Curriculum Inspirations

Inspiring students with rich content from the MAA American Mathematics Competitions

Curriculum Burst 46: Dealing with Averages

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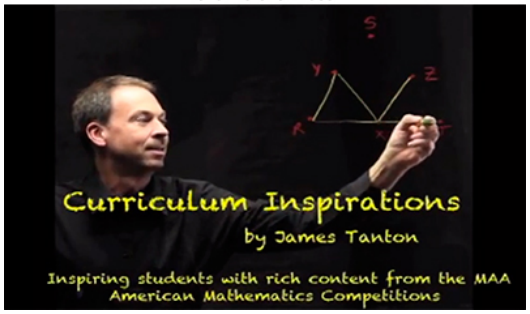
There is a list of seven numbers. The average of the first four numbers is 5, and the average of the last four numbers is 8. If the average of all seven numbers is $6\frac{4}{7}$, then the number common to both sets of four numbers is …?

THE QUICK STATS:

Click here for video



MAA AMC GRADE LEVEL

This question is appropriate for the 8th grade level.

MATHEMATICAL TOPIC

Statistics

COMMON CORE STATE STANDARDS

6.SP.B.5c Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered**.**

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 4: **DRAW A PICTURE**

SOURCE

This is question # 23 from the 2000 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The way to start is …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question feels overwhelming! There is some list of seven numbers:

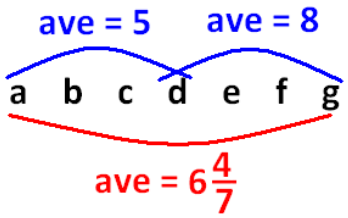
a b c d e f g

and we're told something about the first four, the last four and about all seven. It feels like I will have three big messy equations with seven unknowns that I should somehow solve. As an algebra problem, this is scary.

Does it need to be an algebra problem?

Let me read the question and record the information given in it with a diagram:

The average of the first four numbers is 5.
The average of the last four numbers is 8.
The average of all seven numbers is $6\frac{4}{7}$.



Does this help? Hmm. Not really.

Alright. What does "average" mean?

The first four numbers have average 5 means:

$$\frac{a+b+c+d}{4}=5$$

Oh. This is algebra! I am trying to avoid that. But I do see with a tiny bit of manipulation that the sum of the first four numbers must be 20. In the same way, the sum of the last four numbers, with average 8, is $4\times 8=32$.

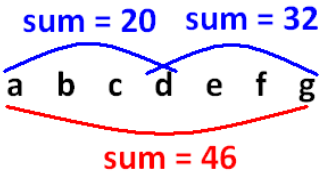
From

$$\frac{a+b+c+d+e+f+g}{7}=6\frac{4}{7}$$

we get that the sum of all seven numbers is

$$7\times 6\frac{4}{7}=42+4=46.$$

Let me try recording all this information.



Oh! Stare at the picture! We see:

$$\begin{aligned} 20+32 &= \text{sum of all seven numbers with } d \text{ repeated} \\ &= 46+d \end{aligned}$$

The middle number d must be 6. Wow!

Thought Questions: In statistics the average of a list of numbers is called the *mean* of those numbers. (Here we are thinking of the numbers in the list as data values.) This is one way of possibly describing a "middle" or "central" value of the data.

One might use instead the *median* of the data or the *mode* of the data (if there is one) for a measure of a central value.

a) If the word "average" in this question was replaced with the word *median* would it, for certain, still be possible to determine the value of the middle number d ?

b) What if the word *mode* was used instead?

Extension: a) The product of the first four numbers in a list of seven numbers is A, the product of the last four numbers is B, and the product of all seven numbers is C. Find a formula for the middle number in the list.

b) Find a list of seven numbers for which the numbers A, B and C described in a) have values 6, 15 and 9, respectively.

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