Curriculum Burst 105: Trapezoidal Perimeter

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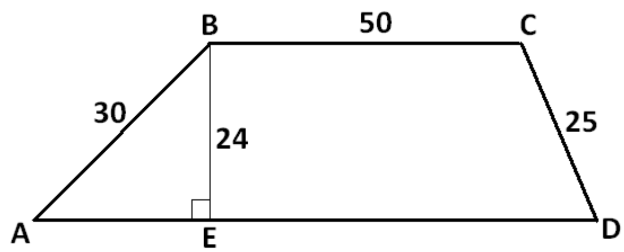
What is the perimeter of trapezoid $ABCD$?



QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry: the Pythagorean Theorem

COMMON CORE STATE STANDARDS

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 1: ENGAGE IN SUCCESSFUL FLAILING

SOURCE: This is question # 19 from the 2005 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

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STEP 1: Read the question, have an
emotional reaction to it, take a deep
breath, and then reread the question.
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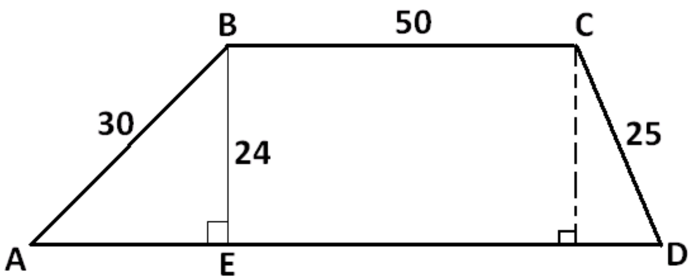
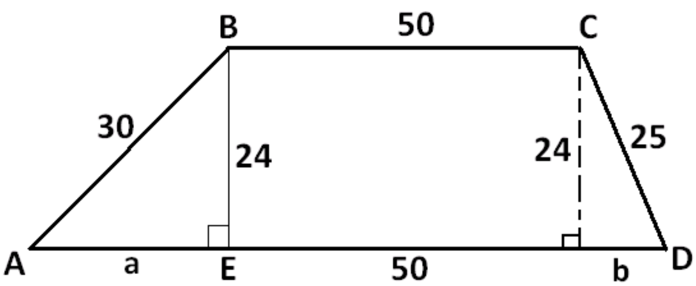
I see a right triangle in a picture. I not sure why, but I feel the compelled to draw in a second vertical line and create a second right triangle.



I can label some lengths. In fact, since there is a rectangle in the picture, there are only two lengths I don't know. I've called them a and b.



Well … now it feels compelling to use the Pythagorean Theorem, twice, once for the left right-triangle and once for the right right-triangle.

$$a^2 + 24^2 = 30^2$$
$$a^2 + 576 = 900$$
$$a^2 = 324$$
$$a = 18$$

(Oh! 30 is 5 times six, 24 is 4 times six. This is basically a $3-4-5$ triangle, so the answer had to be $3 \times 6 = 18$. Noticing that first would have saved me some arithmetic!)

$$b^2 + 24^2 = 25^2$$
$$b^2 + 576 = 625$$
$$b^2 = 49$$
$$b = 7$$

(Is $7-24-25$ a standard example of a right triangle?)

Umm. What was the question?

What is the perimeter of trapezoid $ABCD$?

Okay. That's $30+50+25+7+50+18=180$.

Cool!

Extension: The following video shows how to use an ordinary multiplication table to find triples of numbers that satisfy the famous relation $a^2 + b^2 = c^2$. Can you see why the method works? (Can you explain it before the answer is given away in the video?)

http://www.jamestanton.com/?p=628

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