Consider a 1×3 grid, the maximum amount we can earn is 6.

1	2	3

sum = 6

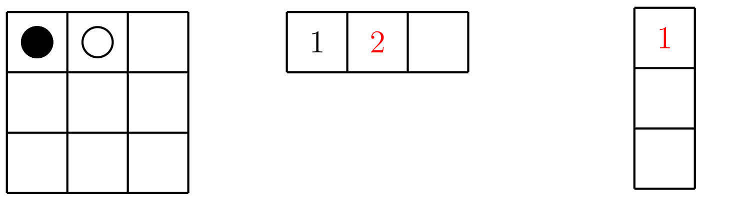
1	3	1

sum = 5

2	1	3

sum = 6

Note that in the case we earn 6, the second token has to be placed next to the first token. Similarly for a 3×1 grid. Therefore, $6 \times 3 \times 2 = 36$ is an upper bound for the amount we can earn in a 3×3 grid. However, we won't be able to achieve 36. This is because when an isolated token in its row/column is placed, we earn the money twice (row and column). For example, in the figure below, when placing the white token, we earn only $(2 + 1) - 1$.



As a result, the maximum amount we can achieve is when the isolated token in its row happens to be the isolated token in its column. In addition, the second token in its row/column has to be placed next to the first token in the same row/column. Here are two different examples.

①	2	3
3	①	2
3	2	①

row score

①	2	3
2	①	2
3	3	①

column score

①	4	6
5	①	4
6	5	①

sum = 33

①	2	3
3	2	①
2	①	3

row score

①	3	2
2	2	①
3	①	3

column score

①	5	5
5	4	①
5	①	6

sum = 33

Therefore, the maximum amount we can achieve is

$$36 \text{ (upper bound)} - 3 \text{ (count twice)} = 33.$$

By the observation on a 1×3 grid, if the second token is not placed next to the first token, we will earn 1 less. In addition, we have to subtract 1 for any isolated tokens (in its row/column) due to counting twice. For example,

3	2	①
①	3	①
2	①	3

row score

3	①	①
①	3	2
2	①	3

column score

6	②	①
①	6	②
4	①	6

sum = 29

$$36 \text{ (upper bound)} - 2 \text{ (6 becomes 5)} - 5 \text{ (count twice)} = 29.$$

As a result, the minimum amount we can achieve is when we have two isolated tokens in its row and in its column if possible. If it is not possible, then we make two separated tokens at least isolated in its row or in its column. Here is the example.

①	3	①
①	3	①
①	3	①

row score

①	①	①
3	3	3
①	①	①

column score

①	③	①
③	6	③
①	③	①

sum = 22

Therefore, the minimum amount we can achieve is

$$36 \text{ (upper bound)} - 6 \text{ (6 becomes 5)} - 8 \text{ (count twice)} = 22.$$

The password is **11223366**.

After reading the note inside the crate, we can update the diagram as follows.

Row 1	A1	C1	C2	O2
Row 2	C1	A1	A3	3
Row 3	1	O1	O	3
Row 4		1	A1	A

Note that there is only one copy of Issue 3 in each era. Since Archival Issue 3 is in row 2 and only the two magazines in the middle in row 3 are Older, it follows that the magazine on the right in row 3 is Current. Consequently, the magazine on the right in row 2 is Older. In addition, since only the two magazines on the left in row 3 are Issue 1, it follows that the unknown Older magazine in row 3 is issue 2.

Row 1	A1	C1	C2	O2
Row 2	C1	A1	A3	O3
Row 3	1	O1	O2	C3
Row 4		1	A1	A

Note that there are only three copies of Issue 1 in each era (and only one copy of Issue 3 in each era), so the magazine on the right in row 4 is Archival Issue 2. In addition, since only the two magazines in the middle in row 3 are Older, it follows that the magazine on the left in row 3 is Current. Consequently, the Unknown Issue 1 in row 4 is Older.

Row 1	A1	C1	C2	O2
Row 2	C1	A1	A3	O3
Row 3	C1	O1	O2	C3
Row 4		O1	A1	A2

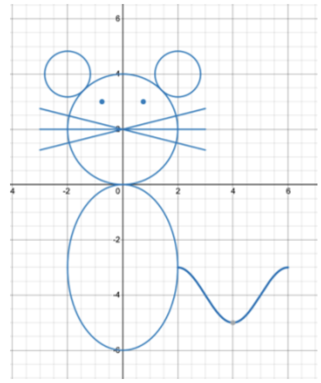
Note that there are only two copies of Issue 2 in each era. In row 4, since only the two magazines in the middle are Issue 1 and only the two magazines on the right are Archival, it follows that the magazine on the left in row 4 is Current Issue 2.

