Curriculum Burst 125: Circle Graphs

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> Amanda Reckonwith draws five circles with radii $1, 2, 3, 4$, and 5. Then for each circle she plots the point (C, A), where C is its circumference and A is its area. Which of the following could be her graph?
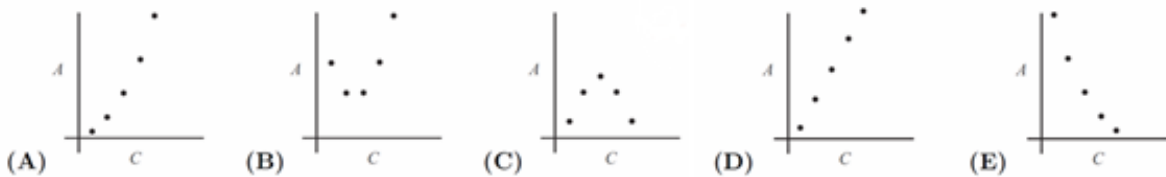
QUICK STATS:

MAA AMC GRADE LEVEL
This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS:
Geometry: circle formulas. Functions: graphing.

COMMON CORE STATE STANDARDS
7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

MATHEMATICAL PRACTICE STANDARDS
MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY
ESSAY 2:   **DO SOMETHING!**

SOURCE: This is question #16 from the 2007 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question is weird! What exactly are we plotting? As I read though the question I am confused.

Reading the question again:

Amanda Reckonwith draws five circles with radii 1, 2, 3, 4, and 5.

Okay, Amanda draws five circles, one of each of the radii mentioned.

Then for each circle she plots the point (C, A), where C is its circumference and A is its area.

This is the part that is confusing.

This sentence mentions the circumference of each circle and the area of each circle. I know, in general, that $C = 2\pi r$ and $A = \pi r^2$. I can work these out for each circle:

	C	A
r=1	2π	π
r=2	4π	4π
r=3	6π	9π
r=4	8π	16π
r=5	10π	25π

So I guess we have the points $(2\pi, \pi)$, $(4\pi, 4\pi)$, $(6\pi, 9\pi)$, and so on. Hmm. These are weird points.

Which of the following could be her graph?

I don't know!

Hmm.

I could actually plot the points myself! Since π is close-ish to the number 3, let me just use three for π just to get a sense of what the points are. We have the points:

$(6,3), (12,12), (18,27), (24,48),$ and $(30,75),$ approximately.

Ugh! These are hard to plot!

But look at the first coordinates of these points: $6, 12, 18, 24, 30$ (or really, $2\pi, 4\pi, 6\pi, 8\pi, 10\pi$). They are increasing by a constant amount. And the second coordinates are getting larger and larger. Okay, the graph is at least increasing (that is, going up). Options (B), (C), and (E) are out.

Also, the second coordinates are not going up by a steady amount: $3, 12, 27, 48, 75$ (or really, $\pi, 4\pi, 9\pi, 16\pi, 25\pi$). The graph is not a straight line graph. That means option (D) is out and the answer must be (A).

Extension: Justin Case is wondering if there a formula for the area A of a circle in terms, not of π and the circle radius r, but instead in terms of π and the circle circumference C. Is there one? If so, does that formula match the graph given in option (A)?