Curriculum Burst 120: A Sailing Ship

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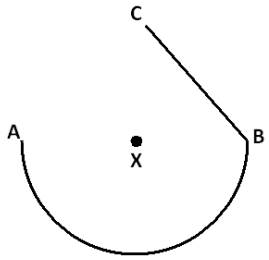
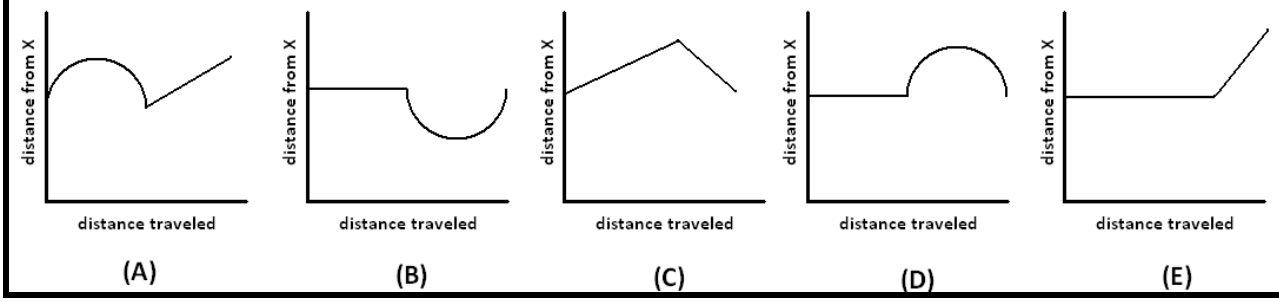
A ship travels from point A to point B along a semicircular path, centered at Island X. Then it travels along a straight path from B to C. Which of these graphs best shows the ship's distance from Island X as it moves along its course?





QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS:

Graphs of functions; Geometry: circles.

COMMON CORE STATE STANDARDS

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.

PROBLEM SOLVING STRATEGY

ESSAY 6: ELIMINATE INCORRECT CHOICES

SOURCE:

This is question # 24 from the 2003 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Okay. I need to get a feel for this question.

The ship moves along a funny path and we're drawing a graph its distance from the point X as it goes along. (The height of the graph at a particular point on its journey corresponds to its distance from the point X .)

Well … Since the ship is moving along that circular arc to start off with, its distance from X does not change for a while. This means that the matching graph should be a constant steady value for its beginning. Graphs B, D, and E start off this way. The answer must be one of those options.

What next?

After the ship reaches point B, it actually starts to move closer to point X. That is, its distance gets smaller. Only graph B shows the distance going down right after the period the distance was constant for a while.

Graph B must be the answer!

Extension: Graph B suggests that the ship gets to some point fairly close to X and then the distance increases again. Does that make sense in terms of the path the ship is following?

Graph B also suggests that its second part is in the shape of a semi-circle. Is it a true semicircle? If a ship travels by a point X along a straight path, would the graph of its distance from X indeed be the arc of perfect circle?

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