Curriculum Burst 110: Overlapping Circle and Square

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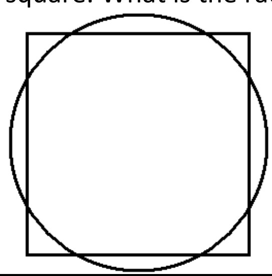
A square with side length 2 and a circle share the same center. The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square. What is the radius of the circle?



QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry: Area of a circle

COMMON CORE STATE STANDARDS

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 9: **AVOID HARD WORK**

SOURCE: This is question # 25 from the 2005 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

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STEP 1: Read the question, have an
emotional reaction to it, take a deep
breath, and then reread the question.
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This question is hard to understand!

A square with side length 2 *and a circle share the same center.*

That's fine.

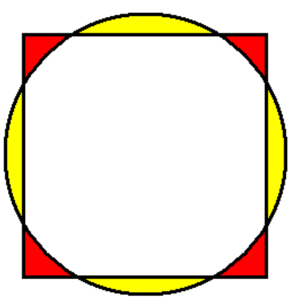
The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square.

That's confusing!

Deep breath!

The area of the regions that are inside the circle and outside the square …
The area of the regions that are outside the circle and inside the square …

Okay, I think I have it! The first description, the areas inside the circle and outside the square, are the regions in yellow. The second description, the regions inside the square but outside the circle, are the ones in red.



Okay. So what are we doing?

The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square.

Okay. The yellow areas equal the red areas!

What is the radius of the circle?

Oh dear. Just when I was feeling like I was getting somewhere I am asked a question that seems impossible!

What can I do?

Well we were told that the square has side length 2, and hence area 4. Can I work out the radius of the circle? Can I work out the area of the middle white region, and then subtract that from 4 and get the areas of the red, and hence the areas of the yellows, and then do something to get the radius of the circle? That seems complicated!

Hmm. I need to just stare at the picture.

The area of the square, the white plus the red, is 4.
The area of the circle, the white plus the yellow, is πr^2, but I don't know what r is.

Hmm.

$$white + red = 4$$

$$white + yellow = \pi r^2$$

Oh. But $red = yellow$. This means the two equations are the same!

$$4 = \pi r^2$$

And so

$$r^2 = \frac{4}{\pi},$$

giving $r = \frac{2}{\sqrt{\pi}}$. A nasty number, but there it is!

Extension: Did this question need the circle and the square to have the same center?

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