Curriculum Burst 76: Row and Columns Sums

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> Andy and Bethany have a rectangular array of numbers with 40 rows and 75 columns. Andy adds the numbers in each row. The average of his 40 sums is A. Bethany adds the numbers in each column. The average of her 75 sums is B. What is the value of $\dfrac{A}{B}$?

QUICK STATS:

MAA AMC GRADE LEVEL
This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS
Counting methods

COMMON CORE STATE STANDARDS
8.EE.8c   Solve real-world and mathematical problems leading to two linear equations in two variables.

MATHEMATICAL PRACTICE STANDARDS

MP1	Make sense of problems and persevere in solving them.
MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.
MP7	Look for and make use of structure.

PROBLEM SOLVING STRATEGY
ESSAY 5:   **SOLVE A SMALLER VERSION OF THE SAME PROBLEM**

SOURCE: This is question # 21 from the 2009 MAA AMC 8 Competition.
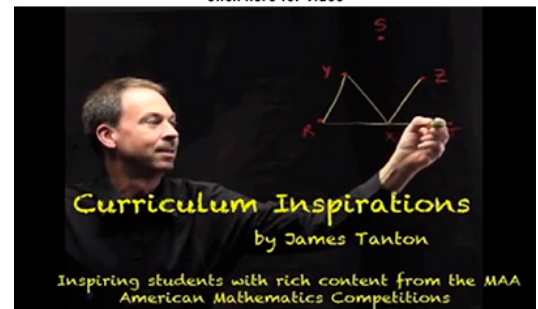
THE PROBLEM-SOLVING PROCESS:

As always, the best start is …

> **STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question is confusing - and hard. There are a lot of numbers in a 40×75 grid and adding all the numbers in each row or each column are big sums in and of themselves. There are too many big sums!

Let me just try the problem with a smaller example, say, with a 2×3 rectangle of numbers.

a	b	c
d	e	f

Andy adds the rows and averages his answers to get an average sum of A.

$$A = \frac{(a+b+c)+(d+e+f)}{2}$$

Bethany sums each column and averages the sums to get B.

$$B = \frac{(a+d)+(b+e)+(c+f)}{3}.$$

Okay. I see that each person gets a numerator which is the sum of <u>all</u> the entries in the table. The denominator is the number of rows or the number of columns. Notice:

$$a+b+c+d+e+f = 2A$$
$$a+b+c+d+e+f = 3B$$

For the 40×75 case, Andy has a denominator of 40 and Bethany a denominator of 75, so we'd have:

ANDY: Sum of all entries $= 40A$
BETHANY: Sum of all entries $= 75B$

I've forgotten what the question wants.

What is the value of $\dfrac{A}{B}$?

Alright, let's divide the two equations we have:

$$\frac{sum\,of\,all}{sum\,of\,all} = \frac{40A}{75B}$$

That is,

$$1 = \frac{40A}{75B}.$$

Thus $\dfrac{A}{B} = \dfrac{75}{40}$. Phew!

Extension: A rectangular array of numbers is *semi-magic* if each row of numbers and each column of numbers all sum to the same value. (A square array is *fully* magic if its two diagonals sum to this common value too.) Find an example of a non-square semi-magic rectangle of numbers.

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