Solution Guide for A Complex Recipe for Rich Roll Cookies

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Note: When applicable, use principal values of ingredients.

- $\displaystyle \frac{1}{\pi i}\oint_{|z|=4} \frac{z}{(z+2)(z-1)}\,dz = 2\left(\mathrm{Res}(f(z);-2) + \mathrm{Res}(f(x);1)\right) = 2(2/3+1/3) = 2.$

- $\displaystyle \frac{1}{\pi i}\mathrm{Log}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = \frac{1}{\pi i}\left(\ln 1 + i\frac{2\pi}{3}\right) = \frac{2}{3}.$

- $\displaystyle \int_{-\infty}^{\infty} \frac{\cos x}{x^2+1}dx = \mathrm{Re}\left[\int_{-\infty}^{\infty}\frac{e^{ix}}{x^2+1}dx\right]$. Let $\Gamma = \Gamma_1 + \Gamma_2$ be a semi-circular contour where Γ_1 is the line segment from $-R$ to R and Γ_2 is parameterized by $z = Re^{it}$ for $t \in [0,\pi]$ for $R > 1$. By the Residue Theorem

  $$\oint_{\Gamma} \frac{e^{iz}}{(z^2+1)}\,dz = 2\pi i\left(\frac{e^{i(i)}}{2i}\right) = \frac{\pi}{e}.$$

  The estimate $|e^{iz}| = |e^{i(R\cos t + iR\sin t)}| = e^{-R\sin t} \leq 1$ for $z \in \Gamma_2$ implies

  $$\left|\int_{\Gamma_2} \frac{e^{iz}}{(z^2+1)}\,dz\right| \leq \frac{(1)\cdot \pi R}{R^2-1} \to 0 \qquad \text{as } R \to \infty,$$

  which implies

  $$\frac{\pi}{e} = \int_{-\infty}^{\infty}\frac{\cos x}{x^2+1}dx + i\int_{-\infty}^{\infty}\frac{\sin x}{x^2+1}dx.$$

  The integral with $\sin x$ vanishes since the integrand is an odd function. Therefore,

  $$\frac{e}{\pi}\int_{-\infty}^{\infty}\frac{\cos x}{x^2+1}dx = 1.$$

- $-1/f''(0) = 1/4$ since $f''(0) = 4i^2 = -4$.

- The function has a simple pole at $z = 0$, hence

$$\operatorname{Res}\left(\frac{z^2+3}{z^7+3z^3+2z}; z=0\right) = \lim_{z\to 0} z \cdot \frac{z^2+3}{z^7+3z^3+2z} = \lim_{z\to 0} \frac{z^2+3}{z^6+3z^2+2} = \frac{3}{2} = 1\frac{1}{2}.$$

- $1/(i\sqrt{2})^4 = 1/4$ since $i^4\sqrt{2}^4 = i^2 i^2 4 = 4$.

- $(1+i) = \sqrt{2}e^{i\frac{\pi}{4}}$ implies $(1+i)^3 = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = -2+2i$. Hence

$$2\ \mathrm{Im}[(1+i)^3] + \frac{3}{4}\ \mathrm{Re}[(1+i)^3] = 2(2) + \frac{3}{4}(-2) = 4 - \frac{3}{2} = 2\frac{1}{2}.$$

- $\Delta u = 0$, so $\epsilon > 0$ (a dash) of cinnamon.

- $\frac{175}{i}\left(e^{i\frac{\pi}{2}} - e^{-i\frac{\pi}{2}}\right) = 175\,(2)\,\frac{e^{i\frac{\pi}{2}} - e^{-i\frac{\pi}{2}}}{2i} = 350\ \sin\frac{\pi}{2} = 350^\circ$ F.