

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 101: Running Lemming

By Dr. James Tanton, MAA Mathematician in Residence

A lemming sits at a corner of a square with side length 10 meters. The lemming runs 6.2 meters along a diagonal toward the opposite corner. It stops, makes a 90° right turn and runs 2 more meters. A scientist measures the shortest distance between the lemming and each side of the square. What is the average of these four distances in meters?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry, Averages

COMMON CORE STATE STANDARDS

6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

MATHEMATICAL PRACTICE STANDARDS

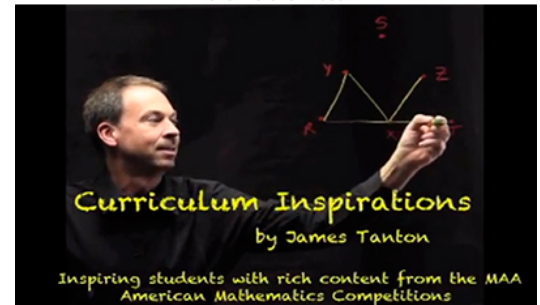
- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 9: [AVOID HARD WORK](#)

SOURCE: This is question # 22 from the 2007 MAA AMC 8 Competition.

[Click here for video](#)



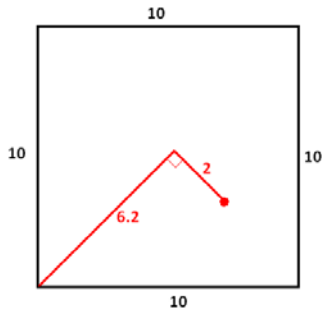
THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question looks hard!

Let me draw a picture of the square and the motion of the lemming.



Does it matter in which corner the lemming starts? I guess all the pictures from different starting corners would basically be the same. And since the question didn't mention a starting corner it probably doesn't matter.

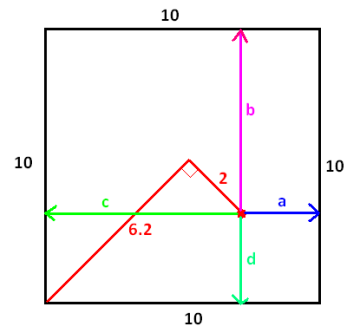
Okay. So I need to figure out where the lemming is now – how many meters it sits from each side of the square.

Ohh! That seems really hard!

What does the question want?

A scientist measures the shortest distance between the lemming and each side of the square. What is the average of these four distances in meters?

Let me draw in those four distances the scientist is actually measuring.



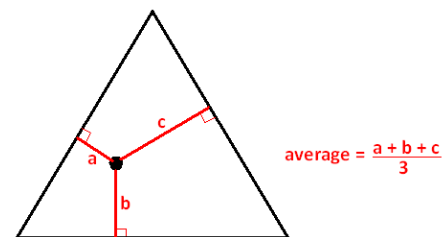
We want the average of the four distances I've just marked: $\frac{a + b + c + d}{4}$.

Working out the values of a , b , c , and d still seems too hard!

BUT WAIT! $a + c$ is 10, this is a side of the square. And $b + d = 10$ as well. So $a + b + c + d = 10 + 10 = 20$ and the average is $\frac{20}{4}$, which is 5 meters! Whoa!

Extension 1: We should check: Does the answer change if the lemming started in a different corner? In fact, does the answer change if the lemming did an entirely different set of motions and ended up at an entirely different point in the square?

Extension 2: A point is chosen at random inside an equilateral triangle. The shortest distances from it to each of the sides are measured and their average value is computed.



Explain why all points give the same average value!

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