Curriculum Burst 140: A Ferris Wheel Ride

By Dr. James Tanton, MAA Mathematician at Large

Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grades.

MATHEMATICAL TOPICS

Geometry: Circles, the Pythagorean Theorem (or congruent triangles).

COMMON CORE STATE STANDARDS

G-C.2 Identify and describe relationships among inscribed angles, radii, and chords.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 4: **DRAW A PICTURE**

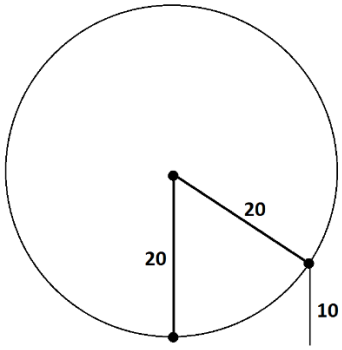
SOURCE: This is question # 24 from the 2002 MAA AMC 10B Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

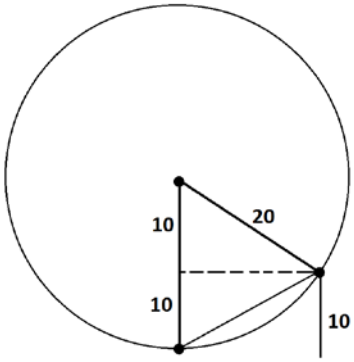
I think I get this question. We have a wheel of radius 20 feet that rotates one full turn every minute and we want to know how long it takes for a rider to go from the bottom of the wheel to 10 feet off of the ground. (Did I get that right?) So we are dealing with a picture like this:



Hmm. We need to find out how much time it takes for the wheel to turn this fraction of a rotation.

Okay. If the angle between the two radii of 20 is 60°, for example, which is one-sixth of a full turn, the time taken would be one-sixth of a minute. Or if the angle is 45°, one-eighth of a full turn, that would be one-eighth of a minute. And so on. We need the angle between the two radii.

I feel compelled to draw a triangle. And I feel then to also draw the height of 10 feet at a second location in the diagram.



Is this helpful?

Now I am flailing!

By the Pythagorean Theorem, the dashed line has length $\sqrt{20^2 - 10^2} = \sqrt{300}$. I am not sure if knowing this is helpful.

Umm. By the Pythagorean Theorem again, the length of the third side of the large triangle is:

$$\sqrt{\sqrt{300}^2 + 10^2} = \sqrt{400} = 20.$$

Oh, the big triangle is an equilateral triangle. All three of its angles is 60°, and so the wheel did make one-sixth of a turn. The time taken is one-sixth of a minute, that is, 10 seconds.

Extension: It takes the rider 30 seconds to ride from the bottom of the wheel to the top, 40 feet off the ground. At 15 seconds (half this time), the rider is 20 feet off the ground (half this height). But as we have just seen, at 10 seconds (one third of this time), the rider is 10 feet off the ground (which is not one-third of 40 feet).

What shape is the graph of the rider's height from the ground as a function of time?

MAA acknowledges with gratitude the generous contributions of
the following donors to the Curriculum Inspirations Project:

The TBL and Akamai Foundations
for providing continuing support

The Mary P. Dolciani Halloran Foundation for providing seed
funding by supporting the Dolciani Visiting
Mathematician Program during fall 2012

MathWorks for its support at the Winner's Circle Level