Curriculum Burst 147: Lots of Chimes

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A clock chimes once at 30 minutes past each hour and chimes on the hour according to the hour. For example, at 1 PM there is one chime and at noon and midnight there are twelve chimes. Starting at 11:15 AM on February 26, 2003, on what date will the 2003rd chime occur?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grades.

MATHEMATICAL TOPICS

Number Sense: Arithmetic sums

COMMON CORE STATE STANDARDS

No Common Core State Standard (Connects with the Standards for Mathematical Practice)

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 2: **DO SOMETHING!**

SOURCE: This is question # 22 from the 2003 MAA AMC 10B Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question gives me the heebie-jeebies! Do we really want the date of the 2003rd chime?

This feels overwhelming!

In the spirit of "Do Something" I can see that just after noon up to and including midnight, and just after midnight up to and including noon, there will be 12 half-hour chimes and

$$\begin{aligned} &1+2+3+4+5+6+7+8+9+10+11+12 \\ &=13+13+13+13+13+13 \\ &=6\times 13 \\ &=60+18 \\ &=78 \end{aligned}$$

on-the-hour chimes. That is, on each day there will be $2\times(12+78)=180$ chimes.

So on February 27 there will be 180 chimes. (Starting just after midnight and ending on midnight of that day.)

On February 28 there will be 180 chimes.

And so on.

I can see that there will be $1+12+(12+78)=103$ chimes from 11:15 AM onwards on February 26 (up to and including midnight.)

Okay. All that was indeed "something" and it feels mighty helpful.

We want the date of the 2003rd chime.

We'll have completed the 103rd chime on midnight of February 26th, leaving 1900 chimes to go. And after ten more days we'll be down to 100 chimes to go.

Oh! All those 100 chimes will occur on that 11th day. So all I have to do is figure out the date of the 11th day!

The February 27th is the first day and February 28th is the second Day. March 1st is the third. (No leap year!) March 9th is the 11th.

Wow! That's it. The answer is March 9!

Extension: Suppose the clock is running fast and completes each of its twelve-hour runs in eleven hours. If no one corrects the clock, on which date will the 2003rd chime occur?

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