Curriculum Inspirations
Inspiring students with rich content from the MAA American Mathematics Competitions
MAA

# Curriculum Burst 24: Frog Leaping

By Dr. James Tanton, MAA Mathematician in Residence

A frog located at $(x, y)$, with both $x$ and $y$ integers, makes successive jumps of length $5$ and always lands on points with integer coordinates. Suppose that the frog starts at $(0,0)$ and ends at $(1,0)$. What is the smallest possible number of jumps the frog makes?

**SOURCE:** This is question # 11 from the 2011 MAA AMC 12b Competition.

YouTube
Click here for video

## QUICK STATS:

### MAA AMC GRADE LEVEL

This question is appropriate for the 12th grade level.

### MATHEMATICAL TOPICS

Geometry: Coordinate planes, the distance formula (the Pythagorean Theorem), equidistance and perpendicular bisectors.

### COMMON CORE STATE STANDARDS

**G-GPE.4:** Use coordinates to prove simple geometric theorems algebraically.
**8.G.8:** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

### MATHEMATICAL PRACTICE STANDARDS

**MP1** Make sense of problems and persevere in solving them.
**MP2** Reason abstractly and quantitatively.
**MP3** Construct viable arguments and critique the reasoning of others.
**MP7** Look for and make use of structure.

### PROBLEM SOLVING STRATEGIES

ESSAY 2: **DO SOMETHING!**
ESSAY 4: **DRAW A PICTURE**

## THE PROBLEM-SOLVING PROCESS:

The most important step is the first:

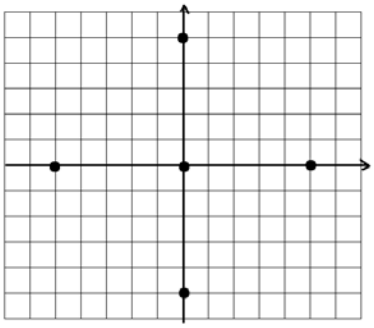
**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I am a little lost as I read this question. It starts by describing the position of the frog as the point $(x, y)$ on the coordinate plane, but then later tells me that the frog starts at $(0,0)$. This seems disjointed. What is that first sentence trying to convey?

The rest of the question is about a leap-frog journey that starts at $(0,0)$ and ends at $(1,0)$, both points with integer coordinates. So maybe that first sentence is simply a general statement about the motion of the frog. Rereading it, I am now interpreting it as saying:

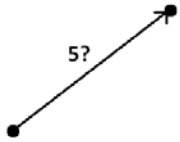
> *Whenever a frog is on a point with integer coordinates, it jumps five units (in some direction) to always land on another point with integer coordinates.*

Okay, we're at $(0,0)$ and we're going to jump five units in some direction to land on another point with integer coordinates. I guess there are four locations it could next jump to: up, down, left or right five units.

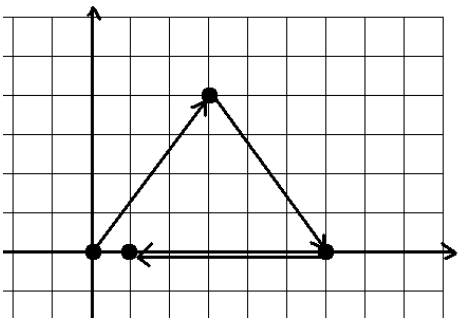


But we need to end up at position $(1,0)$, just one unit to the right from start. By always jumping vertically and horizontally five units, our horizontal (and vertical) displacement from the origin will always be in units of five. I don't think there is any way to land at $(1,0)$!

What am I missing? How else could the frog jump? Is there a diagonal step five units long?

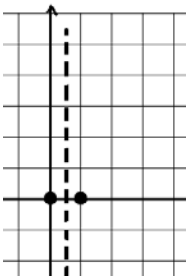



Oh… Just drawing this picture makes me think of a 3-4-5 right triangle! The frog can jump three units horizontally and four units vertically (or the other way round), to make a diagonal leap five units long. I can now see a way to get to position $(1,0)$ in just three leaps!



Can the journey be done in two leaps? Hmm.

Well, if there is a journey of two leaps, the first landing point , call it $P$, must be five units from the start $S=(0,0)$ and five units from the end $E=(1,0)$. That is, $P$ is equidistant from $(0,0)$ and $(1,0)$, and so must lie on the perpendicular bisector of $\overline{SE}$.



There are no points with integer coordinates on this line. A path with just two jumps is impossible.

So that's it: A journey of just three jumps is indeed the best that can be done!

**Challenging Extension:** Prove that it is impossible for the frog to return to start in an odd number of jumps!

*Curriculum Inspirations is brought to you by the Mathematical Association of America and MAA American Mathematics Competitions.*