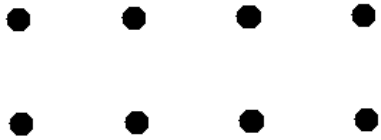
Curriculum Burst 77: Counting Non-Congruent Triangles

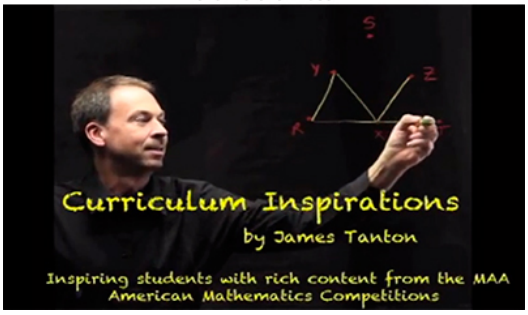
By Dr. James Tanton, MAA Mathematician in Residence

How many non-congruent triangles have vertices at three of the eight points in the array shown below?



QUICK STATS:

Click here for video



MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry; Counting methods

COMMON CORE STATE STANDARDS

8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 7: **PERSEVERANCE IS KEY**

SOURCE: This is question # 20 from the 2009 MAA AMC 8 Competition.

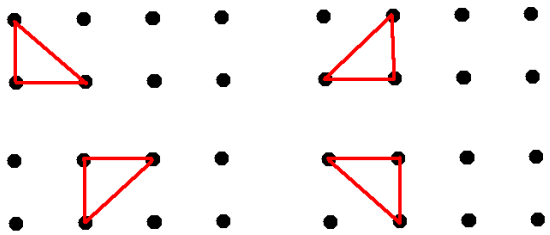
THE PROBLEM-SOLVING PROCESS:

As always, the best start is …

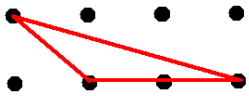
STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I feel like I can get started on this question, well, at least get a feel for it by drawing some examples of the triangles.

Here are four congruent examples:



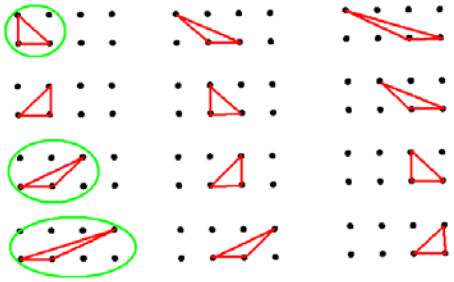
Here's an example not congruent to any of these:



We are being asked to count the number of different non-congruent triangles we can make.

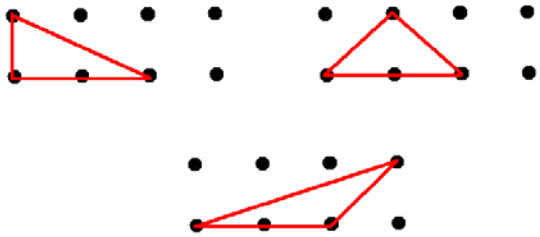
Since a reflection of any triangle gives a congruent triangle we might as well assume each triangle has a base along the bottom row of dots.

Here are all the triangles we can make with a base one unit long sitting on the bottom row. We see there are only 3 non-congruent triangles of this type.



(I guess I didn't really need to draw all of these to see this!)

Of all the triangles with a base of length two, there are 3 non-congruent types:



Of all with a base of length three, there are 2 non-congruent types:

That gives a total of $3+3+2=8$ non-congruent types!

Extension 1: How many triangles, congruent and non-congruent, in total can one draw on two rows of four dots? On two rows of N dots?

Extension 2: How many non-congruent triangles can one draw on a 4×4 square array of dots?

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