Curriculum Burst 4: Area in a Polygon

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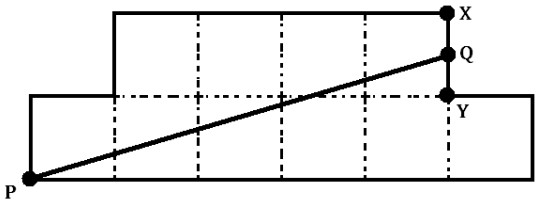
The diagram shows an octagon consisting of 10 unit squares. The portion below $\overline{PQ}$ is a unit square and a triangle with base 5. If $\overline{PQ}$ bisects the area of the octagon, what is the ratio $\frac{XQ}{QY}$?



SOURCE: This is question # 17 from the 2010 MAA AMC 8 Competition.

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the 8th grade level.

MATHEMATICAL TOPICS

Geometry and its notation. Area. Ratios.

YouTube
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COMMON CORE STATE STANDARDS

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ration relationship between two quantities.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.

PROBLEM SOLVING STRATEGY

ESSAY 2: **DO SOMETHING: LIST WHAT YOU KNOW**

THE PROBLEM-SOLVING PROCESS:

Always the first step …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question looks scary. For starters, it has the word "octagon" in it, but I don't see an octagon. And there are all these symbols $\overline{PQ}$ and $\frac{XQ}{QY}$, and comments about unit squares and triangles and areas being "bisected." I am going to have to really take my time rereading this question and taking it in.

Deep breath. Let's go line by line.

> *The diagram shows an octagon consisting of* 10 *unit squares.*

I don't see an octagon. Do I at least see 10 unit squares?

Yes. Oh ... An octagon doesn't have to be a regular octagon like a stop sign. It can be any shape with eight sides. Now I see an octagon!

That makes me feel a better.

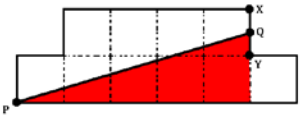
> *The portion below* $\overline{PQ}$ *is a unit square and a triangle with base* 5.

The notation "$\overline{PQ}$" scared me at first, but if I look at the picture I see a point P and a point Q, and so $\overline{PQ}$ must be the line that connects the two.

> *… is a unit square and a triangle with base* 5.

Do I see any of this?

There is certainly a triangle <u>below</u> $\overline{PQ}$ (and it does have base 5 units long).



Is there also a unit square below $\overline{PQ}$? The question says there is. Do they mean the single square sticking out to the right? I guess they do. It really can't be any other unit square. (But do I agree with it being described as "below"?)

> *If* $\overline{PQ}$ *bisects the area of the octagon, …*

What does "bisect" mean? If I recall correctly it means "chop into two equal parts," and if I think about it, it is probably the only sensible thing the line $\overline{PQ}$ could be doing to the area in this question. So the area of the octagon above the line $\overline{PQ}$ is 5 square units, and the area "below" it is 5 square units. The area of the shaded triangle must be 4 square units then.

> *… what is the ratio* $\frac{XQ}{QY}$ *?*

I see the points X, Q and Y. This ratio must be talking about the distances between X and Q, and between Q and Y. Can we figure out these distances?

What do we know? We have unit squares (so their side lengths are 1). We have a shaded triangle with base 5 and area 4. Oh! The area of the triangle is four:

$$\frac{1}{2}\times 5\times \textit{height} = 4$$

The height of the triangle is $\frac{8}{5}$. And if I look at the picture the distance between Q and Y is $8/5-1=3/5$. This is QY. And since the side-lengths of the squares are 1, this means $XQ = 2/5$. We're done!

$$\frac{XQ}{QY}=\frac{2/5}{3/5}=\frac{2}{3}.$$

Extension: Does the bisecting line $\overline{PQ}$ go through the center of the figure? Which lines through the center bisect the area of the figure? Bisect the perimeter of the figure?