Curriculum Burst 108: A Triangle Area

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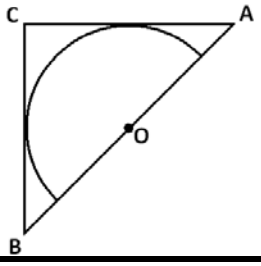
Isosceles right triangle ABC encloses a semicircle of area 2π. The circle has its center O on hypotenuse AB and is tangent to sides $\overline{AC}$ and $\overline{BC}$. What is the area of triangle ABC?



QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry

COMMON CORE STATE STANDARDS

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 3: ENGAGE IN WISHFUL THINKING

SOURCE: This is question # 23 from the 2005 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

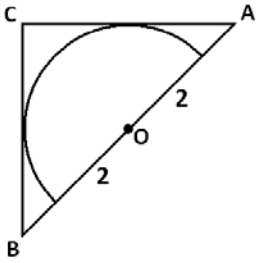
STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Okay. I can think of something relevant I can say right away.

The area of a circle is πr^2, so the area of a semicircle is $\frac{1}{2}\pi r^2$. We need:

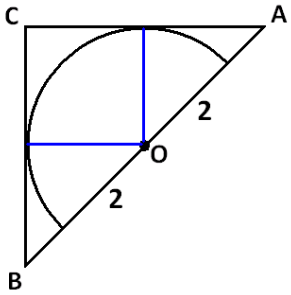
$$\frac{1}{2}\pi r^2 = 2\pi .$$

This means $\pi r^2 = 4\pi$ and so $r = 2$. The radius of the circle is 2.

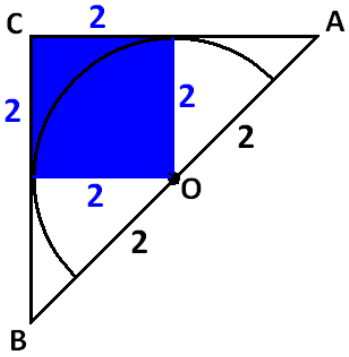


That felt good. Now what?

I don't know why but I want to draw in two more lines.



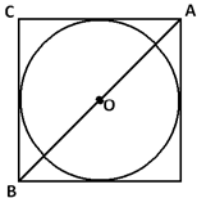
Oh. They are radii and so have length 2 as well. I bet we have just made a square in the corner of the figure.



Oh. It looks like triangle ABC is made of a square (of area 4) and two triangles that, together, make another copy of that square. If this is true, then the area of ΔABC is $4+4=8$ and we're done!

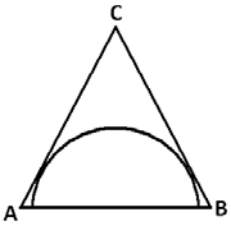
Extension 1: Is any of this wishful thinking true? Is the blue region actually a square? Is each small triangle indeed half that square? What results from geometry class do we need to – hopefully – prove all that we wish for is true?

Extension 2: When Angie answered this question she reflected the figure across the diagonal line.



Does this give another way to think about the question?

Extension 3: An equilateral triangle encloses a semicircle as shown.



If the area of the semicircle is 2π, what is the area of the triangle?

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