Curriculum Burst 61: Overlapping Triangles

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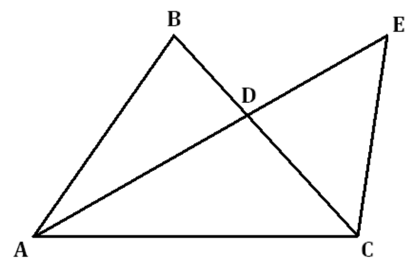
Triangle ABC is an isosceles triangle with $AB = BC$. Point D is the midpoint of both $\overline{BC}$ and $\overline{AE}$ and $\overline{CE}$ is 11 units long. Triangle ABD is congruent to triangle ECD. What is the length of $\overline{BD}$?



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QUICK STATS:

MAA AMC GRADE LEVEL
This question is appropriate for the 8th grade level.

MATHEMATICAL TOPICS
Geometry

COMMON CORE STATE STANDARDS
8-G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

MATHEMATICAL PRACTICE STANDARDS
MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.

PROBLEM SOLVING STRATEGY
ESSAY 1: ENGAGE IN SUCCESSFUL FLAILING

SOURCE: This is question # 19 from the 2006 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

As always, start with …

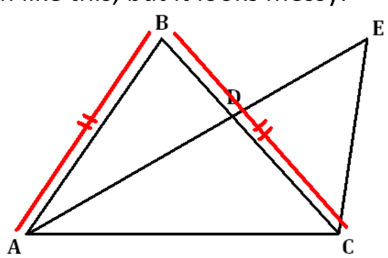
STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question feels complicated. There are a lot of words and only one number, and I am being asked to compute a length in a picture that is essentially blank!

Okay … Deep breath. Maybe the thing I need to do is go though the question slowly and make sure I understand all the words that are in it. (As I said, there are a lot of them!)

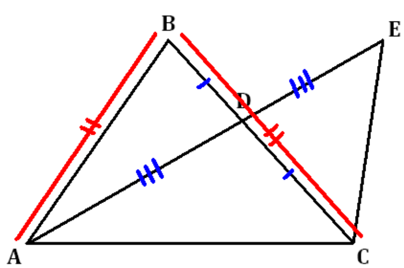
Triangle ABC is an isosceles triangle with $AB = BC$.

That makes sense. I suppose I could mark this information on the diagram like this, but it looks messy.



Point D is the midpoint of both $\overline{BC}$ and $\overline{AE}$ …

So D chops each of the segments on which it sits into two equal parts. My diagram is getting messier.



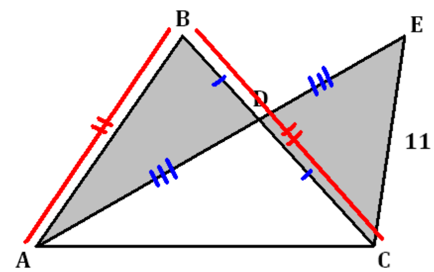
… and $\overline{CE}$ is 11 units long.

Okay. I'll mark that in a moment.

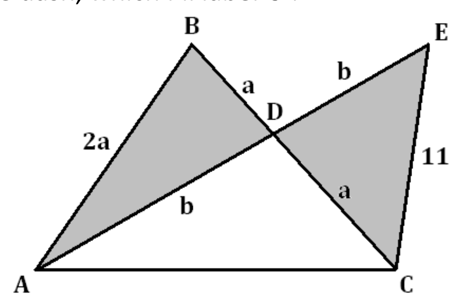
Triangle ABD is congruent to triangle ECD.

I recall that "congruent" means that one can put one triangle on top of the other, that they are the same size and shape.

Let me mark this on the diagram by shading the two triangles that are the same.



This is hard to read. Let me use letters instead of dashes to mark lengths. We have two single dashes, which I'll call a, a double dash, which is the same as two single dashes $2a$, and a triple dash, which I'll label b.



Okay … that's easier to look at and take in. Now, I haven't been paying attention to where I am meant to be going! What was the question?

What is the length of $\overline{BD}$?

Sure. We need to find a. And that's fine! Since the shaded triangles are congruent, $2a = 11$ giving $a = \frac{11}{2}$. Done.

Wow!

Extension: Prove that if we draw $\overline{BE}$, the quadrilateral $ABEC$ is a parallelogram.

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