

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 127: 99-digit Products

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The product of the two 99-digit numbers $303,030,303,\dots,030,303$ and $505,050,505,\dots,050,505$ has thousands digit A and units digit B . What is the sum of A and B ?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Pre-Algebra.

COMMON CORE STATE STANDARDS

Connected to 8.EE.3

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 9: [AVOID HARD WORK!](#)

SOURCE: This is question #18 from the 2007 MAA AMC 8 Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question looks impossible! There is now way I am going to write out all 99 digits of $303,030,303, \dots, 030,303$ and all 99 digits of $505,050,505, \dots, 050,505$ and then do the long multiplication. There has got to be an easier way!

Well, the question says something about the thousands digit and the units digits. It doesn't seem to care about any of the other digits. So maybe I should think of the number $303,030,303, \dots, 030,303$ as:

$$303,030,303, \dots, 030,000 + 0303,$$

where I can really see the thousands digit (which is zero) and the units digit (which is three). Let me do the same for the other number too:

$$505,050,505, \dots, 050,000 + 0505.$$

Now I have to multiply these out. Let me draw out a rectangle for the product:

	505...50000	500	5
303...30000	BIG!	BIG!	BIG!
300	BIG!	150000	1500
3	BIG!	1500	15

Everything labeled BIG! In this table is a multiple of 10,000 or more and has all final four digits 0. So the product we seek is:

$$\text{Some multiple of ten thousand} \\ + 150000 + 1500 + 1500 + 15$$

This is:

$$\text{Some multiple of ten thousand} + 3015$$

The thousands digit is 3 and the units digit is 5.

Ummm. What was the question again?

The product of the two 99-digit numbers $303,030,303, \dots, 030,303$ and $505,050,505, \dots, 050,505$ has thousands digit A and units digit B . What is the sum of A and B ?

Oh! So $A + B = 3 + 5 = 8$. We're done!

Extension 1: What is the ten-thousands digit in this product?

Extension 2:

Consider the product of the two numbers $303,030,303,030,303$ and $505,050,505,050,505$, and the product of the two numbers $303,030,303,030,303,030,303$ and $505,050,505,050,505,050,505$.

These two products certainly agree on their final four digits. What is the first digit from the right at which the two products disagree?

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