Curriculum Burst 68: Hypotenuse Please

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> A right triangle has perimeter 32 and area 20. What is the length of its hypotenuse?

QUICK STATS:

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MAA AMC GRADE LEVEL

This question is appropriate for the junior high-school grade levels.

MATHEMATICAL TOPICS

The Pythagorean Theorem; Systems of Equations

COMMON CORE STATE STANDARDS

8.G.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
A.REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

MATHEMATICAL PRACTICE STANDARDS

MP1	Make sense of problems and persevere in solving them.
MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.
MP7	Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 1: ENGAGE IN SUCCESSFUL FLAILING

SOURCE: This is question # 18 from the 2008 MAA AMC 10A Competition.

THE PROBLEM-SOLVING PROCESS:

As always, one starts the problem-solving process with …

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STEP 1: Read the question, have an
emotional reaction to it, take a deep
breath, and then reread the question.
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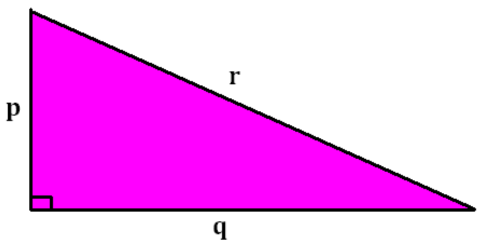
This question doesn't feel too scary to me. If I draw and label a right triangle as follows:



we are told:

$$p+q+r=32$$
$$\frac{1}{2}pq=20$$

and we are being asked to find r. Since it is a right triangle, we also have the Pythagorean formula up our sleeve:

$$p^2+q^2=r^2.$$

Surely with these three equations in three unknowns we can work out the value of r? (As I say this, I am starting to feel a little queasy. These equations are straightforward!)

Before I leap into substituting equations into one another, I am wondering if there is a clever way to deal with all the squared values?

We have $p^2+q^2=r^2$ and $pq=40$. My mind is drawn to these two equations. They remind me of…

$$(p+q)^2=p^2+2pq+q^2.$$

Aha! $p+q=32-r$ and $p^2+q^2+2pq=r^2+80$. We have:

$$(32-r)^2=r^2+80.$$

Alright. Expanding gives:

$$32^2-64r+r^2=r^2+80$$
$$1024-64r=80$$
$$64r=944$$

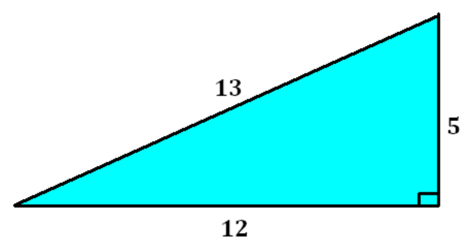
So

$$r=\frac{944}{64}=\frac{450+22}{32}=\frac{225+11}{16}$$
$$=\frac{236}{16}=\frac{118}{8}=\frac{59}{4}$$

We're done!

Extension 1: Does a right triangle of perimeter 32 and area 20 actually exist? (Maybe we just answered an imaginary question!)

Extension 2: The $5-12-13$ right triangle has perimeter 30 and area 30.



There is only one other right triangle with integer sides with the property that, when measured in the appropriate units, its perimeter and its area have the same numerical value. What is it? (And prove for yourself that there are no more examples of these integer right triangles!)

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