Curriculum Burst 36: A Rectangular Region

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A rectangular region is bounded by the graphs of the equations $y = a$, $y = -b$, $x = -c$, and $x = d$, where a, b, c, and d are all positive numbers. Which of the following represents the area of this region?

(A) $ac + ad + bc + bd$
(B) $ac - ad + bc - bd$
(C) $ac + ad - bc - bd$
(D) $-ac - ad + bc + bd$
(E) $ac - ad - bc + bd$

QUICK STATS:

Click here for video

MAA AMC GRADE LEVEL

This question is appropriate for the 10th grade level.

MATHEMATICAL TOPICS

Functions and their Graphs

COMMON CORE STATE STANDARDS

F-IF.C7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 4: DRAW A PICTURE

SOURCE: This is question # 9 from the 2011 MAA AMC 10A Competition.

THE PROBLEM-SOLVING PROCESS:

As always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question feels abstract. No numbers appear in it- only symbols – and the formulas offered look complicated.

Deep breath!

We're being asked something about the graphs of equations, the graphs of these four equations:

$$y = a$$
$$y = -b$$
$$x = -c$$
$$x = d$$

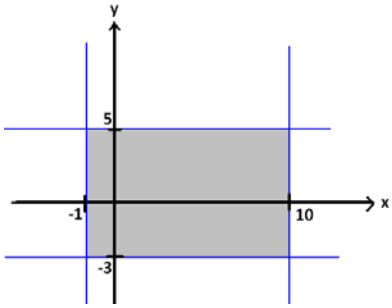
Here a, b, c, and d positive numbers.

It seems appropriate to try to draw graphs of the equations as best we can. Hmm. But there are no numbers!

Would it hurt to make some specific choices of numbers and just graph those equations? At least it might provide a feel for things.
What if the equations were, say, the following?

$$y = 5$$
$$y = -3$$
$$x = -1$$
$$x = 10$$

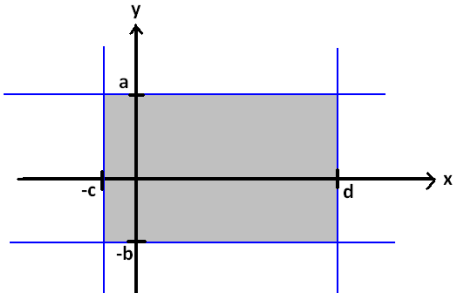
Their graphs consist of two horizontal lines and two vertical lines.



We see a rectangle 11 units wide and 8 units high. It has area 88 square units.

In this example we chose $a = 5$, $b = 3$, $c = 1$ and $d = 10$. Plugging these numbers into each of the five formulas offered (it doesn't take too long to do) we see that only option (A) gives the answer 88. That must be the answer to the question. We can stop now if our only goal is to select the right option and move on!

But, for the sake of exploring the mathematics, we see in general now that the four equations $y = a$, $y = -b$, $x = -c$, and $x = d$ give a picture as follows:



This rectangle extends horizontally c units to the left and d units to the right, and so has width $(c+d)$. Similarly, its height is $(a+b)$ units. And so its area is given by:

$$(a+b)(c+d)$$

Note, when expanding, no negative signs will appear. So again, without doing the work we deduce that only option (A) can be the answer.

But just to check, expanding gives:

$$(a+b)(c+d) = ac + ad + bc + bd$$

Yep! It really is option (A).

Extension: Consider the four equations:
$y = x + a$, $y = x - b$, $y = -x - c$, and $y = -x + d$, with a, b, c and d positive numbers.

a) Explain why the graphs of these equations also bound a rectangular region.

b) Can you find a formula for the area of that region?

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