# Curriculum Burst 26: Areas of Nested Squares

By Dr. James Tanton, MAA Mathematician in Residence

> Square $EFGH$ has one vertex on each side of square $ABCD$. Point $E$ is on $\overline{AB}$ with $AE = 7 : EB$. What is the ratio of the area of $EFGH$ to the area of $ABCD$?

**SOURCE:** This is question # 11 from the 2010 MAA AMC 10a Competition.

YouTube
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## QUICK STATS:

### MAA AMC GRADE LEVEL

This question is appropriate for the 10th grade level.

### MATHEMATICAL TOPICS

Geometry: Area; the Pythagorean Theorem.

### COMMON CORE STATE STANDARDS

**G-MG.3:** Apply geometric methods to solve design problems

### MATHEMATICAL PRACTICE STANDARDS

**MP1** Make sense of problems and persevere in solving them.
**MP2** Reason abstractly and quantitatively.
**MP3** Construct viable arguments and critique the reasoning of others.
**MP7** Look for and make use of structure.

### PROBLEM SOLVING STRATEGY

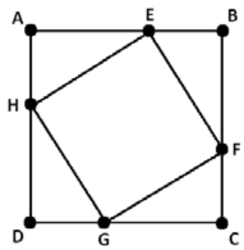
ESSAY 4: **DRAW A PICTURE**

## THE PROBLEM-SOLVING PROCESS:

As always …

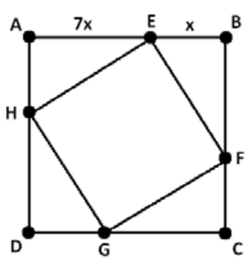
**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

My only real concern with this question is making sure I understand the notation. In reading the question slowly I see that have two squares, one with vertices labeled $EFGH$ and one with vertices labeled $ABCD$. The first square sits inside the second with vertex $E$ on the side $\overline{AB}$ (and the remaining vertices $F$, $G$ and $H$ on the remaining three sides of $ABCD$.) We must have the picture:




(**Question:** I've placed each set of vertex names $A$, $B$, $C$ and $D$, and $E$, $F$, $G$ and $H$ in clockwise order. I know that they must be placed in some consistent order, but would it matter if I set them both in counter-clockwise placement? Or one one direction and the other the other?)

The notation "$AE = 7 : EB$" is ratio notation from middle-school days. It says that if length $EB$ is $x$ units long, then length $AE$ is $7x$ units in length. We can mark this on the diagram:



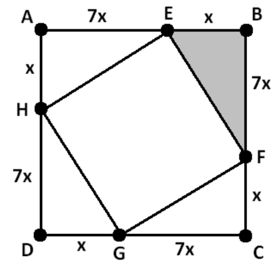

Now … what is the question?

> *What is the ratio of the area of $EFGH$ to the area of $ABCD$?*

In our notation, square $ABCD$ has side-length $8x$ units and so its area is $(8x)^2 = 64x^2$. The trouble is, I don't know the value of $x$ and so do not know the number for this area. This might be a problem. But let's push on.

Do we know the side-length of square $EFGH$, or at least some expression for it?

It seems irresistible to note that by the symmetry of the situation, each side of the large square is portioned into segments of the same $7x : x$ ratio:




And I can't help but notice the right triangles sitting in each corner of the large square. Since we want the side-length of $EFGH$, which is a hypotenuse of any one of these right triangles, the Pythagorean theorem leaps to mind. We have:

$$EF^2 = (x)^2 + (7x)^2$$

(Or should that be $EF^2 = x^2 + 7x^2$?)

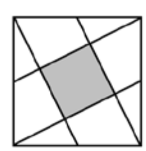
This gives $EF^2 = 50x^2$, which is the area of the small square. (No need to write $EF = \sqrt{50}x$ if we are only going to square it to compute the area!) Alas, again, this is not an actual number.

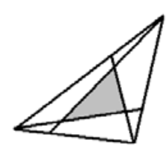
But the question is not asking for the values of the areas of the two squares, but their ratio. We have:

$$\frac{area\ EFGH}{area\ ABCD} = \frac{50x^2}{64x^2} = \frac{25}{32}.$$

And that's an actual number!

**Extension:** a) Lines are drawn from each vertex of a square to the midpoint of a side as shown. What fraction of the total area is the area of the central square formed?





b) Lines are drawn from each vertex of a triangle to a point one-third of the way along the opposite side. What fraction of the total area is the area of the central triangle formed?

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