our dreams.

Geometric Group Theory (Invited Paper Session)

Ted Turner The University at Albany

Curvature in Dimension Two: a Case Study in Geometric Group Theory

Geometric group theory is an approach to the study of discrete groups using ideas from geometry. We will discuss what is meant by 'curvature' and 'dimension' for discrete groups and describe the very different properties of groups of dimension two that have negative curvature as opposed to those that have zero curvature.

Kim Ruane Tufts University

Infinite Groups and Geometry — a Match Made in Heaven!

Peanut butter and jelly, popcorn and a movie, beer and pizza — some of the more familiar matches made in heaven. Add to this list — free groups and trees, free abelian groups and Euclidean space, surface groups and hyperbolic space, just to name a few! There are several important theorems that illustrate just how beautifully these two fields complement each other. In this talk, we will discuss some of these theorems to help introduce techniques used in Geometric Group Theory.

Jennifer Taback Bowdoin College

Dead End Words and Other Anomalies

Geometric group theory is a fascinating field of mathematics which utilizes geometric descriptions and properties of groups to obtain both algebraic and geometric consequences. For example, given a group G, with a finite set of generators S, there is a canonical way to create a "picture" of this group, called a Cayley graph. If you change the set of generators, you might get a different picture! I will give some examples of Cayley graphs of groups, and of interesting phenomena that can occur in Cayley graphs. Examples of groups I will discuss are the Baumslag-Solitar groups and the lamplighter group L_2. An element of the lamplighter group can be envisioned as an infinite string of light bulbs, some of which are illuminated, and a cursor which tells you the light bulb you are considering.

Ken Brown Cornell University

Amenability of Groups

The concept of amenability was introduced into group theory in 1929 by von Neumann, in an effort to understand the Banach-Tarski paradox. Now, 75 years later, the concept remains mysterious in many ways. For example, it can be quite difficult to decide whether or not a given group is amenable. In this talk I will give a survey of amenability, leading up to some open questions.