Row 1	A1	C1	C2	O2
Row 2	C1	A1	A3	O3
Row 3	C1	O1	O2	C3
Row 4	C2	O1	A1	A2

The password is **C2O1A1A2**.

The Flip Chart of Graph Paper

Start by graphing all of the equations on the same graph.
Here is a graph using Desmos.



1) The slope of the whisker with a positive derivative multiplied by 8.
Slope = ¼. Multiplied by 8 yields 2.

2) The slope of the shortest whisker.
The shortest whisker is the horizontal one. Its slope is 0.

3) The *y*-coordinate of the center of the ear in the first quadrant.
The center of that ear has a height of 4.

4) The absolute value of the *y*-coordinate where the tail is attached.
The tail is attached at y = −3. The absolute value is 3.

5) The slope of the line tangent to the face at the point $(-\sqrt{2}, 2+\sqrt{2})$.
Note that this point is where the left ear is attached to the face, on the upper half of the face's circle. The face comes from $x^2 + (y-2)^2 = 4$. Then, $(y-2)^2 = 4 - x^2$ and $y - 2 = \sqrt{4-x^2}$ for the top half of the circle. Then, $y = 2 + \sqrt{4-x^2}$. The derivative of this function is $\frac{1}{2}\,(4-x^2)^{-\frac{1}{2}}(-2x) = -\frac{x}{\sqrt{4-x^2}}$. When $x = -\sqrt{2}$, the slope is $\frac{\sqrt{2}}{\sqrt{4-(-\sqrt{2})^2}} = \frac{\sqrt{2}}{\sqrt{4-2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$. So the slope there is 1.

6) The second derivative of any whisker.
The whiskers have no concavity, so the second derivative of any of them is 0.

7) The largest *y*-value of a point along the face with *dy* / *dx* = 0.
The top of the face is at (0,4), and the slope there is 0. So the y value is 4.

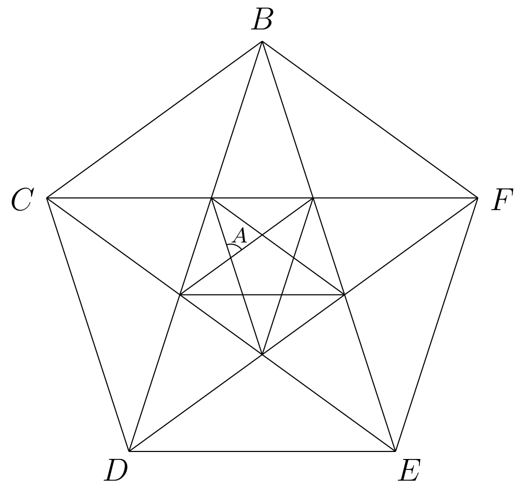
8) The period of the waves in the tail.
This is calculated using $\frac{2\pi}{coefficient\ of\ x} = \frac{2\pi}{\pi/2} = \frac{4\pi}{\pi} = 4$.

9) The amplitude of the tail.
The tail oscillates between – 3 and – 5, with a difference between max and min of 2. The amplitude is half of that, or 1.

10) Suppose the character in the graph ate too many pies making its body twice as big as before, but its height remains the same. In the fourth equation, what is the new constant below x^2?

The volume of the ellipse $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ is $AB\pi$. Using equation 4 here, the volume is $2 * 3 * \pi = 6\pi$. If we double that, we have 12π. We know the height (B) remains the same, so the new area is $(New\ A) * 3\pi = 12\pi$, so $(New\ A) = \frac{12\pi}{3\pi} = 4$. The value below x^2 is $(New\ A)^2 = 16$.

The password is: 20431044116.



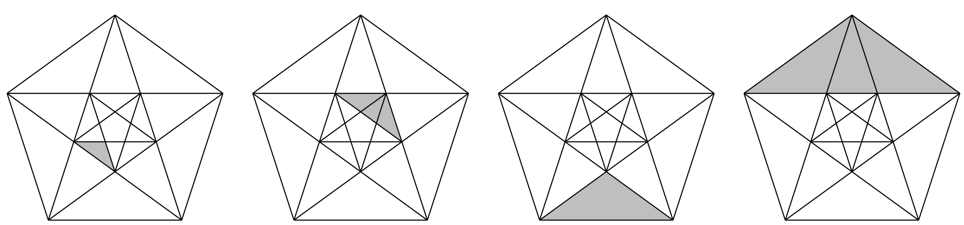
The diagram on the scroll is a regular pentagon. Thus, $\angle CBF = \dfrac{180^\circ \times 3}{5} = 108^\circ$. In addition, $\triangle CBF$ is an isosceles triangle, so

