Curriculum Burst 100: Matching Cards

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> Two cards are dealt from a deck of four red cards labeled $A\,;B\,;C\,;D$ and four green cards labeled $A\,;B\,;C\,;D$. A winning pair is two of the same color or two of the same letter. What is the probability of drawing a winning pair?
>
> (A) $\dfrac{2}{7}$ (B) $\dfrac{3}{8}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{4}{7}$ (E) $\dfrac{5}{8}$

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QUICK STATS:

MAA AMC GRADE LEVEL
This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS
Probability

COMMON CORE STATE STANDARDS
7.SP.5   Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.8   Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
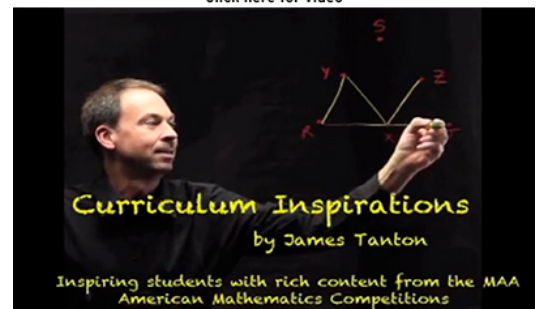
MATHEMATICAL PRACTICE STANDARDS
MP1   Make sense of problems and persevere in solving them.
MP2   Reason abstractly and quantitatively.
MP3   Construct viable arguments and critique the reasoning of others.
MP7   Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 6:   **ELIMINATE INCORRECT CHOICES**

SOURCE: This is question # 21 from the 2007 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

> **STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Probability questions always confuse me, especially when you are meant to do things simultaneously: roll three dice at the same time, or pick five objects at the same time. I find it much simpler to think of doing tasks one at a time.

In this question we are meant to pick two cards. Is there a problem with this question if, instead, we pick one card first, look at it, and then pick a second card? Does choosing the cards one at a time change the question being asked?

No! As long as I have a pair of cards, the question doesn't care how I obtained them, in what order I got them, or how I hold them in my hand.

So let's pick a card. It will have a color and it will have a label.

What do I want for a "win"? I want another card of the same color or of the same label.

Well, there are seven cards left, so the answer is going to be some fraction out of seven. The answer is either (A) or (D).

In fact, no matter what card I first draw, there is one card left of the same label and three cards left of the same color. That is, four out of seven of the remaining cards give me a win. The answer must be (D).

> **Extension 1:** I happen to be holding, right now, a winning pair in my hand. I tell you that one of my cards is red. What is the probability that the other card is also red?

> **Extension 2:** What is the probability of selecting three cards so that each pair of cards amongst the three either has the same color or the same label?

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