

Curriculum Inspirations

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MAA American Mathematics Competitions



Curriculum Burst 38: The Sum of the Ends

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In the eight-term sequence

$$A, B, C, D, E, F, G, H$$

the value of C is 5 and the sum of any three consecutive terms is 30.

What is $A + H$?

THE QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the 10th grade level.

MATHEMATICAL TOPIC

Number Sense

COMMON CORE STATE STANDARDS

A-SSE.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.
- MP8** Look for and express regularity in repeated reasoning.

PROBLEM SOLVING STRATEGY

ESSAY 7: [PERSEVERANCE IS KEY](#)

SOURCE

This is question # 17 from the 2011 MAA AMC 10A Competition.



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THE PROBLEM-SOLVING PROCESS:

The most important step, each and every time:

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question doesn't feel too scary. We are told that C equals 5, so the sequence reads:

A B 5 D E F G H

And any three consecutive terms add to 30. So:

$$A + B + 5 = 30$$

$$B + 5 + D = 30$$

$$5 + D + E = 30$$

$$D + E + F = 30$$

$$E + F + G = 30$$

$$F + G + H = 30$$

From this we need to figure out the value of $A + H$. (Hmm. An actual value!) What can we do?

Let's just play with the equations.

It might be helpful to rewrite the first three as:

$$A + B = 25$$

$$B + D = 25$$

$$D + E = 25$$

(Hmm. Is it helpful?)

Ooh! Look at the fourth equation: $D + E + F = 30$. We just showed $D + E = 25$ so it follows that $F = 5$! That's something.

The final three equations can thus be written:

$$D + E = 25$$

$$E + G = 25$$

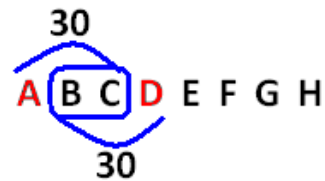
$$G + H = 25$$

But where are we going with this?

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Let's step back a bit.

We are told the first three terms sum to 30, as do the next three terms, and so on.



So $A + (B + C)$ has the same sum as $(B + C) + D$.

$$A + (B + C) = (B + C) + D$$

It follows that $A = D$. Sliding along the sequence we must also conclude that any two terms four places apart are equal. So $B = E$ (using $B + (C + D) = (C + D) + E$) and $C = F$ and $D = G$ and $E = H$. So our sequence is really:

A B 5 A B 5 A B

We're looking for $A + H$, the sum of the first and last terms. This is $A + B$, which we've already seen is 25!

Aside: Go back to our five equations:

$$A + B = 25$$

$$B + D = 25$$

$$D + E = 25$$

$$E + G = 25$$

$$G + H = 25$$

Summing all these equations gives:

$$A + 2(B + D + E + G) + H = 125$$

Can you see what to do now to deduce that $A + H = 25$?

Extension: Twenty students sit in a circle. Each student has age equal to the average age of his or her two neighbors in the circle. Bertrand and Beatrice both sit in the circle. Bertrand is 11 years old. How old is Beatrice?