Curriculum Burst 74: Counting Jelly Beans

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> On the last day of school, Mrs. Wonderful gave jelly beans to her class. She gave each boy as many jelly beans as there were boys in the class. She gave each girl as many jelly beans as there were girls in the class. She brought 400 jelly beans, and when she finished, she had six jelly beans left. There were two more boys than girls in her class. How many students were in her class?

QUICK STATS:

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MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Algebra: Constructing and solving algebraic equations

COMMON CORE STATE STANDARDS

8.EE.8b (Extended)   Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

MATHEMATICAL PRACTICE STANDARDS

MP1   Make sense of problems and persevere in solving them.
MP2   Reason abstractly and quantitatively.
MP3   Construct viable arguments and critique the reasoning of others.
MP7   Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 3:   **ENGAGE IN WISHFUL THINKING**

SOURCE: This is question # 23 from the 2009 MAA AMC 8 Competition.
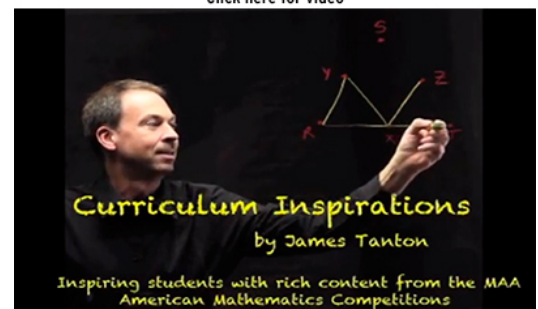
THE PROBLEM-SOLVING PROCESS:

As always, the best start is …

> **STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Just reading and understanding this question feels like a lot of work! Let's take it slowly.

> *On the last day of school, Mrs. Wonderful gave jelly beans to her class.*

Got it.

> *She gave each boy as many jelly beans as there were boys in the class.*

So if there are 10 boys, each boy got 10 beans. If there are 45 boys, each boy got 45 beans.

> *She gave each girl as many jelly beans as there were girls in the class.*

Okay. Same idea as for the boys.

> *She brought 400 jelly beans, and when she finished, she had six jelly beans left.*

So she gave out 394 beans.

> *There were two more boys than girls in her class.*

Okay.

> *How many students were in her class?*

That's the hard part!

Hmm. The only numerical fact we have is that she gave out 394 beans. Apparently that is enough information to deduce the number of students in the class.

Well, if there are 10 boys, then she gives out $10 \times 10 = 100$ beans to the boys. And if there are 10 boys, there are just 8 girls and she gives out $8 \times 8 = 64$ beans to them. That's a total of $100 + 64 = 164$ beans. This is too

few for the question, But now I believe that I could figure the right number of boys and girls to get 394 beans.

One approach: Keep trying more numbers for the count of boys until I hit a number that works. That will certainly work.

But rather than just guess and check, can I be clever in my thinking?

If there are b boys and g girls, then she gives out $b \times b + g \times g$, that is, $b^2 + g^2$ beans. We want this to equal 394 :

$$b^2 + g^2 = 394 \ .$$

The number of boys and girls only differ by two, so the values b^2 and g^2 aren't too different. This means that b^2 and g^2 are basically each equal to half of 394 :

$$b^2 \approx 197 \ \text{ and } \ g^2 \approx 197.$$

This isn't exact, of course, but since $14^2 = 196$ this means b and g are each hovering around the number 14 . Let's try $b = 15$ and $g = 13$. (They differ by two.)

$15^2 + 13^2 = 225 + 169 = 394$. Bingo!

There are $15 + 13 = 28$ students in the class.

> **Extension:** We just saw that the number 394 is the sum of two square numbers: $394 = 13^2 + 15^2$. Is 395 a sum of two square numbers? Is 396? Is 397? Which is the number closest to 394 that is also a sum of two square numbers?

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