Curriculum Inspirations

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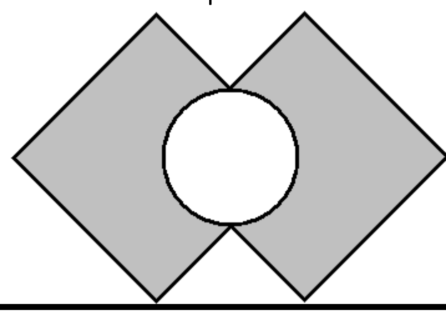
MAA

Curriculum Burst 115: Overlapping Squares

By Dr. James Tanton, MAA Mathematician in Residence

Two 4×4 squares intersect at right angles, bisecting their intersecting sides, as shown. The circle's diameter is the segment between the two points of intersection. What is the area of the shaded region created by removing the circle from the squares?



QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry: Area of a Circle; The Pythagorean Theorem

COMMON CORE STATE STANDARDS

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 5: SOLVE SMALLER VERSION OF SAME PROBLEM

SOURCE: This is question # 25 from the 2004 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

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STEP 1: Read the question, have an
emotional reaction to it, take a deep
breath, and then reread the question.
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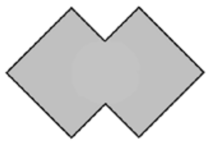
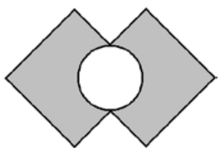
This question feels a little complicated, but I see it really breaks into two parts:

1. Find the area of two intersecting squares:



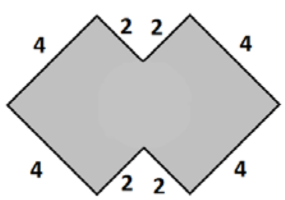
2. Take away the area of a circle:



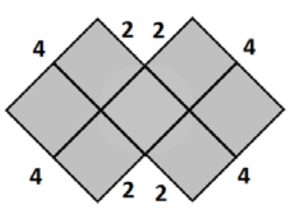
These both feel manageable.

Okay. What are the dimension of these shapes?

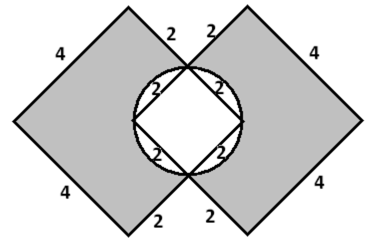
In rereading the question, each square has side-length 4 and they interest in a way that "bisects" the sides. I can't quite remember what "bisects" means (two-sections?) but I bet we're talking about sides cut in half. We must have:



I see that this figure divides into seven 2×2 squares and so has area $7 \times 4 = 28$.

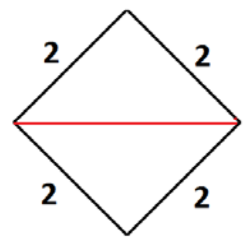


Now we need to take away the area of an inner circle. I am going to trust my intuition here and assume the circle we're taking away circumscribes the inner square. (I think I am right about this. Am I?)



The area of a circle is πr^2. I need the radius of this circle. Hmm.

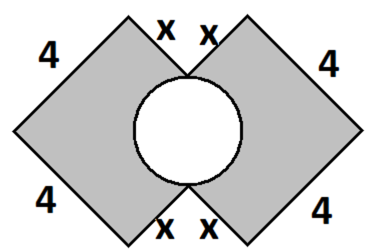
Well I see that the diagonal of the square is the diameter of the circle.



By the Pythagorean Theorem, this diamter is $\sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$. The radius of the circle is half of this and so the area of the circle is $\pi\left(\sqrt{2}\right)^2 = 2\pi$. Great!

The area of the original figure in the question is thus $28 - 2\pi$.

Extension: Suppose the two squares have sides that intersect at a length x as shown. What is the area, in terms of x, of the overlapping squares with a circle removed?



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