Curriculum Burst 28: Stair Climbing

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Every day at school, Jo climbs a flight of 6 stairs.
Jo can take stairs 1, 2 or 3 at a time. For example, Jo could climb 3, then 1, then 2 stairs.
In how many ways can Jo climb the stairs?

QUICK STATS:

MAA AMC GRADE LEVEL
This question is appropriate for the 8th grade level.

MATHEMATICAL TOPICS
Counting.   Recursive formulas.

COMMON CORE STATE STANDARDS

Foreshadows …
F-IF.3   Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

MATHEMATICAL PRACTICE STANDARDS
MP1   Make sense of problems and persevere in solving them.
MP2   Reason abstractly and quantitatively.
MP3   Construct viable arguments and critique the reasoning of others.
MP7   Look for and make use of structure.

PROBLEM SOLVING STRATEGY
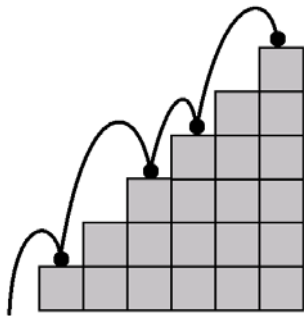ESSAY 5:   SOLVE A SMALER VERSION OF THE SAME PROBLEM

THE PROBLEM-SOLVING PROCESS:

As always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This problem feels easy enough to understand, but as I play with it and try it out, I start to feel overwhelmed by all the possibilities. We have:
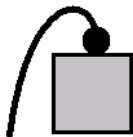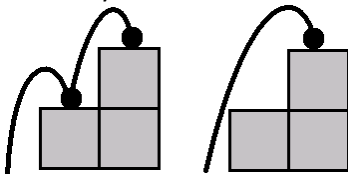

1 + 2 + 1 + 2

$1+2+1+2$ and $3+2+1$ and $1+1+1+1+2$ and $2+2+2$ and $1+3+1+1$ and ….

TRY A SMALLER VERSION OF THE SAME PROBLEM!

The ridiculously small version of this same problem would be Jo climbing just one stair each day at school. There is just **ONE** way to accomplish this task.
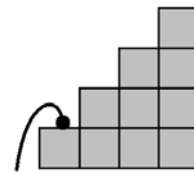

Okay … too easy? Let's try just two stairs then. We can see that there are **TWO** ways Jo can climb them.


With three stairs Jo has **FOUR** possibilities: $1+1+1$ and $1+2$ and $2+1$ and 3. (I don't need the pictures anymore.)

Let's keep building up. With four stairs … Hmm. Well if her first step is a single step (okay, back to pictures!)…

… Jo is left with three steps to climb, and we know she can do them in FOUR ways. If her first step is a double step, she is left with two more stairs to climb and we've seen this can be done in TWO ways. And with a triple step first, she is left with one more stair to go, to be done ONE way.
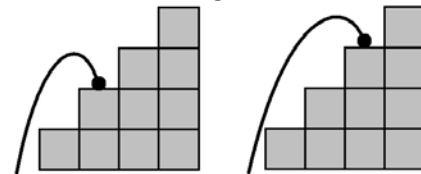
Thus there are FOUR + TWO + ONE = **SEVEN** ways for Jo to climb four steps.

In summary:

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1 stair   = 1 way
2 stairs  = 2 ways
3 stairs  = 4 ways
4 stairs  = 7 ways
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With five stairs … A first single step leaves Jo with four stairs to traverse, which can be done in 7 ways. If her first step is a double, then she is left with three stairs to traverse, in any of 4 ways. And if her first step is a triple, she has three more stairs to traverse in any of 2 ways. Thus she has a total of $7+4+2=13$ ways to traverse five stairs.

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5 stairs  =  13 ways.
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With six stairs…

First step is a single:	Leaves five stairs to traverse. 13 ways.
First step is a double:	Leaves four stairs to traverse. 7 ways.
First step is a triple:	Leaves three stairs to traverse. 4 ways.

There are a total of 24 ways for Jo to traverse thise six stairs!

Extension: Repeat this problem and our method of solution with Jo taking only 1 or 2 (but not 3) stairs at a time and discover a VERY FAMOUS sequence of numbers!

Curriculum Inspirations is brought to you by the Mathematical Association of America and MAA American Mathematics Competitions.