

# Curriculum Inspirations

Inspiring students with rich content from the  
MAA American Mathematics Competitions



## Curriculum Burst 138: Mowing Lawns

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Andy's lawn has twice as much area as Beth's lawn and three times as much area as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower. Suppose they all start to mow their lawns at the same time. Who will finish first?

### QUICK STATS:

#### MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grades.

#### MATHEMATICAL TOPICS

Algebra: Systems of equations

#### COMMON CORE STATE STANDARDS

**A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

#### MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

#### PROBLEM SOLVING STRATEGY

ESSAY 1: [ENGAGE IN SUCCESSFUL FLAILING!](#)

**SOURCE:** This is question # 21 from the 2002 MAA AMC 10B Competition.



## THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Oh heavens! This is a word problem and my brain feels strained just reading through it.

Let's take it slowly.

*Andy's lawn has twice as much as area as Beth's lawn and three times as much area as Carlos' lawn.*

I feel like I should give a name,  $L$ , to the area of someone's lawn. Let's let  $L$  be the area of Andy's lawn. Then we have:

$$\text{Andy's area} = L$$

$$\text{Beth's area} = L/2$$

$$\text{Carlos' area} = L/3$$

*Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower.*

Okay. Let  $R$  be the rate at which Carlos' mower cuts lawn (amount of area per unit time). Then:

$$\text{Carlos' rate} = R$$

$$\text{Beth's rate} = 2R$$

$$\text{Andy's rate} = 3R$$

Now "rate = area/time" so the time taken to mow a lawn is the area divided by the rate of mowing:

$$\text{Andy's time} = \frac{L}{3R}$$

$$\text{Beth's time} = \frac{L/2}{2R} = \frac{L}{4R}$$

$$\text{Carlos' time} = \frac{L/3}{R} = \frac{L}{3R}$$

Whatever the values of  $L$  and  $R$ , we see that Beth completes her mowing task in the least amount of time.

Not too bad after all!

**Extension:** Suppose Beth, as soon as she finishes mowing her lawn, moves over and helps Andy with his lawn. And then when Andy's lawn is complete, both she and Andy help Carlos finish his lawn. In terms of  $L$  and  $R$ , how long does it take to finish mowing all three lawns?

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