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MAA American Mathematics Competitions



Curriculum Burst 97: Avid Reading

By Dr. James Tanton, MAA Mathematician in Residence

Hui is an avid reader. She bought a copy of the best-seller Math is Beautiful. On the first day, Hui read $\frac{1}{5}$ of the pages plus 12 more, and on the second day, she read $\frac{1}{4}$ of the remaining

pages, plus 15 pages. On the third day, she read $\frac{1}{3}$ of the remaining pages, plus 18 pages.

She then realized that there were only 62 pages left to read, which she read the next day.

How many pages are in the book?

(A) 120

(B) 180

(C) 240

(D) 300

(E) 360

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Algebra: Solving linear equations

COMMON CORE STATE STANDARDS

8.EE.7 Solve linear equations in one variable.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others. MP3

Look for and make use of structure. MP7

PROBLEM SOLVING STRATEGY

ESSAY 6: **ELIMINATE INCORRECT CHOICES**

SOURCE: This is question # 25 from the 2011 MAA AMC 10B Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Here's my reaction: Who reads books this way? Who keeps track of pages read in this manner? And why do we want to know the number of pages in the book? Just look at the last page and see what page number it is! (And Hui knows what the page number if because on day one she knows she read one-fifth of the pages!)

I am not sure if a question like this adds to the idea that "math is beautiful."

Okay ... I am over my outburst.

These reading instructions seem complicated. Since I am given five answers to choose from, why not just try those numbers and see if they work?

OPTION (A): There are 120 pages.

If this is the case, then Hui reads

$$\frac{1}{5}120 + 12 = 24 + 12 = 36$$

pages on the first day, leaving 120-36=84 pages for day two. I bet this isn't right because the rest of the question still has Hui reading 15 and 18 and 62 more pages, which is more than 84. Option (A) is out.

OPTION (B): There are 180 pages.

In this case Hui reads 36+12=48 pages on the first day, leaving 180-48=132 pages for day two.

On day two, Hui reads $\frac{1}{4}132 + 15 = 48$ pages, leaving 132 - 48 = 74 pages for day three.

This can't be right as she still has at least 18 and 62 pages left to read. Option (B) is out.

OPTION (C): There are 240 pages.

On day one Hui reads $\frac{1}{5}240+12=60$ pages, leaving 240-60=180 pages for day two.

On day two Hui reads $\frac{1}{4}180 + 15 = 60$ pages, leaving 180 - 60 = 120 pages for day three.

On day three Hui reads $\frac{1}{3}120+18=62\,$ pages, which matches what the question wants!

The answer is option (C).

Extension: When Lucas answered this question he wrote the following:

$$D_{1} - \left(\frac{1}{5}D_{1} + 12\right) = D_{2}$$

$$D_{2} - \left(\frac{1}{4}D_{2} + 15\right) = D_{3}$$

$$D_{3} - \left(\frac{1}{3}D_{3} + 18\right) = 62$$

What do you think he is doing? What do $D_{\rm 1}$, $D_{\rm 2}$, and $D_{\rm 3}$ represent? Do the equations make sense?

Can you solve these equations and find values for $\,D_{\!_{1}}$, $\,D_{\!_{2}}$, and $\,D_{\!_{3}}$?

Curriculum Inspirations is brought to you by the Mathematical Association of America and the MAA American Mathematics Competitions.



MAA acknowledges with gratitude the generous contributions of the following donors to the Curriculum Inspirations Project:

The TBL and Akamai Foundations for providing continuing support

The Mary P. Dolciani Halloran Foundation for providing seed funding by supporting the Dolciani Visiting Mathematician Program during fall 2012

MathWorks for its support at the Winner's Circle Level

