

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 122: One Zero

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How many whole numbers between 99 and 999 contain exactly one 0?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Statistics and Probability (counting)

COMMON CORE STATE STANDARDS

7.SP.8b (The mathematics behind ...) Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 9: [AVOID HARD WORK](#)

SOURCE: This is question # 19 from the 2002 MAA AMC 8 Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Hmm. We're looking for numbers between 99 and 999 that contain exactly one zero. That is, we're looking how many numbers in the list:

100, 101, 102, ..., 997, 998

that, like 101 and 102, contain exactly one zero.

Well, I could try to list them all:

101, 102, 103, ..., 109, 110, 120, 130, ...

but that seems hard! How can I avoid hard work?

Well, all my numbers are three-digits long. (Is that helpful to notice?)

To have exactly one zero the number must either be of the form $X0Y$ or of the form $XY0$, with X and Y each some non-zero digit.

Oh, there are nine possible options for X (one of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9) and nine possible options for Y . This means there are $9 \times 9 = 81$ numbers of the form $X0Y$ and $9 \times 9 = 81$ numbers of the form $XY0$. That makes for $81 + 81 = 162$ three-digit numbers containing exactly one zero. Done!

Extension: a) How many five-digit numbers are there that contain exactly two zeros?

b) How many five-digit numbers are there that contain exactly two zeros and exactly one one?

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