Curriculum Burst 103: Counting Coins

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A box contains gold coins. If the coins are equally divided among six people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meets these two conditions, how many coins are left when equally divided among seven people?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Number Sense

COMMON CORE STATE STANDARDS

No particular standard.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 1: **ENGAGE IN SUCCESSFUL FLAILING**

SOURCE: This is question # 23 from the 2006 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I am going to read the question yet another time … slowly.

A box contains gold coins. If the coins are equally divided among six people, four coins are left over.

So this means the number of coins is four more than a multiple of six. The multiples of six are $6, 12, 18, 24, 30, \ldots$. The number of the coins in the box is thus one of the numbers:

$$10, 16, 22, 28, 34, \ldots$$

If the coins are equally divided among five people, three coins are left over.

The multiples of five are $5, 10, 15, 20, 25, 30, \ldots$ and so the number of coins in the box is also one of the numbers:

$$8, 13, 18, 23, 28, 33, \ldots$$

If the box holds the smallest number of coins that meets these two conditions, how many coins are left when equally divided among seven people?

Okay. I see that "28" is a common number in this list. It is the smallest number in common, so there must be 28 coins in the box. And when we divide them among seven people, 0 coins will be left over. Done! (Wow!)

Extension 1: What is the second-to-smallest number that could be the count of coins in the box? The third-to-smallest? The fourth-to-smallest? Is there anything to be said about these numbers?

Extension 2: Mable, in answering this question, wrote on her piece of paper:

$$6n + 4 = 5m + 3.$$

What do you think she means by this?

Mable reasoned that m must be an odd number. Why must this be the case?

What is the smallest odd number m could be?

Extension 3: Alistair, after reading this question, said:

"Imagine I added two coins to the box. Then no coins would be left over when sharing the loot among six people. Also, no coins would remain when we divvying up the loot five people."

How did this observation lead Alistair to conclude that there are 28 coins in the box?

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