Curriculum Burst 47: One versus Two Coin Tosses

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Keiko tosses one penny and Ephraim tosses two pennies.
The probability that Ephraim gets the same number of heads that Keiko gets is …

THE QUICK STATS:

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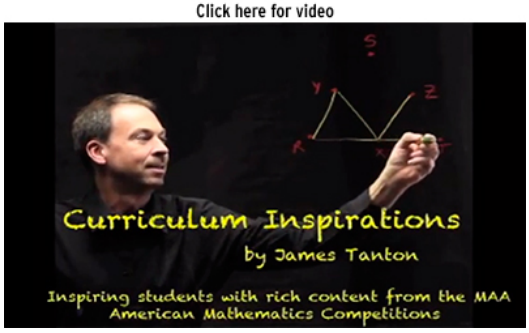

MAA AMC GRADE LEVEL

This question is appropriate for the 8th grade level.

MATHEMATICAL TOPIC

Probability

COMMON CORE STATE STANDARDS

7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.

PROBLEM SOLVING STRATEGY

ESSAY 3: **ENGAGE IN WISHFUL THINKING**

SOURCE

This is question # 21 from the 2000 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The appropriate start, as always, is …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Probability questions always make me nervous. Here we have two actions:

Keiko tosses his one coin.
Ephraim tosses his two coins.

And we're looking for the chance they both observe the same number of heads. Well, Keiko either sees zero heads (when he gets a tail) or one head. Ephraim sees either zero, one or two heads. So that makes for six possibilities?

Keiko: 0 heads and Ephraim: 0 heads.
Keiko: 0 heads and Ephraim: 1 head.
Keiko: 0 heads and Ephraim: 2 heads.
Keiko: 1 head and Ephraim: 0 heads.
Keiko: 1 head and Ephraim: 1 heads.
Keiko: 1 head and Ephraim: 2 heads.

Two of these six options have both lads seeing the same number of heads – so the answer to the question is $2/6 = 1/3$?

Something makes me feel uneasy about this. I am nervous about the counting on for Ephraim. There is certainly only one way for Ephraim to see zero heads (toss tail and tail). But there are two ways for him to see one head (toss head and tail, or toss tail than head) Should the table be …?

Keiko: 0 heads and Ephraim: 0 heads. (TT)
Keiko: 0 heads and Ephraim: 1 head. (TH)
Keiko: 0 heads and Ephraim: 1 head. (HT)
Keiko: 0 heads and Ephraim: 2 heads. (TT)
Keiko: 1 head and Ephraim: 0 heads. (TT)
Keiko: 1 head and Ephraim: 1 heads. (HT)
Keiko: 1 head and Ephraim: 1 heads. (TH)
Keiko: 1 head and Ephraim: 2 heads. (HH)

If this is correct, then the answer to the problem is $3/8$. (Three cases out of eight have the lads with the same number of heads.) Is this right?

But Ephraim is tossing his coins simultaneously, so "HT" is no different from "TH." Maybe the first table is correct! I am confused!

My trouble with probability questions like these is the issue of simultaneity. I think the problem would be much easier if it read something like:

Keiko picks up a penny, tosses it, and notes whether he gets heads or tails. Ephraim then picks up the penny and tosses it. He records the result. He then picks up the penny again and tosses it another time, recording this result too.

What is the probability that both lads saw the same number of heads among their own tosses?

In this scenario it is very clear that we have eight distinct cases to consider, three of which has both lads seeing the same number of heads.

Keiko's toss	Ephraim's 1st toss	Ephraim's 2nd toss
H	T	T
H	H	T
H	T	H
H	H	H
T	T	T
T	H	T
T	T	H
T	H	H

So … is this wishful thinking helpful?

Well, thinking about it. nothing in the world actually happens simultaneously. Even if the two lads tossed three coins at the same time, they do not land on the ground simultaneously. We have Keiko's coin, Ephraim's coin to land first, and Ephraim's coin to land second. So the problem is philosophically the same as the one described in the table above. Great! This means that $3/8$ is the correct answer to the problem.

Extension: Angelique has one less coin than Beatrice. They each toss all the coins they own and count the number of heads they each see. Is it true that Angelique has a 50% chance of seeing at least as many heads among her coins as Beatrice sees among hers?

[Explore the cases: Angelique has zero coins, Beatrice has one; Angelique has 1 coin, Beatrice has 2; Angelique has 2 coins, Beatrice has 3, and so on. Is the answer really always 50%?]

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