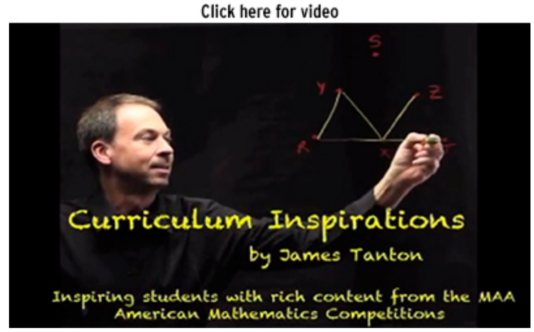
Curriculum Burst 81: Renumbered Dice

By Dr. James Tanton, MAA Mathematician in Residence

> Two cubical dice each have removable numbers 1 through 6. The twelve numbers on the two dice are removed, put into a bag, then drawn one at a time and randomly reattached to the faces of the cubes, one number to each face. The dice are then rolled and the numbers on the two top faces are added. What is the probability that the sum is 7?

QUICK STATS:

Click here for video



MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grade levels.

MATHEMATICAL TOPICS

Probability

COMMON CORE STATE STANDARDS

S-CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 9: [AVOID HARD WORK]

SOURCE: This is question # 22 from the 2009 MAA AMC 10A Competition.

THE PROBLEM-SOLVING PROCESS:

As always, the best start is …

```
STEP 1: Read the question, have an
emotional reaction to it, take a deep
breath, and then reread the question.
```

Oh heavens. Must I consider all the possible ways that the dice could be renumbered and work out the probability of rolling "seven" with each and every one of them? Surely not!

But what else can I do?

It is time to put my feet up on the table, stare at the ceiling and mull on this one. There has to be a much easier way to approach this problem.

What are we really doing in this question?

There are twelve stickers, numbered $1, 1, 2, 2, 3, 3, \ldots, 6, 6$.

We pull them out one at a time and place them on the faces of the two cubes.

We roll the two new dice and look at the top two numbers.

Hmm.

We want to know if those two numbers sum to a seven.

So, really, in the end, we're only ever looking at two of the stickers and asking: *What are the chances that those two sticker-numbers sum to seven?*

So the problem might as well be: *Two stickers are chosen at random. What is the probability their two numbers sum to seven?*

Okay. That's much, much easier – but it is still a little bit tricky to think through.

So we pull out a sticker and get a number. We pull out a second sticker and get a second number. We hope this second number is the right one to get a sum of seven.

Oh! If the first sticker pulled is 5, say, then I need to pull one of the two 2s from the remaining eleven stickers. Or if the first sticker is 1, then I need to pull one of the two 6s. No matter the number of the first sticker pulled, there are two out of the eleven of the remaining stickers that give a sum of seven.

Wow! That's it. The chance of seeing a sum of seven is $\frac{2}{11}$.

Extension 1: What are the chances of seeing a sum of six?

Extension 2: Octavia placed the twelve stickers back on the two cubes so as to produce two dice that have the same chances of rolling any particular sum as ordinary dice. (The probability of rolling a seven with ordinary dice is $6/36$. The same is true for Octavia's dice. The probability of rolling a twelve with ordinary dice is $1/36$. The same is true for Octavia's dice. And so on.) Must each of Octavia's dice have each of the numbers 1 through 6 on it?

MAA acknowledges with gratitude the generous contributions of the following donors to the Curriculum Inspirations Project:

The TBL and Akamai Foundations
for providing continuing support

The Mary P. Dolciani Halloran Foundation for providing seed funding by supporting the Dolciani Visiting Mathematician Program during fall 2012

MathWorks for its support at the Winner's Circle Level