Curriculum Burst 44: Additional License Plates

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Bicycle license plates in Flatville each contain three letters. The first is chosen from the set $\{C, H, L, P, R\}$, the second from $\{A, I, O\}$, and the third from $\{D, M, N, T\}$.

When Flatville needed more license plates, they added two new letters. The new letters may both be added to one set or one letter may be added to one set and one to another set. What is the largest possible number of ADDITIONAL license plates than can be made by adding two letters?

THE QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the 8th grade level.

MATHEMATICAL TOPIC

Combinatorics

COMMON CORE STATE STANDARDS

6.NS.B   Compute fluently with multi-digit numbers and find common factors and multiples.
S-CP.B9   (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

MATHEMATICAL PRACTICE STANDARDS

MP1   Make sense of problems and persevere in solving them.
MP2   Reason abstractly and quantitatively.
MP3   Construct viable arguments and critique the reasoning of others.

PROBLEM SOLVING STRATEGY

ESSAY 1:   **ENGAGE IN SUCCESSFUL FLAILING**

SOURCE
This is question # 15 from the 1999 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best start …

> **STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Okay … This question starts off in a familiar way. We are to make license plate "numbers" by filling in three slots with letters:

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There are 5 choices of letter for the first slot, 3 for the second, and 4 for the third. This makes for a total of $5 \times 3 \times 4 = 60$ possibilities. Ooh! Are there repeat letters I have to worry about? Actually … no. All the letters are given are different. (And if letters were repeated among the choices available for each slot, I don't think it would affect this answer in any case.)

The second part of the question is a little strange. Flatville will add two more letters – but the question is deliberately not telling us to which slots those additional letters will be added. (Actually … that is the point of the question! We want to know where to add those two additional letters to get the largest number of new license plates.)

Well, I really don't know what to do for this question other than try possibilities.

What if the two new letters are added as choices for the first slot? Then there would be 7 choices of letters for the first slot, and still 3 and 4 for the second and third slots, giving a total of $7 \times 3 \times 4 = 84$ license plates. That's pretty good I suppose.

What if both were added as choices for the second slot? That would give $5 \times 5 \times 4 = 100$ license plates. Even better!

Both added as options for the third? This gives $5 \times 3 \times 6 = 90$ license plates. Not as good.

Let's just keep plugging on and consider the cases of one letter added as an option for one slot, and one for the another. We can see this will change the count of plates from $5 \times 3 \times 4 = 60$ to either:

$$6 \times 4 \times 4 = 96$$
or
$$6 \times 3 \times 5 = 90$$
or
$$5 \times 4 \times 5 = 100$$

Okay … the best we can do is get 100 plates (in two different ways).

A quick reread of the question, noting the word "additional" in capital letters, tells me that this is <u>not</u> the answer they seek. Originally we could make 60 different plates. With the addition of two more letters we can get 100 different plates. That's an increase of 40 plates, and that number is the answer to the question!

> **Extension:**
> a) Show, that with the addition of three more letters, one can gain a maximum of 65 more plates.
> b) (**TOUGH**) Which three numbers a, b, c adding to 30 give the largest product abc? How could you convince others that your answer is correct (even if a, b and c need not be whole numbers!)?

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