Curriculum Inspirations

Inspiring students with rich content from the MAA American Mathematics Competitions

Curriculum Burst 41: Shaded Triangles

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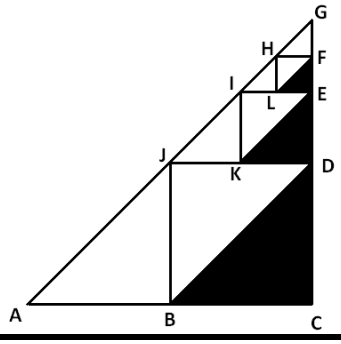
Points B, D, and J are midpoints of the sides of right triangle ACG. Points K, E, I are midpoints of the sides of triangle JDG, etc. If the dividing and shading process is done 100 times (the first three are shown) and $AC = CG = 6$, then the total area of the shaded triangles is nearest

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10



Click here for video

THE QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the 8th grade level.

MATHEMATICAL TOPIC

Geometry

COMMON CORE STATE STANDARDS

7-G.A Draw, construct, and describe geometrical figures and describe the relationships between them.
7-G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 9: **AVOID HARD WORK**

SOURCE

This is question # 25 from the 1999 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

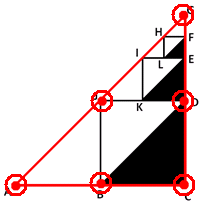
Always the first step…

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Just reading this question once, and then twice, is hard work. My first instinct after doing so is to give up on it! But I am going to persevere. Let's read the question again, one line at a time, and try to take in each line.

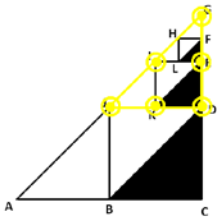
Points B, D, and J are midpoints of the sides of right triangle ACG.

Okay, I see triangle ACG. The three points mentioned in this line are the midpoints of the sides of that triangle.



Points K, E, I are midpoints of the sides of triangle JDG, etc.

Got it!



If the dividing and shading process is done 100 times (the first three are shown)…

Whoa! I guess we keep stacking up those squares and shading those black triangles: 100 times!

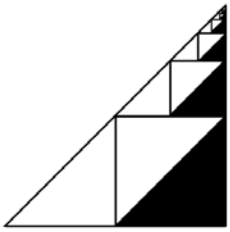
… and $AC = CG = 6$ …

Okay. The big outer triangle is 6 units wide and 6 units high. It has area $(1/2)\times 6\times 6 = 18$ squared units.

… then the total area of the shaded triangles is nearest…

And now I am lost. I don't know what to do. We're being asked about the area of 100 shaded triangles all added together.

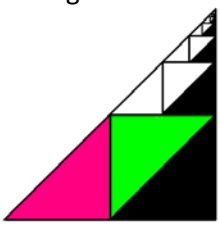
Let me doodle in a few more shaded triangles. I'm just filling in time while I think.



Hmm. Still don't know what to do.

I know that the largest shaded triangle is 3 units high and 3 units wide (midpoints, remember), and I could work out its area. I can probably work out the areas of all 100 triangles and add them up. But that seems too hard! Is there another way to see the area? Hmm.

Well there are two triangles in the diagram the same size as the largest shaded triangle.



Ooh! That's actually the case for each shaded triangle.

black = $\frac{1}{3}$ of each level

Oh! 100 black triangles basically go all the way to the top of the figure. And I see that from this picture that the black areas basically count for $1/3$ of the area of the entire triangle. Their area must add (almost) to $(1/3)\times 18 = 6$.

The answer is (A)!

Extension: Jenny tears a piece of paper into fifths. She gives a piece to each of her four friends. She then tears the fifth piece into fifths and gives a piece of it to each of her friends. She then tears the remaining piece into fifths and shares again. She does this over a million times, tearing and sharing. How much paper does each friend receive from Jenny?

See the video www.jamestanton.com/?p=723 for more on this paper-tearing idea.

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