$$\angle BCF = \angle BFC = \frac{180^\circ - 108^\circ}{2} = 36^\circ.$$

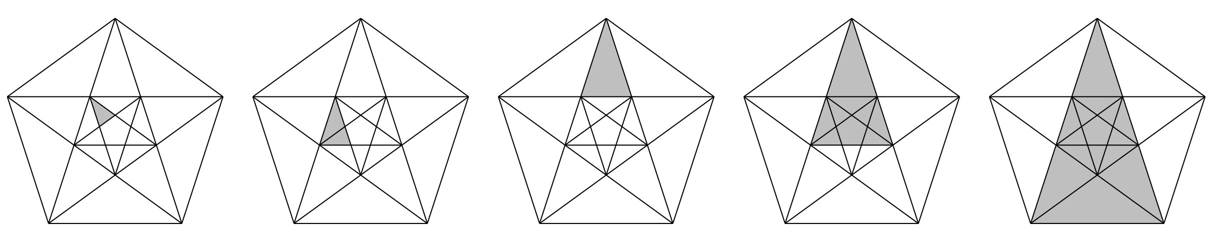
Note that $\triangle CBF$ and $\triangle DCB$ are congruent (**SAS**). Thus, $\angle CDB = \angle BCF = 36^\circ$. Hence, $\angle BDE = \angle CDE - \angle CDB = 108^\circ - 36^\circ = 72^\circ$. Similarly, $\angle BED = 72^\circ$. Therefore, in $\triangle BDE$, $\angle DBE = 180^\circ - 2 \times 72^\circ = 36^\circ$.

As a result, there are 2 non-similar triangles and 9 non-congruent triangles,

(1) those similar to a $108^\circ - 36^\circ - 36^\circ$ triangle and



(2) those similar to a $36^\circ - 72^\circ - 72^\circ$ triangle.



It follows that $\angle A = \angle BDE = 72^\circ$.

The password is **2972**.

The Urn

Without looking anything up, we can reasonably match these data sets as follows.

1. The average temperature in Washington DC during March, May, July, September, and November should start low, get warmer, and then go down again with the seasons. This has to be graph F.
2. The percentage of students who went to college has been increasing overall since 1970, so this has to be graph D.
6. Most awards have the same number of winners each year, so we would expect that the number of people who have won writing awards each year. So, this would be graph E.
5. We would expect the most expensive membership to be the one with print copies of all journals. The next largest would be regular memberships, since the remaining ones of retired, student and transitional should all be smaller. That would have to be graph B, with the second bar highest (print) and the first bar the second highest (regular). None of the other remaining graphs have that property.

The last two are more difficult without looking up data. Here is a reasonable guess toward the right answer.

4. Of China, Germany, Sweden, UK, and USA, the larger countries are more likely to have more Math Olympiad Golds, and they actually do. China is first, followed by the US. That is graph A.
3. That leaves graph C for the numbers of PhD students for some famous mathematicians.

The Password is: FDCABE

The actual data was generated from the following web sites.

1. Temperatures in DC. https://www.usclimatedata.com/climate/district-of-columbia/united-states/3178
2. High school students who went to college. https://educationdata.org/college-enrollment-statistics/#high-school-to-college
3. Number of PhD Students. Look up each person through the Mathematics Genealogy project https://www.genealogy.math.ndsu.nodak.edu/
4. Math Olympiad Medal Counts https://en.wikipedia.org/wiki/List_of_countries_by_medal_count_at_International_Mathematical_Olympiad
5. MAA Membership Costs https://www.maa.org/membership/membership-categories
6. Trevor Evans Award for exceptional writing https://www.maa.org/programs-and-communities/member-communities/maa-awards/writing-awards/trevor-evans-awards

There is a page for each one of the clues. Scroll down.

The final clue is to get to that last certificate is:
LAMBDAMUNUXIOMICRONPI

The Scroll Clue (Password to open it is 2972)

Congratulations on solving the scroll puzzle!

In order to get the password to open the door, you must decipher three words that describe the password. The clues below will give you one letter from the first word of this description, and one letter from the second two words. Once you collect all the clues from all the puzzles in the article, unscramble the three words, and use that to determine what collection of words make up the password. The password itself is a concatenation of several words, all in caps. Use that to open the door.

To obtain a letter in the first word of the code, go to the following article and write down the first letter in the body of the article. That letter is larger than the rest.

Fighting an Epidemic with an Epidemic by Evelyn J. Lamb (April 2019)
Note that this article is available for free at:
https://www.maa.org/press/periodicals/math-horizons/math-horizons-sample-articles.

To obtain a letter in the second or third word of the code, go to the following article in THIS issue of Math Horizons and write down the first letter in the body of the article. That letter is larger than the rest.
Old COAT Measures by Andrew Simoson

The Crate Clue (Password is C2O1A1A2)
Note: That is the letter "O", not the number zero. Also, it is the number "1" and not an upper case letter i or lower case letter L.

