Curriculum Burst 73: Abstract Sums

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The letters A, B, C and D all represent different digits. If

$$\begin{array}{r} AB \\ +\ CA \\ \hline DA \end{array}$$

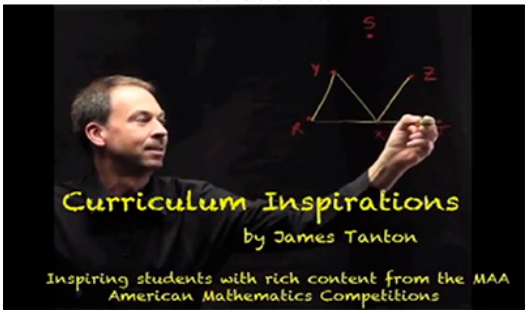
and

$$\begin{array}{r} AB \\ -\ CA \\ \hline A \end{array}$$

what digit does D represent?

QUICK STATS:

Click here for video



MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Number Sense, Linear Equations

COMMON CORE STATE STANDARDS

8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 1: ENGAGE IN SUCCESSFUL FLAILING

SOURCE: This is question # 24 from the 2009 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

As always, the best start is …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question seems hard. I am given an addition and a subtraction problem, but only symbols of the actual numbers. How could I possibly work out what the numbers are?

First we have the sum:

$$\begin{array}{r} AB \\ +\ CA \\ \hline DA \end{array}$$

Hmm. Well, if I imagine myself actually doing the sum we'd start at the right, add the B and the A, write the answer at the bottom but carry any tens digits to the left column. Then we'd add the A and the C and any carried digits, and write down the answer.

Okay … so in the ones column we have $B+A$ equals a number that ends with the digit A again. If A is 3 (I don't know, I am just making this up for now), we have $B+3$ is an answer that ends in 3.

$$B+3 = \ 3 \ or \ 13 \ or \ 23 \ or \ 33 \ or \ \ldots$$

But B is a single digit, so $B+3=13$ or higher is impossible. We must have $B+3=3$ giving $B=0$. And this would still be the case if I chose $A=5$ or $A=9$ or $A=0$ instead.

Alright. We've got $B=0$.

$$\begin{array}{r} A0 \\ +\ CA \\ \hline DA \end{array}$$

This is feeling good!

What about the subtraction?

$$\begin{array}{r} A0 \\ -\ CA \\ \hline A \end{array}$$

To compute it, we do have to borrow a one. It looks weird, but here's what we'll have:

$$\begin{array}{r} A\text{-}1\ \ {}^{1}0 \\ -\ C\ \ A \\ \hline A \end{array}$$

So $10-A=A$ and this shows $A=5$!

$$\begin{array}{r} 4\ \ {}^{1}0 \\ -\ C\ \ 5 \\ \hline 5 \end{array}$$

So $C=4$.

That's everything, except D!

But look at the first sum:

$$\begin{array}{r} AB \\ +\ CA \\ \hline DA \end{array} \longrightarrow \begin{array}{r} 50 \\ +\ 45 \\ \hline D5 \end{array}$$

It is now clear that $D=9$.

Extension: All the numbers and arithmetic problems in this question are assumed to be written in base ten. What if this question is actually a question in base eight? Can we still work out the single digit D? (This time the digits could only be 0, 1, 2, 3, 4, 5, 6 or 7, and "254," for example, is the number $2\times 8^2+5\times 8+4\times 1$.)

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