# Curriculum Burst 1: Two Trees

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The top of one tree is $16$ feet higher than the top of another tree. The heights of the two trees are in the ratio $3:4$. In feet, how tall is the taller tree?

**SOURCE:** This is question # 11 from the 2010 MAA AMC 8 Competition.

## QUICK STATS:


### MAA AMC GRADE LEVEL

This question is appropriate for the 8th grade level.

### MATHEMATICAL TOPICS

Ratio and Proportion

### COMMON CORE STATE STANDARDS

**6.RP.1** Understand the concept of a ratio and use ratio language to describe a ration relationship between two quantities.
**7.RP.2** Recognize and represent proportional relationships between quantities.
**7.RP.3** Use proportional relationships to solve multistep ratio and percent problems.

### MATHEMATICAL PRACTICE STANDARDS

**MP1** Make sense of problems and persevere in solving them.
**MP2** Reason abstractly and quantitatively.
**MP3** Construct viable arguments and critique the reasoning of others.

### PROBLEM SOLVING STRATEGY

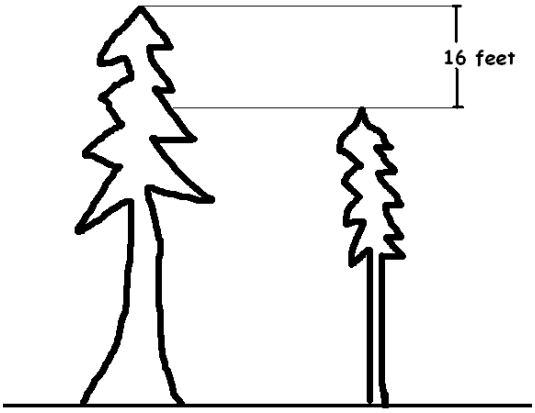
ESSAY 4: **DRAW A PICTURE**

## THE PROBLEM-SOLVING PROCESS:

As always …

**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I don't feel I can really "take in" this question unless I draw a picture. Here are two trees, one $16$ feet taller than the other.




Okay, the question now feels manageable. Except, as I read it again, the sentence:

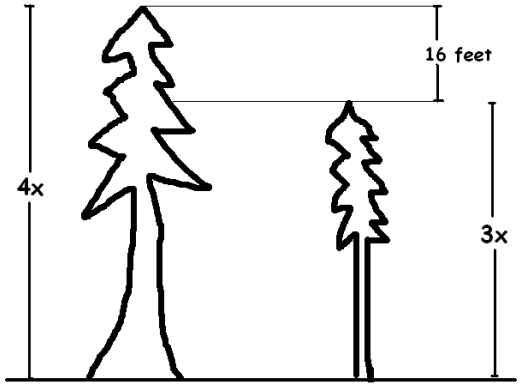
*The heights of the two trees are in the ratio* $3:4$.

is cryptic. Does it mean one tree is three feet tall and the other is four feet tall? No.

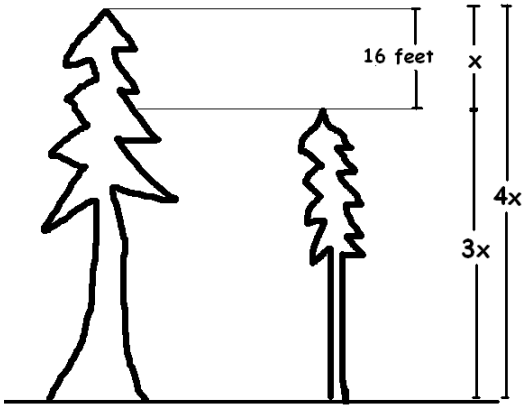
Hmm. What does the "ratio $3:4$" mean?

To get a feel for it, let's ask: What are some possible tree-heights that would be in a $3:4$ ratio? Well, $30$ feet and $40$ feet are in a $3:4$ ratio. So are $60$ feet and $80$ feet. And $120$ feet and $160$ feet.

Okay. Heights being in a $3:4$ ratio means one tree is of height $3x$ and the other of height $4x$ for some number $x$. Let's mark this on the diagram.




Oh. Look at the picture! It is now obvious that $x$ is $16$!




So the tallest tree is $4x = 4 \times 16 = 64$ feet tall!

**Extension:** Would the question be significantly harder if were told that one tree is still $16$ feet taller than the other but that their heights now come in a $31:32$ ratio? How about a $117:201$ ratio? (These might lead to very tall trees!)