Curriculum Inspirations

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MAA

Curriculum Burst 114: A Tricky Length

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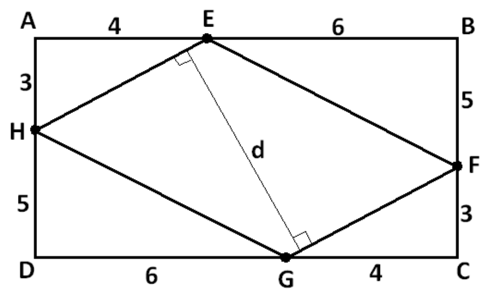
In the figure, $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?



QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry: Areas of rectangles and parallelograms.

COMMON CORE STATE STANDARDS

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 1: ENGAGE IN SUCCESSFUL FLAILING

SOURCE: This is question # 20 from the 2004 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

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STEP 1: Read the question, have an
emotional reaction to it, take a deep
breath, and then reread the question.
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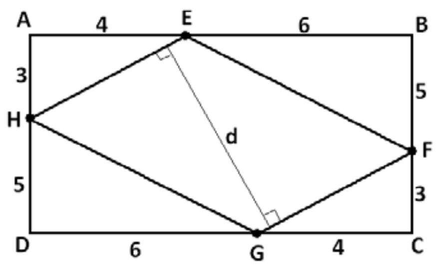
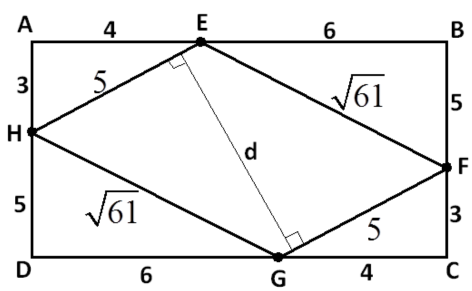
Hmm. This question feels a bit tricky. I see lots of triangles in the picture, but that length d is sitting in a strange position. It is not immediately obvious to me how I am going to figure out that length.



Well, I do see right triangles. By the Pythagorean theorem I see the lengths $\sqrt{3^2+4^2}=\sqrt{25}=5$ and $\sqrt{6^2+5^2}=\sqrt{61}$. I can write in those measurements.



Hmm. I am not sure if that was helpful.

What else can I do? Now I am groping for ideas.

The big rectangle is 10 units wide and 8 units high. It has area 80 square units. Helpful?

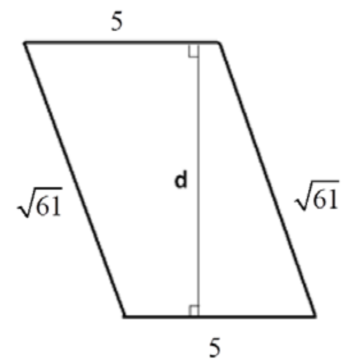
The $3-4-5$ right triangles in the corners each have area $\frac{1}{2}\cdot 3\cdot 4=6$ square units. The other two right triangles each have area $\frac{1}{2}\cdot 5\cdot 6=15$ square units. (I have no idea where this is going!)

The area of the middle parallelogram must be:

$$80-6-6-15-15=38 \text{ square units.}$$

Alright, the middle parallelogram has side of lengths 5 and $\sqrt{61}$, and has area 38 . Surely that tells me something helpful!

Oh … turn the picture of the parallelogram this way:

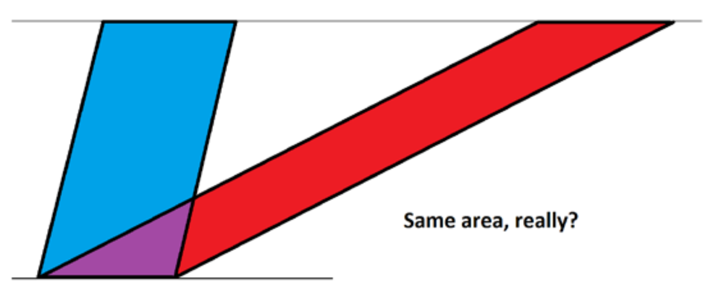


We see that d is the height of the parallelogram, and 5 is the base. The area of a parallelogram is "base times height" so we must have:

$$5\times d=38$$

giving $d=\frac{38}{5}=\frac{76}{10}=7.6$. That's it! (And we didn't need the number $\sqrt{61}$ after all.)

Extension: Is it obvious that if two parallelograms have the same base and the same height they are sure to have the same area? (The formula "area = base times height" says it will be so. But should I believe it?)



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