Topological Crosswords

Sam Eastridge and J. Tanner Slagel

Sometimes a topological space can be created by gluing a simpler space to itself. For example, we can create a torus—a hollow doughnut shape—by taking a square sheet of paper (or perhaps something more malleable), gluing the right and left edges together to make a cylinder, and then gluing the top and bottom of the cylinder together. In technical terminology, the torus is the *quotient space* obtained by taking a square and *identifying* opposite edges with each other, oriented in the same direction.

We can construct other topological spaces by taking quotients of the square. When we made the cylinder by gluing opposite edges of the paper together, we could have first twisted the paper, resulting in a Möbius strip. So the Möbius strip is the quotient space obtained by taking the square and identifying two opposite edges with different orientations.

Below are crossword puzzles on the torus, Klein bottle, and sphere. They are formed by taking the square and identifying edges in different ways. The colored arrows and the numbers and letters on the sides convey which edges are identified and with which orientations (whether we twist before gluing). Be warned, the edge of the puzzle does not mean the end of a word! Enjoy!

The solutions to these puzzles are found below. ∎

Sam Eastridge is an algebraist who credits his love for crosswords to his great-grandmother and his love for the torus to Krispy Kreme.

J. Tanner Slagel is a second-year PhD student at Virginia Tech. When he is not developing stochastic approximation methods for solving very large least squares problems, he is hiding from his advisers, making topological crossword puzzles.

Torus

Across

1. $(1 - \sqrt{x^2 + y^2})^2 + z^2 = 1^2$ and this puzzle
3. Sodium: Abbr.
4. Knot theorist's hair decoration
5. Spasm
8. $2x^2 + y^2 = 1$
10. Something a goalie makes
12. 400 meters
14. Lopsided squares
15. Inverse of exp
18. Prefix related to the atmosphere
20. First Bond film, *Dr. ___*
21. Vehicle-slowing device
23. She has a way with words
24. Inanimate object pronoun
26. Word following "if, then"

Down

1. It stops an NBA game
2. Exclamation of pain
4. Different form of *is* or *was*
6. Cubs' state: Abbr.
7. $x^2 + y^2 + z^2 = 1$ and $2x^2 + 2y^2 + 2z^2 = 8$
8. Axis of ___
9. $y = 3 - x^2$
10. Companion of mirrors
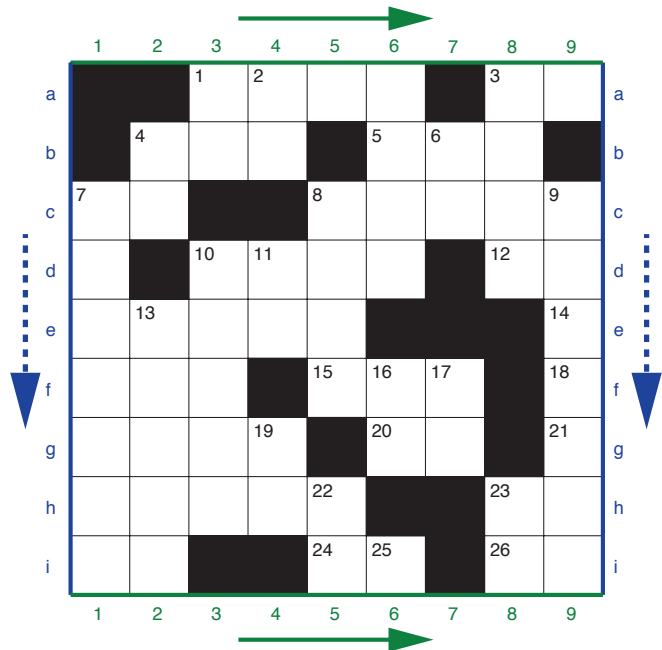
11. Stomach contractor, for short
13. Speak at length
16. Atop
17. Chinese game with stones
19. Present tense linking verb in Spain
22. Respectful pronoun for a mister
23. Euler said his ___ seemed to surpass himself in intelligence
25. *A Beautiful Mind* or *Elements*

