Curriculum Burst 106: Circle Moves

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Alice and Bob play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise, from 1 to 12. Both start on point 12. Alice moves clockwise and Bob, counterclockwise. In a turn of the game, Alice moves 5 points clockwise and Bob moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take?

(A) 6 (B) 8 (C) 12 (D) 14 (E) 24

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QUICK STATS:

MAA AMC GRADE LEVEL
This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS
The Number System

COMMON CORE STATE STANDARDS
Connected to …
6.NS.4   Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36+8 as 4(9+2).

MATHEMATICAL PRACTICE STANDARDS
MP1   Make sense of problems and persevere in solving them.
MP2   Reason abstractly and quantitatively.
MP3   Construct viable arguments and critique the reasoning of others.
MP7   Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 9:   **AVOID HARD WORK**

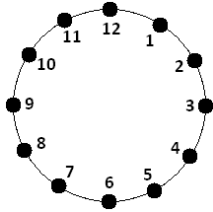**SOURCE:**   This is question # 20 from the 2005 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

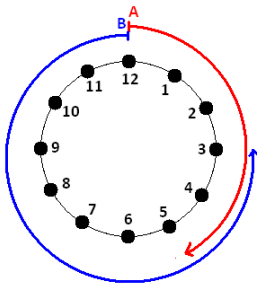The best, and most appropriate, first step is always …

> **STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

For starters, I think this question is basically about a clock.

(It feels better just to call this circle divided into twelve parts a clock!)

Alice and Bob both start as "12" and at each turn of the game, Alice moves clockwise five places, and Bob counterclockwise nine places. That's complicated!

Let me try out a few moves.

After the first turn Alice is at 5 and Bob is at 3.

After the second turn:
  Alice $= 10$
  Bob $= 6$
(Yep! It is hard counting Bob's backward moves.)

After the third turn:
  Alice $= 3$
  Bob $= 9$

And so on.

Hang on! Bob is going "up three" each time. And that makes sense: moving 9 places counterclockwise is the same as moving 3 places clockwise!

Okay, the problem could just as well have been:

> *At each turn Alice moves 5 places clockwise and Bob moves 3 places clockwise.*

That feels less complicated!

What was the question?

> *The game ends when they stop on the same point. How many turns will this take?*

Well, here are the numbers Alice lands on in turn:

$$12, 5, 10, 3, 8, 1, 6, 11, 4, ...$$

Here are the numbers Bob lands on:

$$12, 3, 6, 9, 12, 3, 6, 9, 12, 3, 6, ...$$

Let's line them up:

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12  5  10  3  8   1  (6) 11  4 ...
12  3   6  9  12  3  (6)  9  12 ...
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They both land on the number "6" on the sixth turn. The answer is (A).

> **Extension 1:** What number will Bob be on after 100 turns of the game? What number will Alice be on?

> **Extension 2:** At each turn of the game Alice moves 5 places clockwise, Bob 3 places clockwise, Charles 7 places, Dwayne 6 places, and Edwina 10 places. If they all start at position 12, on what number will the players be when next all five land in the same location?

Curriculum Inspirations is brought to you by the Mathematical Association of America and the MAA American Mathematics Competitions.