Congratulations on solving the crate puzzle!

In order to get the password to open the door, you must decipher three words that describe the password.   The clues below will give you one letter from the first word of this description, and one letter from the second two words. Once you collect all the clues from all the puzzles in the article, unscramble the three words, and use that to determine what collection of words make up the password. The password itself is a concatenation of several words, all in caps. Use that to open the door.

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The Paintball Party by James Propp (November 2017)
Note that this article is available for free at:
https://www.maa.org/press/periodicals/math-horizons/math-horizons-sample-articles.

To obtain a letter in the second or third word of the code, go to the following article in THIS issue of Math Horizons and write down the first letter in the body of the article. That letter is larger than the rest.
Honeycombs in the Hosoya Triangle by Blair, Florez, and Mukherjee

The Bag of Marbles Clue (Password is 96)

Congratulations on solving the marbles puzzle!

In order to get the password to open the door, you must decipher three words that describe the password.   The clues below will give you one letter from the first word of this description, and one letter from the second two words. Once you collect all the clues from all the puzzles in the article, unscramble the three words, and use that to determine what collection of words make up the password. The password itself is a concatenation of several words, all in caps. Use that to open the door.

To obtain a letter in the first word of the code, go to the following article and write down the first letter in the body of the article. That letter is larger than the rest.

The Bingo Paradox by Arthur Benjamin, Joseph Kisenwether, and Ben Weiss (September 2017)
Note that this article is available for free at:
https://www.maa.org/press/periodicals/math-horizons/math-horizons-sample-articles.

To obtain a letter in the second or third word of the code, go to the following article in THIS issue of Math Horizons and write down the first letter in the body of the article. That letter is larger than the rest.
A Fold and Cut Pi and e by Curtis and Todd Nelson
(Do the Math column)

The Urn Clue   (Password to open it is FDCABE)

Congratulations on solving the urn puzzle!

In order to get the password to open the door, you must decipher three words that describe the password.   The clues below will give you one letter from the first word of this description, and one letter from the second two words. Once you collect all the clues from all the puzzles in the article, unscramble the three words, and use that to determine what collection of words make up the password. The password itself is a concatenation of several words, all in caps. Use that to open the door.

To obtain a letter in the first word of the code, go to the following article and write down the first letter in the body of the article. That letter is larger than the rest.

Hidden Figures Light Up Screen: Black Women Who Helped America Win the Space Race by Jenna D. Carpenter (February 2017)
Note that this article is available for free at:
https://www.maa.org/press/periodicals/math-horizons/math-horizons-sample-articles.

To obtain a letter in the second or third word of the code, go to the following article in THIS issue of Math Horizons and write down the first letter in the body of the article. That letter is larger than the rest.
The Hidden and Surprising Symmetry of Ordered Lists by Lara Pudwell

The Flip Chart of Graph Paper Clue (Password to open it is 20431044116)

Congratulations on solving the flip chart puzzle!

In order to get the password to open the door, you must decipher three words that describe the password. The clues below will give you one letter from the first word of this description, and one letter from the second two words. Once you collect all the clues from all the puzzles in the article, unscramble the three words, and use that to determine what collection of words make up the password. The password itself is a concatenation of several words, all in caps. Use that to open the door.

To obtain a letter in the first word of the code, go to the following article and write down the first letter in the body of the article. That letter is larger than the rest.

How to Make a Torus By Laszlo C. Bardos (November 2015)
Note that this article is available for free at:
https://www.maa.org/press/periodicals/math-horizons/math-horizons-sample-articles.

To obtain a letter in the second or third word of the code, go to the following article in THIS issue of Math Horizons and write down the first letter in the body of the article. That letter is larger than the rest.
Parabolic Properties from a Piece of String by John Quintanilla
(Illuminating Illustration column)

The Briefcase Clue   (Password to open it is 11223366)

Congratulations on solving the briefcase puzzle!

In order to get the password to open the door, you must decipher three words that describe the password. The clues below will give you one letter from the first word of this description, and one letter from the second two words. Once you collect all the clues from all the puzzles in the article, unscramble the three words, and use that to determine what collection of words make up the password. The password itself is a concatenation of several words, all in caps. Use that to open the door.

To obtain a letter in the first word of the code, go to the following article and write down the first letter in the body of the article. That letter is larger than the rest.

Epic Math Battles: Counting vs. Matching by Jennifer Quinn (February 2015)
Note that this article is available for free at:
https://www.maa.org/press/periodicals/math-horizons/math-horizons-sample-articles.

To obtain a letter in the second or third word of the code, go to the following article in THIS issue of Math Horizons and write down the first letter in the body of the article. That letter is larger than the rest.
The Muffin Curse by James Propp