Curriculum Burst 8: Stacking Coins

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> In the United States, coins have the following thicknesses:
>
> penny, 1.55 mm;
> nickel, 1.95 mm;
> dime, 1.35 mm;
> quarter, 1.75 mm.
>
> If a stack of these coins is exactly 14 mm high, how many coins are in the stack?
>
> (A) 7 (B) 8 (C) 9 (D) 10 (E) 11

SOURCE: This is question # 13 from the 2004 MAA AMC 10b Competition.

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the 10th grade level.

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MATHEMATICAL TOPICS

Decimals

COMMON CORE STATE STANDARDS

None specific to the HS standards

5.NBT *Numbers* in the K-8 standards.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

PROBLEM SOLVING STRATEGIES

ESSAY 6: ELIMINATE INCORRECT CHOICES

ESSAY 2: DO SOMETHING

THE PROBLEM-SOLVING PROCESS:

Okay … here goes:

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I have a slight feeling of panic. I've read the question three times now and I am worried that I don't know how many coins of each type there are in that stack 14 mm high.

DO SOMETHING

Okay, I'll just pretend that I know what the coins are. We have a sum something like:

$$\begin{array}{r} 1.55 \\ +1.55 \\ +1.95 \\ +1.95 \\ +1.95 \\ +1.35 \\ +1.75 \\ +1.75 \\ +1.75 \\ +1.75 \\ +1.75 \\ \hline =14.00 \end{array}$$

Since I made this up, it is most likely wrong!

Actually, is it wrong?

Let's add the 5s in the hundredths place. There are eleven of them. Even with carrying, adding them up is going to give a 5 in the hundredths place … not a zero.

Ooooh! Adding up any odd number of numbers like these will give a 5 in the hundredths place. (Would I have still noticed this if I happened to choose an even number of coins in the sum?)

We can't have an odd number of coins in the stack!

This means choices (A), (C) and (E) are out.

We've eliminated more than half the options presented to us!

There are only two choices left to consider:

(B) There are 8 coins.
(D) There are 10 coins.

Can we eliminate one of these?

AVOID HARD WORK - if you can

Since it is easier to do arithmetic with the number 10 rather than the number 8, let's explore option (D) first. (If this gets hard, we'll go back and study option (B) instead!)

Can there be 10 coins in the stack?
We have ten coins whose heights are from the list 1.35, 1.55, 1.75 and 1.95 mm. So what?

DO SOMETHING

The shortest the stack can be is

$$10\times 1.35 = 13.5 \text{ mm}$$

and the tallest is

$$10\times 1.95 = 19.5 \text{ mm.}$$

Umm. The average thickness of the coins in the stack must be $14 \div 10 = 1.40$ mm. (So the stack is mostly the 1.35 mm coins?)

I don't know where this is going. I am stuck!

Can there be 8 coins in the stack?
For the sake of DOING SOMETHING, the shortest the stack can be is … 8×1.35. Ugh! This is too hard to calculate.

Okay... The average size of the coins in the stack is $14\div 8$ mm. That's not too hard to calculate, we just have to halve the number 14 three times:

$$14 \to 7 \to 3.5 \to 1.75 \text{ mm}$$

Oh, we can have EIGHT QUARTERS!
The answer is (B).

Extension 1: Are eight quarters the only possibility?
Extension 2: We didn't actually rule out the possibility of ten coins. Look at the tenths digits. We need ten digits from the set $\{3,5,7,9\}$, plus the carry of adding the ten 5s in the hundredths place, all to add to the zero in the tenths place of 14.00. Possible?

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