Klein Bottle

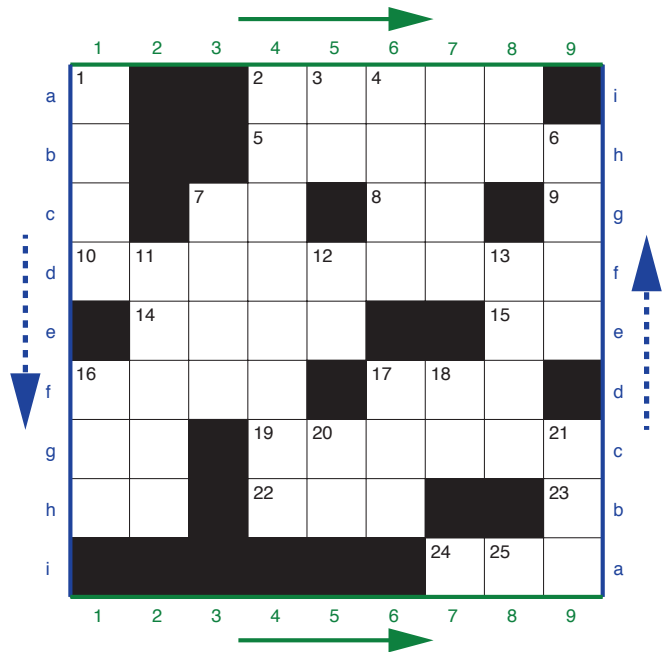**Across**

2. Discourage, as an action
5. Element in every group
7. Contented sigh
8. Stu. dorm supervisor
9. Where PC firmware is stored
10. Matrix group
14. Vertex
15. Software to run apps
17. DVR button
19. Commutative group
22. Didn't follow
23. Pa's partner
24. Lin-Manuel Miranda does this in *Hamilton*

Down

1. Made beautiful sounds with voice
2. Group of reflections and rotations
3. Witten or Halley, familiarly
4. 10^{12} prefix
6. Makes the pupil work
7. Small, deerlike water buffalo
11. Not an ally
12. Prefix on a return email
13. Conic sections can be defined as these

16. Negation word
17. Angry people see it
18. Right-angle letter
20. "To be, or not to ___"
21. Device to increase volume, for short
24. Relating to the kidneys
25. Benjamin or Cayley, familiarly

Sphere

Across

1. Having more beauty
2. Control-Z
4. Every mathematician in the 1700s did this
5. Donald's second wife
7. TV medical drama
9. A representation of a function using 16 down, but without its tail
11. Linking verb
13. Type of product in a Hilbert space
14. Down for the count in calc. class?
15. Someone who loves calculus, perhaps a little too much
17. What 2 is for $f(x) = x^2 - 6x + 11$
20. For ex.
22. Between one and million
23. Period of time
24. In an n-gon they sum to $180(n-2)$

Down

3. Consumed liquid
6. English brew
8. Necessary tool to compute derivatives
10. Boolean logic word that is not too picky
12. Exclaimed at aha! moment: "I ___!"
15. A giant word
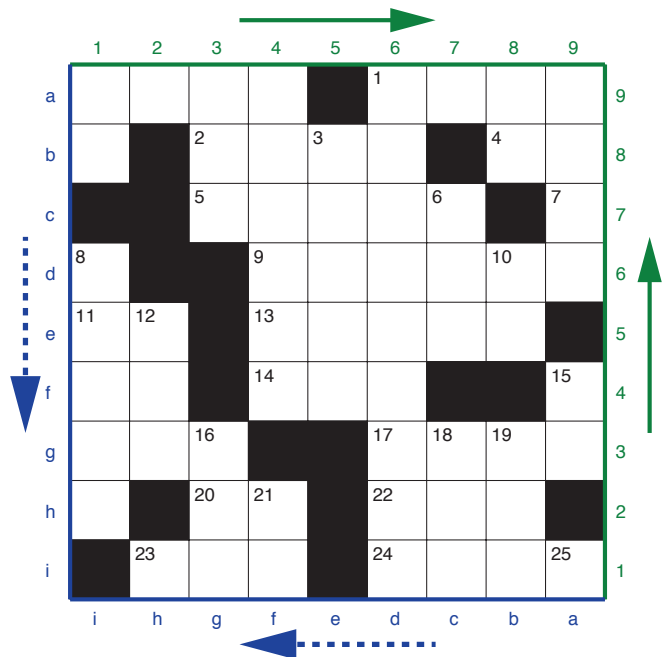
16. Sum of the terms in a sequence
18. Place where travelers stay
19. Annoys
21. Theory that studies rational decision-makers
23. S. Spielberg's alien
25. Useful for proving a theorem

Crossword solution: Torus

	1 T	2 O	R	I		3 N	A	
	4 B	O	W		5 T	6 I	C	
7 S	E		8 E	L	L	I	9 P	
P		10 S	11 A	V	E		12 L	A
H	13 O	M	B	I				14 R
E	R	O		15 L	16 O	17 G		18 A
R	A	K	19 E		20 N	O		21 B
E	T	E	S	22 S			23 P	O
S	E		24 I	25 T		26 E	L	

Crossword solution: Klein Bottle

1 S		2 D	3 E	4 T	E	R	
A		5 I	D	E	N	T	6 I
N		7 A	H		8 R	A	9 R
10 G	11 E	N	E	R	12 A	L	L	13 I
	14 N	O	D	E			15 O	S
16 N	E	A	R		17 R	18 E	C	
O	M		19 A	20 B	E	L	I	21 A
T	Y		22 L	E	D			23 M
					24 R	25 A	P	

Crossword solution: Sphere

E	M	M	A		1 P	R	E	T
S		2 U	N	3 D	O		4 D	I
		5 M	A	R	L	6 A		7 E
8 L			9 T	A	Y	L	10 O	R
11 I	12 S		13 I	N	N	E	R	
M	E		14 C	K	O			15 F
I	E	16 S		17 M	18 I	19 N	I	
T		20 E	21 G		22 I	N	A	
	23 E	R	A		24 A	N	G	25 L