Curriculum Inspirations

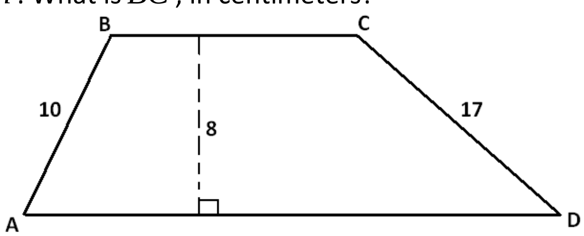
Inspiring students with rich content from the MAA American Mathematics Competitions

MAA

Curriculum Burst 118: A Trapezoid Area

By Dr. James Tanton, MAA Mathematician in Residence

The area of trapezoid $ABCD$ is $164\ cm^2$. The altitude is $8\ cm$, AB is $10\ cm$, and CD is $17\ cm$. What is BC, in centimeters?



QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry

COMMON CORE STATE STANDARDS

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 2: **DO SOMETHING**

SOURCE: This is question # 21 from the 2003 MAA AMC 8 Competition.

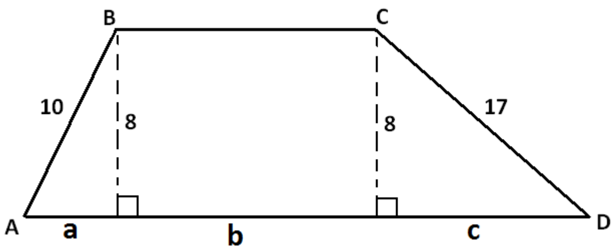
THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question has the feel of a typical textbook geometry question. I wonder if it is as straight-forward to solve as a textbook question?

I can't help but think to redraw the altitude – twice. The desire to create right triangles is strong!



Now the pull to the Pythagorean is strong! (I wonder if any of these pulls will prove to be helpful?)

Using the notation I added to the picture we have:

$$a^2 + 8^2 = 10^2$$

giving $a^2 = 100 - 64 = 36$ and so $a = 6$, and

$$c^2 = 17^2 - 8^2 = 289 - 64 = 225$$

giving $c = 15$.

Okay. Now what?

The question would like the length BC , which is the same as my b . That's the only length we haven't figured out! Hmm.

But we haven't used that fact that the area of the trapezoid is 164 square centimeters. Oh! The areas of the two right triangles and the central rectangles sum to 164 :

$$\frac{1}{2}\cdot 6\cdot 8 + b\cdot 8 + \frac{1}{2}\cdot 15\cdot 8 = 164$$
$$24 + 8b + 60 = 164$$
$$8b = 80$$
$$b = 10$$

Fabulous! The length BC is ten centimeters.

Extension: This question was clever in its construction: it used the fact that the number 8 appears as a side of two different integer right triangles, the $6-8-10$ right triangle and the $8-15-17$ right triangle.

a) Does the number 8 appear as the side of yet another integer right triangle?
b) Can you find other examples integers that appear as a side of two or more integer right triangles?

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